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New directions for proof theory in linguistics

Chris Barker and Anna Szabolcsi
New York University, Linguistics
`chris.barker@nyu.edu`, `anna.szabolcsi@nyu.edu`
`homepages.nyu.edu/∼cb125`,
`homepages.nyu.edu/∼as109`

Abstract:

Proof theory has been fruitfully applied in several areas of linguistics, including theories of syntactic well-formedness, ellipsis resolution, and cross-linguistic variation. We concentrate in this course on new work by various authors using proof theory for characterizing aspects of the syntax/semantics interface, including proof-theoretic accounts of phrase subtyping for scope interactions; negative polarity and other types of licensing; the differential behavior of quantificational elements; and more. The research question addressed is: What insights does the proof theoretic approach offer in this particular empirical domain and to what extent should linguistic competence be viewed as a system of logical inference? Prerequisites should be familiarity with formal approaches to natural language semantics, including some exposure to formal logic.

Reader contents:

Altman, Alon, Ya'acov Peterzil, and Yoad Winter. 2005. Scope dominance with upward monotone quantifiers. *Journal of Logic, Language and Information* **14.4**:445-455. [preprint]

Barker, Chris. 2004. Continuations in Natural Language. In Hayo Thielecke (ed). *Proceedings of the Fourth ACM SIGPLAN Continuations Workshop (CW'04).* Technical Report CSR-04-1, School of Computer Science, University of Birmingham, Birmingham B15 2TT, United Kingdom. 1–11.

Barker, Chris. 2007. Direct compositionality on demand. In Chris Barker and Pauline Jacobson (eds). *Direct Compositionality*. Oxford University Press. 102–131. [preprint]

Barker, Chris and Pauline Jacobson. 2007. Introduction: Direct compositionality. In Chris Barker and Pauline Jacobson (eds), *Direct Compositionality*. Oxford. [excerpt]

Bernardi, Raffaella, and Anna Szabolcsi. 2007. Optionality, scope,and licensing. Manuscript, ESSLLI 2007 CD version.

de Groote, Philippe. 2001. Type raising, continuations, and classical logic. 13th Amsterdam Colloquium. [preprint]

Moortgat, Michael. 2002. Categorial grammar and formal semantics. *Encyclopedia of Cognitive Science*. [preprint]

Szabolcsi, Anna. 2007. Questions about proof theory vis-à-vis natural language semantics. Manuscript, ESSLLI 2007 CD version.

Plan:

(1) Day One: Basic concepts: proof theory (inference) versus model theory (entailment). In what ways can proof theory provide an account of natural language meaning? Finite representations, sense as algorithm, reasoning in psychology and with natural language structures. The lambda calculus as pure inference. A hybrid theory, and a hypothesis about semantically-flavored syntactic features.
(2) Day Two: Logical symmetries. Lambek grammar. Adverbial modification and non-constituent coordination. The Curry-Howard isomorphism. Continuations and natural language quantification.
(3) Day Three: Direct compositionality. Binding, quantification.
(4) Day Four: Partially ordered categories: Optionality, scope, and licensing.
(5) Day Five: A case study: Quantifier scope and negative polarity in Hungarian.

Proof theory in linguistics

- What is Proof Theory?
- What is linguistics?
- What is the connection between Proof Theory and Linguistics?

What is Proof Theory?

Proof theory is concerned with the properties of the inference system of a logic.

So, what is a logic?

A logic is a set of **formulas** along with a set of **inference rules**.

Example: The Propositional Calculus

Formulas

Given a set of atomic formulas (p, q, ...), new formulas are licensed by rules such as:
If α and β are formulas, then $(\alpha \wedge \beta)$ is a formula.

$$(p \wedge p)$$
$$(p \wedge (p \wedge p))$$
$$((p \wedge p) \wedge p)$$

etc.

(Also $\neg\alpha, (\alpha \vee \beta), (\alpha \rightarrow \beta), (\alpha \leftrightarrow \beta)$ are formulas.)

Inference

In addition, a logic has a set of *inference* rules.

$$\frac{A \wedge B}{A}$$

Example with two premises:

$$\frac{A, \qquad A \rightarrow B}{B}$$

Intuition: given certain antecedent propositions, infer/deduce/conclude/believe/derive the conclusion.

Proof ($\vdash$)

Stringing inferences together:

$$p, p \rightarrow q, q \rightarrow r \vdash r$$

From $p, p \rightarrow q$, infer q; from $q, q \rightarrow r$, infer r.

$$\dfrac{\dfrac{p \qquad p \rightarrow q}{q} \qquad\qquad q \rightarrow r}{r}$$

derivation

Proof theory versus Model theory

A **model** is a set of objects along with a function M mapping each formula into the set of objects.

E.g., for the Propositional Logic: $M : \{\mathtt{t}, \mathtt{f}\}, [\![\cdot]\!]$

$[\![A]\!]$	$[\![\neg A]\!]$	$[\![B]\!]$	$[\![A \vee B]\!]$	$[\![A \wedge B]\!]$	$[\![A \rightarrow B]\!]$
`t`	`f`	`t`	`t`	`t`	`t`
`t`	`f`	`f`	`t`	`f`	`f`
`f`	`t`	`t`	`t`	`f`	`t`
`f`	`t`	`f`	`f`	`f`	`t`

The “theory” part of “model theory”: how the denotation of complex expressions depends on the denotation of its parts.

Entailment ($\models$)

$$p, p \rightarrow q, q \rightarrow r \models r$$

A sequence Γ of formulas **entails** a formula F iff every model under which each formula of Γ is true is a model under which F is also true.

$$p, p \vee q \models q?$$

Choose a model such that $[\![p]\!] = \mathtt{t}$, $[\![q]\!] = \mathtt{f}$. Then $[\![p \vee q]\!] = \mathtt{t}$.

Since there is at least one model that verifies the antecedent formulas but not the consequent, in this case $\Gamma \not\models F$.

Connecting Proof Theory with Model Theory

Soundness: If $\Gamma \vdash F$, then $\Gamma \models F$.

Soundness is non-negotiable.

Proof:

$$\frac{A \wedge B}{A}$$

Denotation:

$[\![A]\!]$	$[\![B]\!]$	$[\![A \wedge B]\!]$
`t`	`t`	`t`
`t`	`f`	`f`
`f`	`t`	`f`
`f`	`f`	`f`

Since $A \wedge B \models B$, $A \wedge B \vdash B$ is sound.

NB: soundness is always with respect to a specific class of models!

Completeness

If $\Gamma \models F$, then $\Gamma \vdash F$.

$$\neg\neg p \models p$$

In order for the Propositional Calculus to be complete, it must be possible to prove

$$\neg\neg p \vdash p$$

Logic for Natural Language

Formulas: formation rules (syntax: S → DP VP)

Inference: [missing!]

Models: set of denotations, $[\![S]\!] = [\![VP]\!]([\![DP]\!])$

“Entailment”:

(1) a. All big dogs are smart.
    b. All dogs are smart.

Direct fit: Logical inference is natural language inference

$$p \wedge q \vdash p$$

(2) a. Snow is white and it is raining.
    b. Snow is white.

So far, so good.

$$p \rightarrow q \vdash (p \wedge r) \rightarrow q$$

(3) a. If I visit Spain I will be happy.
    b. If I visit Spain and I am sick, I will be happy.

Sideways fit: logical inference is syntactic derivation

Formulas are modeled by sets of expressions.

I.e., formulas are category labels.

Let $[\![DP]\!] = \{\textit{John}, \textit{two dogs}, ...\}$.
$[\![DP \rightarrow S]\!] = \{\textit{left}, \textit{barked}, ...\}$

$$DP, DP \rightarrow S \vdash S$$

Sound only if
$[\![S]\!] = \{\textit{John left, John barked, Two dogs left, Two dogs barked,...}\}$

Soundness vs. overgeneration;
Completeness vs. undergeneration.

Semantics?? Curry-Howard...

Is NL inference independent of model theory entailment?

(4) Geurts & van der Slik 2005:

a. If a farmer owns a donkey, he beats it.
b. If a farmer owns a big donkey, he beats it.

(5) a. John is seeking a big unicorn.
b. John is seeking a unicorn.

Moss: why not let the proof theory BE the semantics?

Larry Moss, LSA Workshop on Proof Theory in Linguistics, 20 March 2005:

> But then after providing semantics for various fragments, one rarely goes back and does the hard work of determining the complete logics for validity in those fragments... A model-theoretic semantics could, and should, lead to complete proof systems for fragments of natural language... [I]f one is seriously interested in entailment, why not study it axiomatically instead of building models? In particular, **if one has a complete proof system, why not declare it to *be* the semantics?** Indeed, why should semantics be founded on model theory rather than proof theory?

The λ-calculus: pure proof theory

Formulas:

$$x$$

$$(\lambda x y)$$

$$(\lambda y (\lambda x y)$$

$$(x y)$$

$$((\lambda x y) z)$$

etc.

Add logical constants such as $\mathbf{j}$, $\mathbf{love}$, $\mathbf{talk}$, etc.

Inference in the λ-calculus:

$$\frac{(\lambda x M)}{(\lambda y M[x := y])}\,\alpha \qquad \frac{(\lambda x((\mathbf{love}\,x)\,x))}{(\lambda y((\mathbf{love}\,y)\,y))}\,\alpha$$

$$\frac{((\lambda x M)N)}{M[x := N]}\,\beta \qquad \frac{((\lambda P(P\,\mathbf{j}))\mathbf{talk})}{(\mathbf{talk}\,\mathbf{j})}\,\beta$$

$$\frac{(\lambda x(Mx))}{M}\,\eta \qquad \frac{(\lambda x(\mathbf{talk}\,x))}{\mathbf{talk}}\,\eta$$

How inference in the λ-calculus expresses meaning

Compositional meaning is a recipe for building complex meanings out of simpler parts.

Copy: $(\lambda x(x\,x))$
Reorder: $(\lambda x(\lambda y(y\,x)))$
Delete: $(\lambda x(\lambda y x))$

An ideal word processor: pure syntactic manipulation.

Models for the λ-calculus

What do we expect from a model of the λ-calculus?

- **Soundness**: $M \vdash_\lambda N$ only if $[\![M]\!] = [\![N]\!]$.
- **Completeness**: $M \vdash_\lambda N$ if $[\![M]\!] = [\![N]\!]$.

Term algebras, Scott domains

No one feels the need to learn about Scott domains before they feel as if they truly understand the λ-calculus—the proof theory is enough!

Scott domains in linguistics

Turner 1982 et seq., Chierchia 1984: nominalization

(6) Most of his **attempts to solve the problem** failed.

(7) Being crazy is crazy. [Chierchia]

Proof theoretic alternative:

$[\![\textit{attempt to solve the problem}_{\textit{VP}}]\!] = (\lambda x(\mathbf{attempt}((\mathbf{solve}\,\mathbf{p})x)x));$

$[\![\textit{attempt to solve the problem}_{\textit{Nom}}]\!] = (\lambda n(\lambda x(\mathbf{attempt}((\mathbf{solve}\,\mathbf{p})x)x)))$