Lecture Notes on Dynamic Demand Estimation I

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Introduction

- Extensions to the standard BLP demand estimation framework
  - Freqent sales of storable goods
  - Fast changing product characteristics and prices of new durable goods
  - Learning about the unobserved charateristics of products
- Need to model consumer's dynamic optimization problem
- Introduce additional assumptions to identify model's structural parameters

Three papers we will focus on:

- Hendel and Nevo (2006): Sales and consumer inventory behavior
- Gowrisankaran and Rysman (2007): Demand for new durable goods
- Grawford and Shum (2005): Learning from drug prescription experience

Similarity and differences in their modeling and estimation strategies?

Hendel and Nevo (2006)

- Temporary price cut may generate a large demand increase. But..if it is because of intertemporal substituion, the own-price elasticity would be smaller than static estimates suggest.
- The distinction between long-run and short-run price effects: need to control for past prices and inventories.
- Features of the scanner data of detergent
  - Nine super-markets in a large midwest city
  - Store-level: for each brand-size in each store in each week–the price charged, quantity sold, promotions
  - Household-level: when a household visited a supermarket, which brand-size was bought, where it was bought, how much paid
  - Abstract from store choice: most of consumers purchase at only one store

Model Setup

- 13 brands j: Tide, All, etc. 5 sizes x: 32-256oz. Promotion a_{jxt}: feature/display.
- consumer h's utility at time t: $u(c_{ht} + \nu_{ht}; \theta_h)$, where $c_{ht} = \sum_j c_{jht}$. Note a useful trick that will be exploited later: brand doesn't matter for consumption!
- let $d_{hjxt} = 1$ be a purchase of consumer h for brand j and size x at t. $x = 0$ means no purchase.
- consumer's dynamic discrete choice problem:

$$V(s_t) = max_{c_h(s_t), d_{hjx}(s_t)} \sum_t \delta^{t-1} E\{u(c_{ht}, \nu_{ht}; \theta_h) - c_h(i_{ht+1}; \theta_h) + \sum_j d_{hjxt}(\alpha_h p_{jxt} + \xi_{hjx} + \beta_h a_{jxt} + \epsilon_{hjxt}) | s_t\}$$

s.t. $i_{ht+1} = i_{ht} + x_{ht} - c_{ht}$

Assumptions

- ν_t i.i.d. overtime and across consumers
- prices follow an exogenous markov process (NO modeling of the supply side)
- ϵ_{jxt} i.i.d extreme value type one overtime and across consumers
- Think about ξ_{hjx}–what is assumed?
- Dimensionality of s_t: individual specific inventory i and shocks ν,ϵ. In addition observables p_t (which includes..?)

Estimation

- NO observable on inventory
- Conditional likelihood for each household:

$$Pr(d_1, ...d_T | p_1, ..., p_T) = \int \prod_t Pr(d_t | p_t, i_t(d_{t-1}...d_1, \nu_{t-1}, ..., \nu_1, i_1), \nu_t) dF(\nu_1, ..., \nu_T) dF(i_1)$$

- The prob of choosing brand j, size x: $Pr(d_{jt} = 1, x_t | p_t, i_t, \nu_t) = Pr(d_{jt} = 1 | p_t, x_t, i_t, \nu_t) Pr(x_t | p_t, i_t, \nu_t)$
- Need to solve the DP in the usual case to get $Pr(d_{jt} = 1 | p_t, x_t, i_t, \nu_t)$.

Three-step estimation procedures

Step One: estimate $Pr(d_{jt} = 1|x_t, p_t, i_t, v_t)$ without solving DP

- Let $M(s_t, j, x) = max_c\{u(c + \nu_t) - c(i_{t+1}) + \delta E(V(s_{t+1})|d_{jx}, c, s_t)\}$. Conditional on x_t, c_t is not brand-specific. So $M(s_t, j, x) = M(s_t, x)$.
- ϵ_{jxt} is i.i.d distributed (even across the SAME brand j)
- It can then be proved that:

$$Pr(d_{jt} = 1|x_t, p_t, i_t, \nu_t) = \frac{exp(\alpha p_{jxt} + \xi_{jx} + \beta a_{jxt})}{\sum_k exp(\alpha p_{kxt} + \xi_{kx} + \beta a_{kxt})}$$
$$= Pr(d_{jt} = 1|x_t, p_t)$$

- So the brand choice given size and other observables is a static problem.

Step Two: further simplify the state space

- dynamic decisons of x depends on $\omega_{xt} = log\{\sum_k exp(\alpha p_{kxt} + \xi_{kx} + \beta a_{kxt})\}$, which is a quality-adjusted price index for all brands of size x
- Another key assumption: $F(\omega_t|s_{t-1})$ can be summarized by $F(\omega_t|\omega_{t-1})$.

Step Three: the simplified DP problem

- Note that these inclusive values can be constructed once we finish with step one.
- The authors further proved that an equivalent DP is:

$$V(i_t, \omega_t, \epsilon_t, \nu_t) = Max_{c,x}\{u(c + \nu_t) - C(i_{t+1}) + \omega_{xt} + \epsilon_{xt} \\ + \delta E[V(i_{t+1}, \omega_{t+1}, \epsilon_{t+1}, \nu_{t+1})|i_t, \omega_t, c, x]\}$$

- The i.i.d assumption on ϵ and FOMP of ω are the key to the proof.

Highlights of Parameter Estimates

Static estimates:

- large effect from feature and display
- demograhics interact with price sensitivity (larger family, nowhite, surburban shoppers more price sensitive)
- brand preference vary by size by interacting brand-size dummy

Evolution of inclusive values:

- estimate a process for all 4 sizes (32-128oz) jointly
- they finally DIDN'T allow the process to differ by household type

Dynamic parameters:

- allow for six type of housholds (size,urban/suburban)
- the storage cost (surburban$<$urban)
- 30oz to 128oz increases storage cost by 0.2-0.75 dollars relative to 64oz, depending on household type.

TABLE IV

FIRST STEP: BRAND CHOICE CONDITIONAL ON SIZE[a]

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Price	−0.51	−1.06	−0.49	−0.26	−0.27	−0.38	−0.38	−0.57	−1.41	−0.75
	(0.022)	(0.038)	(0.043)	(0.050)	(0.052)	(0.055)	(0.056)	(0.085)	(0.092)	(0.098)
*Suburban dummy				−0.33	−0.30	−0.34	−0.33	−0.25	−0.45	−0.19
				(0.055)	(0.061)	(0.055)	(0.056)	(0.113)	(0.127)	(0.127)
*Nonwhite dummy				−0.34	−0.39	−0.38	−0.33	−0.34	−0.33	−0.26
				(0.075)	(0.083)	(0.076)	(0.076)	(0.152)	(0.166)	(0.168)
Large family				−0.23	−0.13	−0.21	−0.22	−0.46	−0.38	−0.43
				(0.080)	(0.107)	(0.080)	(0.082)	(0.181)	(0.192)	(0.195)
Feature			1.06	1.05	1.08	0.92	0.93	1.08		1.05
			(0.095)	(0.096)	(0.097)	(0.099)	(0.100)	(0.123)		(0.126)
Display			1.19	1.17	1.20	1.14	1.15	1.55		1.52
			(0.069)	(0.070)	(0.071)	(0.071)	(0.072)	(0.093)		(0.093)
Brand dummy variable		✓	✓	✓	✓					
*Demographics					✓					
*Size						✓				
Brand–size dummy variable							✓			
Brand-HH dummy variable								✓		
*Size									✓	✓

[a]Estimates of a conditional logit model. An observation is a purchase instance by a household. Options include only products of the same size as the product actually purchased. Asymptotic standard errors are shown in parentheses.

TABLE VIII

AVERAGE RATIOS OF ELASTICITIES COMPUTED FROM A STATIC MODEL TO LONG-RUN ELASTICITIES COMPUTED FROM THE DYNAMIC MODEL[a]

		64 oz.						128 oz.					
Brand	Size (oz.)	All[b]	Wisk	Surf	Cheer	Tide	Private Label	All[b]	Wisk	Surf	Cheer	Tide	Private Label
All[b]	64	1.03	0.13	0.14	0.12	0.13	0.15	0.14	0.17	0.17	0.18	0.21	0.34
	128	0.17	0.24	0.26	0.20	0.28	0.35	1.23	0.09	0.11	0.09	0.15	0.22
Wisk	64	0.14	1.20	0.13	0.17	0.12	0.13	0.16	0.22	0.14	0.22	0.25	0.20
	128	0.25	0.27	0.23	0.31	0.26	0.28	0.08	1.42	0.08	0.13	0.18	0.11
Surf	64	0.14	0.13	0.93	0.16	0.13	0.14	0.18	0.18	0.12	0.18	0.22	0.28
	128	0.25	0.22	0.18	0.27	0.25	0.18	0.12	0.11	1.20	0.08	0.15	0.14
Cheer	64	0.12	0.17	0.16	0.84	0.09	0.13	0.14	0.24	0.16	0.14	0.22	0.24
	128	0.25	0.26	0.26	0.12	0.23	0.22	0.09	0.12	0.06	0.89	0.15	0.07
Tide	64	0.16	0.17	0.13	0.13	1.26	0.15	0.22	0.28	0.16	0.26	0.22	0.37
	128	0.25	0.31	0.22	0.24	0.22	0.31	0.11	0.16	0.08	0.13	1.44	0.31
Solo	64	0.15	0.12	0.15	0.14	0.12	0.14	0.17	0.15	0.15	0.30	0.30	0.28
	128	0.23	0.20	0.24	0.21	0.21	0.25	0.07	0.07	0.06	0.16	0.17	0.21
Era	64	0.21	0.12	0.13	0.13	0.10	0.19	0.43	0.17	0.15	0.22	0.19	0.35
	128	0.31	0.22	0.24	0.25	0.17	0.38	0.19	0.08	0.09	0.11	0.10	0.22
Private	64	0.19	0.15	0.14	0.17	0.17	1.02	0.32	0.22	0.15	0.26	0.31	0.25
label	128	0.29	0.28	0.34	0.30	0.39	0.29	0.16	0.12	0.13	0.10	0.27	1.29
No purchase		2.12	1.13	1.15	1.40	1.27	2.39	1.80	7.60	2.26	14.11	2.38	10.86

[a]Cell entries i and j, where i indexes row and j indexes column, give the ratio of the (short-run) elasticities computed from a static model divided by the long-run elasticities computed from the dynamic model. The elasticities for both models are the percent change in market share of brand i with a 1 percent change in the price of j. The static model is identical to the model estimated in the first step, except that brands of all sizes are included as well as a no-purchase decision, not just products of the same size as the chosen option. The results from the dynamic model are based on the results presented in Tables IV–VI.

[b]Note that "All" is the name of a detergent produced by Unilever.

Gowrisankaran Rysman (2007)

- New durable goods: rapidly falling prices and improving features
- Consumer chooses what to buy AND when to buy
- Biased welfare measure if ignore consumer's dynamic behavior
- A panel of aggregate data for digital camcorders
  - monthly data for 378 models j: units sold, average price (p_{jt}), observabel characteristics (x_{jt})
  - improving features overtime: size,pixel count,etc.
  - nature of competition: number of camcorders available growing steadily overtime

Model

- consumer i at time t chooses among J_t products or nothing. No utility from old product once upgraded
- flow utility of product j follows BLP: $\delta^f_{ijt} = \alpha^x_i x_{jt} + \xi_{jt}$ and net flow utility is $u_{ijt} = \delta^f_{ijt} - \alpha^p_i ln(p_{jt}) + \epsilon_{ijt}$
- no purchase, then $u_{i0t} = \delta^f_{i0t} + \epsilon_{i0t}$ is the utility from product currently owned.
- let Ω_t denote current product attributes, i.e. $(x_{jt}, p_{jt}), j = 1, 2, ...J_t$ and $\epsilon_{it} = (\epsilon_{i0t}, \epsilon_{i1t}, ..., \epsilon_{iJ_t t})$.
- Assumptions: Ω_t FOMP according to $P(\Omega_{t+1}|\Omega_t)$, ϵ_{it} i.i.d. type I extreme value.

- Let $EV_i(\delta^f_{ijt}, \Omega_t) = \int V_i(\epsilon_{it}, \delta^f_{ijt}, \Omega_t) dP_\epsilon$. Then consumer's problem:

$$V_i(\epsilon_{it}, \delta^f_{i0t}, \Omega_t) = max\{u_{i0t} + \beta E[EV_i(\delta^f_{i0t}, \Omega_{t+1})|\Omega_t], \\ max_{j=1,\ldots,J_t}\{u_{ijt} + \beta E[EV_i(\delta^f_{ijt}, \Omega_{t+1})|\Omega_t]\}\}$$

- Face the same difficulty as Hendel and Nevo (2006). Use a similar trick: let $\delta_{ijt}(\Omega_t) = \delta^f_{ijt} - \alpha^p_i ln(p_{jt}) + \beta E[EV_i(\delta^f_{ijt}, \Omega_{t+1})|\Omega_t]$.
- Define the logit inclusive value as: $\delta_{it}(\Omega_t) = ln(\sum_j exp(\delta_{ijt}(\Omega)))$
- Now think about the comparison with Hendel and Nevo (2006):
    - whether to purchase (what size to purchase in HN) determined only by inclusive value
    - but the choice of j is NOT a static estimation, because j enters every period's flow utility.

Simplified Consumer Problem

- Assumption: Inclusive Value Sufficiency $P(\delta_{it+1}|\Omega_t) = P(\delta_{it+1}|\Omega'_t)$ if $\delta_{it}(\Omega_t) = \delta_{it}(\Omega'_t)$
- Then

$$EV_i(\delta^f_{i0t}, \delta_{it}) = ln(exp(\delta_{it}) + exp(\delta^f_{i0t} + \beta E[EV_i(\delta^f_{i0t}, \delta_{it+1})|\delta_{it}]))$$

- Policy: the probability that consumer i purchase good j is

$$\hat{s}_{ijt}(\delta^f_{i0t}, \delta_{ijt}, \delta_{it}) = \frac{exp(\delta_{it})}{exp(\delta_{it}) + exp(\delta^f_{i0t} + \beta E[EV_i(\delta^f_{i0t}, \delta_{it+1})|\delta_{it}])} \times \frac{exp(\delta_{ijt})}{exp(\delta_{it})}$$

- Finally, need $\delta_{it+1} = \gamma_{1i} + \gamma_{2i}\delta_{it} + u_{it}$

Estimation Procedures

Need to estimate $\alpha = (\alpha^x, \alpha^p), \Sigma$

- Inner Loop: Draw $\alpha_i = \alpha + \Sigma^{1/2}\tilde{\alpha}_i$ and solve the simplified consumer's problem. Then use the solved policy function $\hat{s}_{ijt}(\delta^f_{i0t}, \delta_{ijt}, \delta_{it})$, δ_{ijt}, and δ_{it} to calculate market share for this consumer starting at time 0.
- Middle Loop: Sum across diffrent consumers from inner loop, we have model market share of good j at time t as $\hat{s}_{jt}(\bar{\delta}^f_{jt}, \alpha^p, \Sigma)$. Use Berry (1994) inversion to recover $\bar{\delta}^f_{jt}$.
- Outer Loop: $\xi_{jt} = \bar{\delta}^f_{jt} - \alpha^x x_{jt}$. Define the GMM criterion function $G(\alpha, \Sigma) = z'\xi(\alpha, \Sigma)$.

Hightlights of the Estimates

- Use the standard BLP instruments: product characteristics of j, mean product characteristics of all firms, count of products offered by the same firm and by all firms. Intuition: affect price-cost margin or product substitubility.
- The static BLP mode produces unreasonable coefficients, including a positive price coefficient. Because within a time period, the cheapest models were often not the most popular.
- The dynamic model helps to identify a negative price coefficient due to time series variation: once prices fell overtime, a lot more people purchased digital camcorders.

Parameter	Base dynamic model	Dynamic model with extra random coefficients	Dynamic model without repurchases	Static model	Static model aggregated to year	Dynamic model with micro moments
Mean coefficients (α)						
Constant	−.141 (.044) *	−.097 (.195) *	−.087 (1.5)	−8.90 (2e3)	−4.03 (132)	−.243 (.213)
Log price	−2.66 (.576) *	−2.74 (.975) *	−.056 (72.6)	.0247 (19.1)	−.089 (14.5)	−3.01 (.582) *
Log size	−.007 (.001) *	−.007 (.014)	−.002 (7e-4) *	−.152 (.068)	−.340 (.204)	−.019 (.002) *
Log pixel	.095 (.050)	.098 (.028) *	−.002 (.027)	−2.56 (2.43)	−4.52 (5.85)	.241 (.146) *
Log zoom	.007 (.002) *	.007 (.002) *	.007 (9e-4) *	.654 (.086) *	.861 (.269)	.010 (.004) *
Log LCD size	.003 (.001) *	.003 (.001) *	−5e-4 (9e-4)	−.053 (.105)	−.361 (.325)	.011 (.004) *
Media: DVD	.025 (.005) *	.027 (.006) *	−.001 (.004)	−.177 (.344)	.229 (1.35)	.052 (.017) *
Media: tape	.007 (.005)	.007 (.005)	−.007 (.003) *	−.763 (.333) *	−.671 (1.05)	.017 (.016)
Media: HD	.020 (.006)	.023 (.008) *	−.008 (.004)	−.873 (.425) *	−1.32 (1.59)	.039 (.019) *
Lamp	.006 (.001) *	.007 (.001) *	−.002 (.001)	−.209 (.130)	−.351 (.402)	.002 (.004)
Night shot	.009 (.001) *	.009 (.001) *	.007 (6e-4) *	.646 (.073) *	1.20(.199)	.022 (.003) *
Photo capable	−.014 (.002) *	−.015 (.003) *	−.005 (.002) *	−.431 (.205) *	−.432 (.767)	−.022 (.007) *
Standard deviation coefficients ($\Sigma^{1/2}$)						
Constant	.086 (.025) *	.058 (.130)	2e-5 (27)	.007 (4e4)	.036 (1e3)	1e-7 (.082)
Log price	7e-6 (.563)	.043 (8.06)	.0002 (817)	.001 (267)	.011 (67.7)	.651 (.233) *
Log size		5e-09 (.096)				
Log pixel		.0015 (.337)				

Standard errors in parentheses; statistical significance at 5% level indicated with *