Homework 1

For all these problems, present both the code and the results.

1. Maximize the following function using golden search, 3 quasi-Newton methods (steepest descent, BFGS, and DFP), as well as Nelder-Mead. Use starting values of your own choice. Report 1) solution, 2) starting values, 3) number of function evaluations until convergence, 4) number of iterations until convergence, 5) time in seconds until convergence. Use Gauss-Seidel for the golden search method in more than one dimension on the bounds specified below (ignore these bounds for all other methods).

a) $f(x) = x\cos(x^2)$ on [0,3]
b)$f(x, y) = -100(y - x^2)^2 - (1 - x)^2$ on $x \in$[-.2,1.2],$y \in$[-.2,1.2]
c)$f(x, y, z) = x^{-3} - \frac{1}{2}x^{-4} + (y^{-3} - \frac{1}{3}y^{-4}) + (y^{-3} - \frac{2}{3}y^{-4}) - \frac{1}{10}(x-y)^2 - \frac{1}{10}(y-z)^2$ on $x, y, z \in$[0.1,10]

2. Solve for the equilibrium of the following endowment economy with 3 agents and 2 goods. Agents $i = 1, 2, 3$ have preferences over goods $j = 1, 2$ given by

$$U_i(x) = \sum_{j=1}^{2} a_{ij} \frac{x_{ij}^{1+\sigma_{ij}}}{(1 + \sigma_{ij})}$$

The budget constraints are

$$\sum_{j=1}^{2} p_{ij} x_{ij} = \sum_{j=1}^{2} p_{ij} e_{ij}$$

where e_{ij} is agent $i's$ endowment of j. Solve for the equilibrium prices and allocations under the following parameter values

$$\begin{bmatrix} (i,j) & a_{ij} & \sigma_{ij} & e_{ij} \\ (1,1) & 2 & -2 & 2 \\ (1,2) & 1.5 & -0.5 & 3 \\ (2,1) & 1.5 & -1.5 & 1 \\ (2,2) & 2 & -0.5 & 2 \\ (3,1) & 1.5 & -0.5 & 4 \\ (3,2) & 2 & -1.5 & 0 \end{bmatrix}$$

Use bisection (with Gauss-Seidel), Newton's method, Broyden's method, and function iteration. As above, report the 1) solution, 2) starting values, 3) number of function evaluations until convergence, 4) number of iterations until convergence, 5) time in seconds until convergence. Use whatever starting values desired.

3. Solve for the α that satisfies: $\alpha \int_0^\infty \exp(\alpha\lambda - \lambda^2) d\lambda = 1$

4.Evaluate the integral of $\exp(-x)$ and $|x|^{\frac{1}{2}}$ on $[-1, 1]$ analytically and numerically using the trapezoid rule, Simpson's rule, and Gaussian quadrature for $n = 5, 10, 30$ nodes. Compare the errors of the 3 approaches.

5. Approximate $f(x,y) = \frac{1}{1+10x^2+10y^2}$ on $x, y \in$[-1,1] using Chebyshev polynomials of order 3, 7, 15 in each dimension, and the same number of a) equally-space nodes and b) Chebyshev nodes. Report the maximum and average approximation errors by evaluating the errors at 100×100 grid of nodes in the state-space.

6. Approximate $f(x) = p^{1-\theta} - p^{-\theta}$ for $\theta = 2$, and $\theta = 11$ using Chebyshev approximation of order 3,7,15.

7. Solve the following functional equation numerically:

$$V(p_{-1}, a) = \left[\max_p p^{1-\theta} - \frac{\theta - 1}{\theta}\frac{1}{a}p^{-\theta} - \gamma(p - p_{-1})^2 + \beta E V(p, a')\right]$$

where $\log(a) = \log(a_{-1}) + \varepsilon$ and $\varepsilon \sim U[.8, 1.2]$, $\theta = 5$ and $\gamma = 0.1$. You need to figure bounds for p_{-1} and a to use projection methods: the wider the bounds, the less accurate things are, the narrower, the higher the likelihood of extrapolation (evaluating the function outside the state-space for Chebyshev collocation which will likely produce wild results).