Appendices to "Is Firm Pricing State- or Time-Dependent? Evidence from US Manufacturing,"

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July 2007

Appendix 1: Solving the State-Dependent Model

Recall that the firm's problem is:

$$V(\hat{z},g)=\max[V^a(\hat{z},g),V^n(\hat{z},g)]$$

where V^a and V^n denote the firm's value of adjusting and not adjusting its nominal price, respectively that satisfy:

$$V^a(\hat{z}_{-1},g)=\max_{\hat{z}}\left[\hat{z}^{1-\theta}-c\hat{z}^{-\theta}-\xi+\beta\int\limits_{\varepsilon\times u\times\eta}e^{(\theta-1)(\varepsilon+u)}V\left(\hat{z}'_{-1},g'\right)dF(\varepsilon,u,\eta)\right], \quad (1)$$

$$V^n(\hat{z}_{-1},g)=\left[\hat{z}_{-1}^{1-\theta}-c\hat{z}_{-1}^{-\theta}+\beta\int\limits_{\varepsilon\times u\times\eta}e^{(\theta-1)(\varepsilon+u)}V\left(\hat{z}'_{-1},g'\right)dF(\varepsilon,u,\eta)\right],$$

I solve this problem numerically, using collocation.[1] I approximate the two value functions using linear combinations of Chebyshev polynomials:, e.g.,

$$V^a(s)\approx\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}c_{i_1i_2}\phi_{i_1}(\hat{z}_{-1})\phi_{i_2}(g),$$

where $\phi_{i_j}()$ is an i_j-th degree Chebyshev polynomial evaluated at the respective argument, N_j is the degree of the approximation along each dimension, and $c_{i_1i_2}$ the unknown coefficients. This approximation reduces the infinite-dimensional problem of solving the system of two functional equations above to a finite-dimensional non-

[1] This solution method is extensively discussed in Miranda and Fackler (2002).

linear system of $2N_1N_2$ equations in the unknown coefficients $c_{i_1i_2}$. The equations I use to solve for these unknown coefficients arise from the condition that the system of equations above holds exactly at N_1N_2 nodes along the state-space. A Newton routine is used to solve for the unknown coefficients, as well as to solve the maximization problem in the right hand-side of the first equation. I use Gaussian quadrature to form expectations (evaluate the integrals). The essence of this approach is to replace the joint-distribution of technology and monetary shocks using a discrete distribution with K mass points. The weights and nodes of the discrete distribution are chosen to ensure that the first $2K$ moments of the original distribution are equal to those of the approximant[2].

I gauge the accuracy of the approximants I use by calculating the difference between the right and left-hand side of the Bellman equations at points other than the nodes used to solve for the unknown coefficients. The maximum difference is small in absolute value (less than $.5\times10^{-4}$), suggesting the accuracy of the solution method.

Appendix 2: Standard Errors for Two-Stage Estimates

I test the implication of state-dependent pricing models in two stages with residuals estimated in the first stage used as a dependent variable in the second-stage estimation. This appendix discusses how I calculate asymptotic standard errors for second-stage estimates that take into account the two-stage nature of the estimation.

In stage 1, I rely on two-stage least squares regressions in which I retrieve residuals (technology shocks) to be used in the second stage estimation. For each SIC 2-digit industry, I estimate technology shocks as the residuals from the following Fixed Effects 2SLS regressions. Letting Δy_{it} be the growth rate of log output of industry i, c_i be industry-specific effects and $z'_{it} = [\Delta x_{it}, \Delta h_{it}]$ be the regressors (share-weighted growth rate of primary inputs), I estimate:

$$\Delta y_{it} = z'_{it}\alpha + c_i + \tilde{\varepsilon}_{it}$$

[2]See again Miranda and Fackler (2002).

Let ξ_{it} be the instruments I use for z and rewrite the above expression more compactly as

$$Y = Z\alpha + (I_N \otimes \iota_T) c + \varepsilon$$

where $Z' = [z_{11}, ...z_{1T}, ..., z_{N1}, ..., z_{NT}], Y' = [\Delta y_{11}, ..\Delta y_{1T}, ..., \Delta y_{N1}, ..., \Delta y_{NT}], c = [c_1, ..., c_N]\,, I_N$ is the identity matrix, and ι_T is a T×1 vector of ones. The Fixed-effects two stage least squares estimate of α is

$$\hat{\alpha} = \left(\tilde{Z}' P_\xi \tilde{Z}\right)^{-1} \tilde{Z}' P_\xi \tilde{Y}$$

where $\tilde{Z} = QZ, \tilde{Y} = QY$, Q is the matrix that demeans observations of sector-specific time-series means, and $P_\xi = \tilde{\xi}\left(\tilde{\xi}'\tilde{\xi}\right)^{-1}\tilde{\xi}'$,with $\tilde{\xi}$ denoting the demeaned $NT \times j$ matrix of stacked instruments. I consider asymptotic results for the case $N \longrightarrow \infty$, holding constant T. Under standard regularity conditions:

$$\sqrt{N}\left(\hat{\alpha} - \alpha\right) = \left(\frac{1}{N}\tilde{Z}' P_\xi \tilde{Z}\right)^{-1} \sum_{i=1}^{N} \frac{1}{\sqrt{N}} \tilde{Z}' P_\xi \varepsilon \xrightarrow{d} N\left(0, V_1\right)$$

To compute an estimate of V_1, I employ the Arellano (1987) estimator that is robust to arbitrary forms of heteroskedasticity and serial correlation. In particular, I estimate $\hat{V}_1$ using

$$\hat{V}_1 = \left(\tilde{Z}' P_\xi \tilde{Z}\right)^{-1} \tilde{Z}'\tilde{\xi}\left(\tilde{\xi}'\tilde{\xi}\right)^{-1}\left(\sum_{i=1}^{N} \tilde{\xi}_i' \varepsilon_i \varepsilon_i' \tilde{\xi}\right)\left(\tilde{\xi}'\tilde{\xi}\right)^{-1}\tilde{\xi}'\tilde{Z}\left(\tilde{Z}' P_\xi \tilde{Z}\right)^{-1}$$

where, say, $\varepsilon_i = [\varepsilon_{i1}, ..., \varepsilon_{iT}]'$, etc.

My measure of technology shocks is $\varepsilon_{it} = \hat{\varepsilon}_{it} + \bar{y}_i - \bar{z}_i'\hat{\alpha}$, where $\hat{\varepsilon}_{it}$ are the residuals of the regression above, and $\bar{y}_i - \bar{g}_i'\hat{\alpha}$ the estimate of the fixed effect term. In the second stage, I construct $\varepsilon_{it}^{+} = \max(\varepsilon_{it}, 0)$ and $\hat{\varepsilon}_{it}^{-} = \min(\varepsilon_{it}, 0)$ and estimate

$$\pi_{it} = c_i + \alpha_0 \varepsilon_{it}^{+} + \alpha_1 g_t \varepsilon_{it}^{+} + \beta_0 \varepsilon_{it}^{-} + \beta_1 g_t \varepsilon_{it}^{-} + \rho g_t + \varsigma_{it}.$$

where g_t are aggregate shocks and c_i are fixed-effects. Letting $\theta = [\rho' \;\; \alpha_0 \;\; \alpha_1 \;\; \beta_0 \;\; \beta_1]'$, x_{it} collect all the right-hand side covariates and a tilde denote a demeaned variable, the fixed effects estimator is the solution to:

$$\arg\min_{\theta}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{\pi}_{it}-\tilde{x}_{it}'\theta\right)^{2}=\arg\max_{\theta}\sum_{i=1}^{N}\sum_{t=1}^{T}\psi_{it}\left(\theta,\hat{\alpha}\right)$$

where $\psi_{it} = \left(\tilde{\pi}_{it}-\tilde{x}_{it}'\theta\right)^{2}$ and $\hat{\alpha}$ are the coefficient estimates from the first-stage regressions. To calculate its asymptotic distribution I note that $\hat{\theta}$ satisfies:

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{\partial}{\partial\theta}\psi_{it}\left(\hat{\theta},\hat{\alpha}\right)=0$$

A first-order Taylor series expression of this expression around θ and α yields:

$$0\approx\tfrac{1}{\sqrt{N}}\textstyle\sum_{i=1}^{N}\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\psi_{it}\left(\theta,\alpha\right)+B_N\sqrt{N}\left(\hat{\theta}-\theta\right)+J_N\sqrt{N}\left(\hat{\alpha}-\alpha\right),$$

$$\text{where, } B_N=\left(\tfrac{1}{N}\textstyle\sum_{i=1}^{N}\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\tfrac{\partial}{\partial\theta'}\psi_{it}\left(\theta,\alpha\right)\right)\overset{p}{\to}B=E\left[\textstyle\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\tfrac{\partial}{\partial\theta'}\psi_{it}\left(\theta,\alpha\right)\right]$$

$$J_N=\left(\tfrac{1}{N}\textstyle\sum_{i=1}^{N}\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\tfrac{\partial}{\partial\alpha'}\psi_{it}\left(\theta,\alpha\right)\right)\overset{p}{\to}J=E\left[\textstyle\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\tfrac{\partial}{\partial\alpha'}\psi_{it}\left(\theta,\alpha\right)\right]$$

Hence,

$$\left(\tfrac{1}{N}\textstyle\sum_{i=1}^{N}\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\tfrac{\partial}{\partial\theta'}\psi_{it}\left(\theta,\alpha\right)\right)\sqrt{N}\left(\hat{\theta}-\theta\right)=-\left[I\quad J_N\right]\begin{bmatrix}\tfrac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\tfrac{\partial}{\partial\beta}\psi_{it}\left(\theta,\alpha\right)\\ \left(\tfrac{1}{N}\tilde{Z}P_{\xi}\tilde{Z}\right)^{-1}\tfrac{1}{\sqrt{N}}\tilde{Z}P_{\xi}\varepsilon\end{bmatrix}\overset{d}{\to}N(0,V_2)$$

$$\text{where } V_2=\lim_{T\to\infty}\left[I\quad J_T\right]E\begin{bmatrix}\tfrac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\psi_{it}\left(\theta,\alpha\right)\\ \left(\tfrac{1}{N}\tilde{Z}P_{\xi}\tilde{Z}\right)^{-1}\tfrac{1}{\sqrt{N}}\tilde{Z}P_{\xi}\varepsilon\end{bmatrix}\begin{bmatrix}\tfrac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\tfrac{\partial}{\partial\theta}\psi_{it}\left(\theta,\alpha\right)\\ \left(\tfrac{1}{N}\tilde{Z}P_{\xi}\tilde{Z}\right)^{-1}\tfrac{1}{\sqrt{N}}\tilde{Z}P_{\xi}\varepsilon\end{bmatrix}\begin{bmatrix}I\\ J_T'\end{bmatrix},$$

or

$$V_2=\left[I\quad J\right]\begin{bmatrix}A_{11} & A_{12}\\ A_{12} & A_{22}\end{bmatrix}'\begin{bmatrix}I\\ J'\end{bmatrix}$$

Note however that $\frac{\partial}{\partial\theta}\psi_{it}\left(\beta,\alpha\right)$ is proportional to u_{it}, the error term in the second-stage regression, which is by assumption orthogonal to technology shocks. Therefore, $A_{12}=A_{21}=0$. A_{22} is the asymptotic variance of $\hat{\alpha}$ in the first-stage regression (V_1 above) and $V_2=B^{-1}A_{11}B^{-1}+B^{-1}JV_1J'B^{-1}$. I report $\hat{V}_2$, as opposed to $B^{-1}A_{11}B^{-1}$ in the text. I once again use an Arellano (1987)-type estimator to guard against the possibility of heteroskedasticity and serial correlation within industries, in order to estimate A_{11}.