IO Class Notes: A Framework for Dynamic Analysis.

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Review: Static analysis in applied I.O.

Let's review where we have gotten to. Static analysis conditions on

- the goods marketed (or their characteristics) and their cost functions,
- preferences over the goods (or over characteristics tuples).
- "institutional" features, like the type of equilibrium, structure of ownership, e.g. regulatory rules

and then analyzes how prices, quantities, and the distribution of profits and consumer surplus, are determinned (or how they might change if we changed one or the other element of the conditioning set).

To do the static analysis we need the following primitives.

1. the demand system [can estimate it]

2. the cost system [can estimate it]
3. the equilibrium assumption [usually taken as given from knowledge of institutions, but could "test"]

Simplifying Assumptions for Static Analysis.

For the simple class of models IO usually begins with we assume that the current "q" or "p" choice does not have an independent effect on;

- future costs [l.b.d., adjustment costs, networks,...]
- future demand [durable or experience goods, networks,...]
- future equilibrium choices [collusion]

Assuming, then, that there is a unique equilibrium and that we limit strategies to depend on "payoff relevant" random variables (variables which effect either demand or costs), our three primitives will enable us to

- solve for each firms prices (quantities) as a function of
  - that firm's own state variables. These typically include the characteristics of its cost function and the characteristics of its products.
  - the state variables of the other firms active in the industry,
  - and "exogenous" state variables. These typically include factor prices (or factor supply functions if these

have some elasticity), exogenous factors which determine demand conditions, institutional detail like tariffs and taxes, ...

- Substitute these price and quantities into the profit functions and solve for the profits of each firm as a function of all state variables (those of the firm in question, its competitors, and exogenous state variables)

- Similarly we can solve for (the distribution of) consumer surplus as a function of the prices (i.e. the state variables) and the distribution of preferences.

This in turn enables us to do

- Numerical analysis of price, quantity, profitability, and consumer surplus responses to policy & or environmental change, conditional on the state vectors of the firms.

- Allows us to feed the profit function into a dynamic problem that allows us to analyze, entry, exit, investment, and (at least potentially) ownership structure decisions.

Of course once there is, say, a policy or environmental change, it will generally also produce an incentive to change the "state variables" of the system (the goods produced, their cost functions, perhaps even ownership and "preferences").

- E.g. merger activity. Typical analysis focuses on the impact of mergers on prices *conditional* on both the products marketed and their production costs. Ownership changes cause changes in investment (as well as in prices)..., and these can offset the static effects. Similar for collusion.

- Environmental change. Realistic analysis of even intermediate run responses requires some sort of dynamic analysis. Empirical e.g.: Gas prices and average mpg of new car fleet from PLB.

So we want also a model to analyze the responses of the state variables, or a "dynamic" model.

- To be realistic a dynamic model of even the simplest of markets has to be quite complex; too complex to admit analytic results with much applied content.

- This the reason that the theory papers on dynamics work in a highly "stylized" environment; i.e. an environment that is designed more for the clarity it allows in developing intuitions than for its realism.

- We give up on the goal of analytic elegance.

- Instead they provide a framework for obtaining *quantitative* responses for dynamic policy or descriptive analysis in a more realistic, user specified, environment. The goal is to enable us to analyze the equilibrium implications of the more complex institutional structures that we actually observe.

A "Framework" for Analysis.

As noted, alot can happen in dynamic analysis. What does happen depends on the primitives of the problem, and the equilibrium that is established. For the most part we will take the choice of equilibrium here as determined elsewhere, and just analyze dynamics conditional on this choice (e.g., we will decide a priori whether we allow strategies to depend on past play or just current values of "payoff relevant" random variables, ect.). Even though the choice of equilibrium will be treated as exogenous, determinned say by knowledge of an industry's institutions, we will consider ways of checking whether the choices made are consistent with the data.[1]

What the framework does is

- Take realistic
  - primitives and
  - notions of equlibrium

  (and here realism takes precedence over simplicity),
- Compute equilibrium policies,
- Use these policies to generate quantitative responses to environmental or policy changes from a prespecified initial condition.

[1]Of course there is a sense where this makes the framework incomplete. That is one might also want to relate the choice of equilibrium back to primitives (and there is a sense where some of our dynamic models do this (especially when we get to collusion). Still we do not have a useful "megamodel" that nests all alternatives, and we proceed assuming the researcher can make many choices based on knowledge of institutional detail.

The primitives that one has to know before the framework can be used include the primitives of the static analysis

- distribution of cost functions (estimated)
- distribution of preferences (estimated)
- equilibrium assumption for any "static controls"(assumed from knowledge of industry, perhaps tested).

and, in addition, at least some notion of

- sunk costs (entry, exit).
- the cost and impacts of investments.
- the discount rate.

Characteristics of Framework.

Framework is a dynamic game that allows for

- heterogeneity among firms,
- firm and industry sources of uncertainty (so that firms that are relatively "poor performers" in one period can become relative "good performers" in another)
- investment, and entry and exit.

Core Version: Public Program.

Consists of dynamic analogues of standard classroom static models

- differentiated products,
- homogeneous products - capacity constraints, and
- homogeneous products with different marginal costs.

Extensions (largely by others)

- Other profit functions.
- Multiple Controls (both exogenously evolving, and controlled processes)
- Multiproduct firms.*
- Mergers.
- Non-pecuniary externalities ("spillovers" from investments).
- "Static" control has an independent effect on the evolution of state variable
  - "Dynamic" supply (learning by doing, adjustment costs, networks)*,
  - "Dynamic" demand (experience goods, durable goods, networks),
  - Collusion*.

* We will go through examples with these extensions in this part of the notes. Examples of the other extensions are given at the end of the Doraszelski and Pakes (forthcoming) article.

Lectures.

- Describe "core version".
- Describe "backward solution" technique (publicly available program).
- A numerical example.

- Describe basic extensions and some applications (including empirical).

Depending on time constraints we will then do one of more of the following.

- Consider computational burden.
- More powerful computational algorithms;
  - Deterministic Approximations(Judd)
  - Reinforcement Learning, Artificial Intelligence, Stochastic Algorithms (Pakes and McGuire),
  - Continuous Time (Doraszelski and Judd, 2007).
- Weaker Notions of Equilibria.
- Asymmetric Information in Dynamic Games.

Core Version (Ericson and Pakes, 1995).

Note. A more formal treatment of what follows is given in Doraszelski and Pakes (forthcoming, now on both our web sites). From an assumption point of view, the big difference between the material presented here and that in their article is that I will ignore random entry and exit fees. Proofs of existence require those sources of randomness. However most of the examples that I will go over do not have it, and still compute equilibria (or at least "ϵ-equilibria"). I ignore them here largely for pedagogical reasons; they make the notation much more complex.

States.

- $i \in \mathcal{Z}^+$.
- s_i will be the number of firms with efficiency level i,
- $s = [s_i;\ i \in \mathcal{Z}^+]$ is the "industry structure" (the number of firms at each different efficiency level).

Assumption. Given investment decisions, the distribution of future values of the state of the system (of (i, s)) are independent of the choice of prices or quantities.
$\Rightarrow$ changes in price (quantity) affect only current profits.
$\Rightarrow$ $\pi(i, s)$ can be calculated "off-line" (without needing to compute the value function).

Differentiated Product Example

The publically available program gives three examples of markets, and then lets the user chose the one to analyze. Each example contains a demand system, a cost function, and an equilibrium assumption. The program then calculates the "reduced form", $\pi(i, s)$, which can be printed out and examined.

We will focus on one of these three. It is a differentiated product model with a "logit" type demand system, spot market equilibrium which is Nash in prices, and investment in the "quality" of the product marketed (the same example analyzed in Pakes and McGuire, 1994). Later we will describe the other examples on the publically available program.

Static Profit Function.

$$U_{i,j} = v_j - p_j^* + \epsilon(i,j)$$

where v_j is the "quality" of the good and p_j^* is the price (for $j = 1, \ldots, J$.

Consumer i chooses good j if and only if

$$\epsilon(i,j) - \epsilon(i,q) > [v_q - v_j] - [p_q^* - p_j^*]$$
$$\equiv [v_q - \zeta] - [p_q^* - p_0^*] - \{[v_j - \zeta] - [p_j^* - p_0^*]\}$$

where ζ is the quality of the outside alternative,

$$\equiv i_q - p_q - [i_j - p_j]$$

which implicitly defines "real prices" and "real quality" (i.e. both relative to the outside alternative).

Define $C(i_j; s, p) = \{\epsilon\text{: the consumer chooses good } j\}$ Assuming the distribution of ϵ is i.i.d. extreme value, then the market shares have the "logit form"

$$\sigma(i_j; s, p) = \frac{exp[i_j - p_j]}{[1 + \sum exp[i_q - p_q]}.$$

Profits, assuming a constant marginal cost of mc are then

$$[p_j - mc]M\sigma(i_j; s, p).$$

This implies that the Nash pricing equilibrium pricing vector satisfies

$$-[p_j - mc]\sigma_j[1 - \sigma_j] + \sigma_j = 0.$$

There is a unique solution to this system, say $p(i_j; s)$, (see Caplin and Nalebuff,1991) and profits become

$$\pi(i_j; s) = [p(i_j; s) - mc]M\sigma[i_j; s, p(s)].$$

Bellman Equation for Incumbent Behavior.

$$V(i,s) = \max\{\phi, \pi(i,s) +$$
$$\sup_{(x \geq 0)} [-cx + \beta \sum V(i', s') pr(i', s' | x, i, s)]\}.$$

- $pr(i', s'|x, i, s) = pr(i'|i, x)pr(s'|i, x, s)$. Games where my own investment only affects my own state variables are often called "capital accumulation games".
- $i_{t+1} - i_t \equiv \tau_{t+1}$
- $\tau_t \equiv \nu_t - \zeta_t$
- ν = firm's investment outcome. $\mathcal{P} = \{p_\nu(\cdot|x), x \in \mathcal{R}^+\}$, stochastically increasing in x. Further we assume that $Pr\{\nu > 0|x = 0\} = 0$. In the example we use $\nu = 1$ with probability $p(x) = ax/[1 + ax]$ and zero otherwise.
- ζ = common industry shock. Density $\mu(\zeta)$. In the example we assume $\zeta = 1$ with probability δ and zero otherwise.

Let $\hat{s}_i = s - e_i$ provide the states of the competitors of a firm at state i for a particular s, and $q[\hat{s}_i{}'|i, s, \zeta]$ provide the firm's perceived probability of its competitors future states conditional on a particular value of ζ. Then given the above

$$pr(i' = i^*, s' = s^*|x, i, s) =$$

$$\Sigma_\zeta p(\nu = i^* - i - \zeta|x) q[\hat{s}_i{}' = s^* - e(i^*)|i, s, \zeta]\mu(\zeta).$$

where $e(i)$ has a one in the i^{th} slot and zero elsewhere.

Note that $q[\cdot|i, s, \zeta]$ embodies the incumbent's beliefs about entry and exit.

Entry model.

- Must pay an amount x_e ($> \beta\phi$) to enter,
- Enters one period later at state $\imath^e \in \Omega^e \subset \mathcal{Z}^+$ with probability $p^e(\cdot)$.
- Only enters if the expected discounted value of future net cash flows from entering is greater than the cost of entry.
- The cost of entry can be specified as either x_e, or as a random variable which distributes uniformly on $[x_{e,l}, x_{e,u}]$. When random entry costs are used only the potential entrant knows the realization of the entry costs, the other incumbents know only that entry costs will be a random draw from this uniform distribution.

Dynamic Equilibrium:Characterization Results.

Explain "equilibrium".

- Every agent chooses optimal policies given its perceptions on likely future industry structures
- Those pereptions are consistent with the behavior of the agent's competitors.

This framework is due to E-P(1995). Doraszelski and Satterwaite (2003) prove that a Markov Perfect equilibrium exists for this model (at least if we have random entry fees and exit costs), and E-P show that any such equilibria has the following characteristics

1. It is "computable", i.e..

   - Never more than $\overline{n}$ firms active.
   - Only observe "i" on $\Omega = \{1, ..., K\}$ (given the competitors states, there is a lower ω at which the given firm exits, and an upper ω at which the given firm stops investing – since there is a finite number of other firms possible you can then take the min of the lower state and the max of the upper state).

   $\Rightarrow$ need only compute equilibria for $(i, s) \in \Omega \times S$, where

   $$S \equiv \{s = [s_1, ..., s_k] : \sum s_j \leq \overline{n} < \infty\}$$

   so that the number of elements in S or $\#S \leq K^N$.

2. It is Markov. Indeed equilibrium policies generate a homogeneous Markov chain for industry structures [for $\{s_t\}$], i.e.

   $$Pr[s_{t+1} = s' | s^t] = Pr[s_{t+1} = s' | s_t] \equiv Q[s' | s_t].$$

   with the Markov transition "kernel" $Q(\cdot, \cdot)$ on $S \times S$.

3. They provide conditions on the primitives such that insure that any equilibrium $Q[\cdot|\cdot]$ is *ergodic.* [Picture].

   - Note that the nature of states in R, how states cycle in R, and the transitions to R depend on primitives.
   - R is frequently much smaller than S (and the divergence is greatest for large markets with many state variables).

Computation of Policies:Pakes and McGuire,1994.

Assume temporarily that Ω and $\overline{n}$ are known. The algorithm we provide here is a "backward solution" algorithm that computes the value and policy functions pointwise. It is the multiple agent analogue of what we did to compute single agent dynamic problems.

- In memory. Estimates of the value function and policies associated with each $(i, s) \in \Omega \times S$.
- Updating. *Synchronous*; i.e. it circles through the points in S in some fixed order and updates all estimates associated with *every* $s \in S$ at each iteration (here updating estimates at s involves updating estimates at each (i, s) that has $s_i > 0$).
- Convergence. The values and policies from successive iterations are the same. Converged policies and values satisfy all the properties of equilibrium values and policies (see below).

Updating 1: Rewrite Bellman Equation.

$$V(i, s) = \max_{\chi \in \{0,1\}} \{[1 - \chi]\phi + \tag{1}$$

$$\chi\{\pi(i, s) - \sup_{x \geq 0}[-cx + \beta \sum_{\nu} w(\nu; i, s)p(\nu|x_1)]\},$$

where

$$w(\nu;\ i, s) \equiv$$

$$\sum_{(\hat{s}_i',\zeta)} V(i+\nu-\zeta, \hat{s}_i'+e(i+\nu-\zeta)|w)q[\hat{s}_i'|i,s,\zeta]\mu(\zeta), \quad (1a)$$

and

$$q[\hat{s}_i' = s_i^*|i,s,\zeta] \equiv$$
$$Pr\{\hat{s}_i' = \hat{s}_i^*|i,s,\zeta, \text{equilibrium policies }\} \quad (1b).$$

Here $w(\nu;i,s)$ is the expected discounted value of future net cash flow conditional on the current year's investment resulting in a particular value of ν, and the current state being (i,s) (it integrates out over the possible outcomes of both the investment strategies of competitors (the $\hat{s}_i'$), and over the outside alternative (the ζ)).

Note: Just as in the single agent problem $w(\nu;i,s)$ is all the firm needs to know in order to make decisions. It is thus a sufficient statistic for decision making purposes (note that it is sufficient for a very complicated object, the expected discounted value of future net cash flows given a realization for the investment process).

Updating Rules.

- calculates $w^{k-1}(\cdot|i,s)$ from the information in memory, i.e. from (x^{k-1}, V^{k-1}) (as in 1a),
- substitutes $w^{k-1}(\cdot)$ for $w(\cdot)$ in (1) and then solve the resultant *single agent* optimization problem for the j^{th} iter-

ation's entry, exit and investment polices at (i, s). That is

- Incumbents solve for (χ^k, x^k) that

$$\max_{\chi \in \{0,1\}} \{[1-\chi]\phi +$$

$$\chi \sup_{x \geq 0} [\pi(i, s) - cx + \beta \sum_{\nu} w^{k-1}(\nu; i, s) p(\nu|x)]$$

  I.e. we solve the Kuhn-Tucker probem for investment conditional on continuing which in the example works out to be

$$\frac{\partial p(x)}{\partial x}[w(1; i, s) - w(0; i, s)] - c \leq 0,$$

  with stict inequality if and only if $x = 0$. We then substitute the solution for x in the continuation value above and determine whether it is greater than ϕ.

- Potential entrants compute

$$V_e^k(s) = \beta \sum_{\zeta} w^{k-1}(\zeta; i_e, s + e(i_e))\mu(\zeta).$$

  and set $\chi_e^k = 1 \Leftrightarrow V_e^k(s) > x_e$,

- substitutes these policies and the w^{k-1} for the w, x and the max operator in (1), and labels the result $V^k(\cdot)$,
- calculates $V^k(\cdot) - V^{k-1}(\cdot)$ and then substitutes $V^k(\cdot)$ and the policies from iteration k, for the iteration $k-1$ values that were in memory.

Convergence.

At the end of the iteration calculate $||V^{k-1}(\cdot) - V^k(\cdot)||$ and $||x^{k-1}(\cdot) - x^k(\cdot)||$. If both are sufficiently small, stop. Else continue. The program prints out both the L^2 norm and the sup norm.

At fixed point each incumbent and potential entrant

- uses, as its perceived distribution of the future states of its competitors, the actual distribution of future states of those competitors, and
- chooses its policy to maximize its expected discounted value of future net cash flow given this distribution of the future of its competitors.

Star and Ho 1969, provide a proof that this is all that is needed for a MPE.

Setting K and $\overline{n}$

K.
Start with the monopoly problem ($\overline{n} = 1$) and an oversized K; $\rightarrow$ a lowest i at which the monopolist remains active and a highest i at which the monopolist invests. $\rightarrow$ 1 and K in Ω.

$\overline{n}$.
Set $\overline{n} = 2$ and do the iterative calculations again starting at $V^0(i_1, i_2) = V^*(i_1)$. Then set $\overline{n} = 3$ and set $V^0(i_1, i_2, i_3) = V^*(i_1, max(i_2, i_3))$. Continue until we reach an $\overline{n}$ so high that whenever there are $\overline{n} - 1$ firm's active there is no possible structure at which an entrant would want to enter. This is $\overline{n}$.

Notes on the (Updated) Publicly Available Algorithm.

- Choose to compute:
  - Differentiated product model,
  - homogeneous products with capacity constraint,
  - homogeneous products, differences in marginal cost.
- Choose to compute:
  - MPN equilibrium,
  - Social Planner Problem (ignoring information and incentive problems). This is a single agent dynamic programming problem. It sets price=mc, and choose all entry, exit, and investment policies to maximize the discounted value of consumer surplus minus the costs of entry and exit plus the selloff value. Designed to give you some idea if there is "room" for policy intervention to improve social surplus (of course if there is room there is still a question of whether we can find a mechansim that can lead us to an improvement without counterveiling costs).
  - Perfect collusion (ignoring the problem of supporting collusion). Also a single agent problem. Multi-product monopoly with no one else able to enter. Single agent

chooses all prices, entry, exit and investment to maximize the expected discounted value of net cash flow. Designed to give you some idea of how profitable collusion could possibly be (of course the fact that collusion would be profitable were we not required to support it, does not mean that collusion would be profitable).

- Set parameter values.
  - maximum number of firms active ($\overline{n}$).
  - Highest efficiency level available (K).
  - Static profit function parameters (differ somewhat with problem)
    For differentiated products; marginal cost (mc), market size (M), inflection point in utility.
    For homogenous products, investment in marginal cost; demand intercept,fixed cost, minimum marginal cost.
    For homogenous products, investment in capacity; demand intercept, marginal cost, maximum capacity.
  - Dynamic parameters; discount rate, scrap value, entry sunk costs (deterministic or stochastic), sunk costs, efficiency level at which entrant enters, "a" parameter setting $p(x)$, δ parameter setting how the outside alternative moves,
- Compute $\pi(\cdot)$. This has to be done before computing the dynamic equilibrium. You can now go to the descriptive statistics part of the problem and just look at $\pi(\cdot)$.
- Compute entry, exit, and investment. Can now look at part of descriptive statistics which has to do with value

function (which gives you value function, investment, $p(x)$ entry, exit)

- Simulate industry structures and provide descriptive statistics from user specified initial conditions. Currently working part gives you
  - number of periods with; n firms active, entry, exit, entry and exit
  - mean and standard deviation of; investment, price-cost margin, one-firm concentration ratio, lifespan
- Simulate industry structures and compute mean and standard deviations of consumer, producer, and total surplus for runs of user specified length from user specified initial conditions. specified initial conditions.

Numerical Example.

Look to tables.

Descriptive Results.

Parameter values: See table 1.

Table 1

Characteristics of Ergodic Distribution[1]

$\delta = .7 \quad \alpha = 3 \quad \beta = .925 \quad xe = .2 \quad \phi = .1$
$m = 5 \quad c = 5$

No. of Time Periods	10,000				
	MP	$\sigma* = .65$	$x^e = 2$	PP	Coll.
% with 6 firms active	.1	.2	.0	0	0
% with 5 firms active	1.6	3.1	.3	.1	.1
% with 4 firms active	35.3	33.3	1.2	5.8	1.1
% with 3 firms active	63.0	63.3	17.1	44.5	23.1
% with 2 firms active	.0	0.0	81.4	49.6	75.6
% with entry and exit	13.1	11.5	.3	10.1	10.7
% with entry only	4.8	4.8	.7	3.0	1.9
% with exit only	2.0	2.5	.6	2.3	1.7
% with entry or exit	20.1	18.7	1.6	15.4	14.2
Gross job creation	.086	.086	.031	.033	.027
Gross job destruction	.087	.088	.032	.033	.027

1. Legend

MP = Markov Perfect Nash Equilibruim
δ^* = .65, MP with market share constrained to be below .65.
x^e = 2, MP with sunk entry costs increased to 2.
PP = planner's problem (see the text below).
Coll. = perfect cartel (see the text below).

Note: The total current cost of producing the typical output of this industry is about 25. So the cost of production is about 1/125 of the cost of production in a given period. It is not very costly to enter in our base case.

Table 1. Generate 10,000 periods of industry evolution, and describe results.
Table Cases:

- MP is base case.
- $\sigma* = .65$. This is an institutionally created upper bound to the max market share of the largest firm.
- $x^e = 2$. Increase sunk entry costs by a factor of 10.
- PP is planners market.
- Coll. Is colluders market.

Mostly 3 or 4 firms active. Not same firms and entry and exit positively correlated.

Table 2:

- Lifetime Distribution very skewed.
- 1,800 firms in 10,000 periods
- Model lifespan is 1, mean is 18.4, and standard deviation is 77.5
- Analytic reason is approximate "S" shape of sections of the value function.

Table 2

Lifetime Distribution

(Based on 10,000 time periods)

$\delta = .7 \quad \alpha = 3 \quad \beta = .925 \quad \text{xe} = .2 \quad \phi = .1$
$\text{m} = 5 \quad \text{c} = 5$

Life time	Percent			Implied Hazard			Cumulative Percent		
	MP	$x^e = 2$	PP	MP	$x^e = 2$	PP	MP	$x^e = 2$	PP
1	48.3	17.5	25	48.3	17.5	12.2	48.3	17.5	25.0
2	24.7	14.6	44.0	47.7	19.6	24.0	73.0	35.0	68.9
3	6.1	4.7	9.0	22.6	7.3	17.9	79.1	40.7	77.9
4	3.6	8.5	4.4	17.2	14.3	17.9	82.7	49.1	82.3
5	2.5	3.8	1.9	14.4	7.5	10.9	85.2	52.1	84.2
6	1.3	2.8	2.2	8.78	5.9	8.9	86.5	55.7	86.5
7	.8	1.9	1.4	5.9	4.5	4.1	87.3	57.2	87.9
8	.7	.9	.8	5.5	2	4.8	88.1	58.5	88.6
9	.7	.9	.5	5.9	2.9	5.7	88.8	59.5	89.2
10	.3	1.9	.7	2.7	4.6	5.7	89.1	61.3	89.9

	Number Active	Mean	Median	Standard Deviation
MP	1,800	18.4	2	73.2
S	103	191.5	6	428.0
PP	1,301	19.5	2	93.9

1. Legend

MP = Markov Perfect Nash Equilibruim
δ^* = .65, MP with market share constrained to be below .65.
x^e = 2, MP with sunk entry costs increased to 2.
PP = planner's problem (see the text below).
Coll. = perfect cartel (see the text below).

Table 3:

- Value Distribution very skewed.
- Only 10% make positive returns, but those who make it over the initial high mortality period do very well.
- Note the values and rates of returns of firms operating at one time will tend to be “supernormal”, but this is no indication of something being wrong.

Table 3

Realized Value Distribution

$\delta = .7 \quad \alpha = 3 \quad \beta = .925 \quad \text{xe} = .2 \quad \phi = .1$
$\text{m} = 5 \quad \text{c} = 5$

Obs/Num	Realized Values			Life Time	Sum of Realized	
	MP	$\sigma^* = .65$	$x^e = 2$	MP	MP	$\sigma^* = .6$
1	72.8	45.1	83.0	79	72.8	45.0
2	52.6	38.9	34.0	247	125.4	84.0
3	33.1	36.3	32.4	718	158.57	120.3
4	32.7	35.7	28.4	118	191.3	156.0
5	29.3	30.2	20.2	102	220.6	186.3
10	22.8	21.2	15.74	5	343.8	301.3
100	7.11	6.8	-4.43	215	1462.6	1313.9
150	1.81	1.1	—	37	1700.8	1508.5
170	.1	-.06	—	3	1717.6	1514.0
171	-.05	-.07	—	2	1717.5	1514.0
1491	-.10	-.7	—	4	1586.5	1336.1
1800	-4.08	—	—	15	1282.05	—
	Mean	Median	Std. Dev.	# Positive	Mean of Positive	# Negati
MP	.71	-.1	4.09	170	10.0	1630
$\sigma^* = .65$	.68	-.1	3.74	150	9.5	1465
$x^e = 2$	2.37	-1.89	11.28	329	14.2	74

1. Legend

MP	=	Markov Perfect Nash Equilibruim
δ^*	=	.65, MP with market share constrained to be below .65.
x^e	=	2, MP with sunk entry costs increased to 2.
PP	=	planner's problem (see the text below).
Coll.	=	perfect cartel (see the text below).

Table 4 and figures.

- Markups 27 to 44%.
- Most often a reasonably fractured industry (concentration ratio averages .39 when usually 3 or 4 firms active
- but it goes through cycles of concentration that have simply to do with the logic of the technology.

Table 4A[1]

One-Firm Concentration Ratios

	MP	$\sigma^* = .65$	$x^e = 2$	PP	Coll.
.95 quantile	.62	.61	.63	1.0	1.0
.90 quantile	.5	.5	.63	.84	1.0
.75 quantile	.43	.43	.51	.52	.72
.50 quantile	.34	.34	.5	.5	.5
.25 quantile	.33	.33	.5	.46	.5
.10 quantile	.26	.3	.34	.33	.33
.05 quantile	.25	.25	.53	.33	.33
mean	.39	.38	.51	.54	.60
standard deviation	.11	.097	.09	.20	.20

1. Legend

MP = Markov Perfect Nash Equilibruim

δ^* = .65, MP with market share constrained to be below .65.

x^e = 2, MP with sunk entry costs increased to 2.

PP = planner's problem (see the text below).

Coll. = perfect cartel (see the text below).

Table $4B^2$

Price / Cost Ratios

	MP	$\sigma^* = .65$	$x^e = 2$	Coll.
MAX	2.11	2.21	2.18	2.45
.95	1.44	1.48	1.46	2.34
.90	1.40	1.39	1.46	2.33
.75	1.33	1.33	1.40	2.33
.50	1.30	1.30	1.40	2.33
.25	1.30	1.30	1.40	2.24
.10	1.27	1.27	1.30	2.22
.05	1.27	1.27	1.30	2.20
MIN	1.24	1.23	1.27	1.29
Mean	1.33	1.32	1.41	2.30
Standard Deviation	.085	.073	.099	(.089)

1. Legend

MP = Markov Perfect Nash Equilibruim

δ^* = .65, MP with market share constrained to be below .65.

x^e = 2, MP with sunk entry costs increased to 2.

Coll. = perfect cartel (see the text below).

Policy Analysis.

Consumer surplus given by

$$\int max(\omega_j - p_j + \epsilon_j)dG(\epsilon_1, \dots \epsilon_n) \equiv log \sum_j exp[\omega_j - p_j)$$

Note that there are a distribution of welfare results associated with any given set of institutions; it depends on the realizations of the random terms.

We run 100 runs of 100 periods and give mean and variance.

Table 5.

- Initial condition always $s_0 = e(\omega_0)$.
- temporary monopolies correspond to patents. Difference between monopoly and MPN is seen by comparring consumer and producer benefits. Monopoly also decreases welfare by 20%.
- multiproduct monopolist or colluder. Now 10%, and both producer and consumer surplus increase.
- note that the standard error of welfare gains is $\approx$ 20%. Problem for case studies.
- Compare MPN to the planner's problem. Not much difference in welfare but huge difference in market structure. Planner is much more similar to colluder in observables (in entry exit rates, number of firms....). The colluder however charges much higher prices. Its P/C ratio is 2.3 whereas the MPN average is 1.3 and the planner's is one.
- Look at number of firms and investment. MPN has most firms and investment (Mankiw Whinston story). Then comes the planner (it internalizes the consumer surplus generated). Then the colluder.
- $\sigma* = .65$. Here when an unconstrained firm would have gotten a market share greater thean .65, that firm increases its price until the market share goes back to this number. Market structures look much the same as unconstrained MPE but welfare falls 5%.
- Increae sunk costs. Market structures change dramatically, look like colluder, but welfare does not change much (and if we added back in the sunk costs it would be even closer). Actually it is likely that there is a sunk cost inbetween which maximizes welfare from MPE (since when the entry costs are low there is too much entry and investment). Still there is not too much fine tuning here.
- The relationship between market structure and welfare is quite complex.

Table 5

Social Welfare From Alternative Market Structures*

Benefits/ Market Structure	Total Firm Cash Flows		Consumer Benefits		Total Benefits**	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Monopoly	207	66	96	17	303	83
20 Year Monopoly then free entry	180	57	140	21	320	74
10 Year Monopoly then free entry	146	54	186	31	331	71
Markov-Perfect Nash	70	26	301	65	369	68
Perfect Collusion	218	55	115	19	332	74
Social Planner	—	—	—	—	377	—
$\sigma^* = .65$	61.7	15.1	289.7	64.4	349.6	67.4
$\sigma^* = .55$	54.4	12.0	284.8	66.1	337.5	73.1
Sunk Costs = 2	76.5	26.3	293.4	55	361.5	69.8

Table 6

Average Investment and Number of Active Firms Under Alternative Institutional Arrangements

	Investment	Active Firms
Markov Perfect Nash	2.57	3.4
$\sigma^* = .65$	2.56	3.4
$x^e = 2$	1.97	2.2
Planner	1.95	2.6
Perfect Collusion	1.75	2.3