The Estimation of Dynamic Games: Continous Choices

Based on Notes of Mark Roberts

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A Dynamic Model of Entry, Exit, and Investment

- An application of the Bajari, Benkard, and Levin (2007) methodology:

  The Cost of Environmental Regulation in a Concentrated Industry by Stephen Ryan

- Concrete industry - very concentrated regional markets, capital intensive, huge user of fossil fuels and producer of air pollution

- U.S. 1990 Clean Air Act Amendments altered the pollution requirements for plants in this industry.

  existing plants - must install pollution monitoring equipment and develop plans for pollution reductions. \$5 million/plant

  new plants - must provide engineering studies of how they will reduce pollution. \$5-10 million/plant

- Goal: Measure the welfare effect of this increase in entry cost. Reduce profits of potential entrants, increase market power of existing producers

Details of the industry

- Homogeneous product (common market price)
- Spatially separated regional markets in U.S.
- Firms are differentiated by production capacity (size of kilns)
- Short run competition - simultaneous quantity choice s.t. capacity constraint
- Dynamic choices - entry, exit, level of capacity
- Lumpy capacity adjustment is common

Data

- Market Level Data - 27 regional markets in U.S. 1981-1999
  - Price and Quantity
  - Price of coal, natural gas, electricity, wages (IV's)

- Plant Level Data
  - daily capacity level - nameplate rating
  - annual capacity level = output. Recognizes down time for maintenance
  - investment = change in capacity (annual)
  - utilization = output/max capacity using daily capacity rating

N firms (incumbents + potential entrants) in a regional market (each market treated independently).
Market state vector: $S_t = (s_{1t}, s_{2t}, ...s_{Nt})$ where s is the plant capacity

Timing of information and decisions

- All firms observe state vector S_t
- Potential entrants receive *private* entry cost draw and decide to enter, Incumbent faces common scrap value and decides to exit
- All firms make investment decisions and pay adjustment costs
- All firms get *private* MC/productivity shock and choose quantities:
- Short run profits are realized
- Entrants pay entry fee, Exiting firms get scrap value
- State vector is updated as new capacities come on-line

Note: investments are paid for in current period but do not come on-line until next period.

Theoretical Model - Market Demand and Firm Profit

- Market Demand: $P = AQ^{1/\varepsilon}$
- Short run profit for firm i:

$$\pi_i = q_i(P - MC + \omega_i) - I(utilpct_i > \nu) * (CAPCOST * (utilpct_i - \upsilon)^2)$$

MC is a common production cost
CAPCOST is a common cost when nearing capacity
υ is a common threshold where increasing MC begin
$utilpct_i = \frac{q_i}{CAP_i}$ is firm capacity utilization (observed)
ω_i firm MC/productivity shock (private information)

- Short run profit for firm i if there is investment or disinvestment

$$
\begin{aligned}
u_i = \pi_i \quad & -I(INV_i > 0)(\beta_0 + \beta_1 INV_i + \beta_2 INV_i^2) \\
& -I(INV_i < 0)(\gamma_0 + \gamma_1 INV_i + \gamma_2 INV_i^2)
\end{aligned}
$$

Different adjustment cost parameters if investment is positive or negative

- Profits for the firm that exits:

$$u_i = \pi_i + SCRAP$$

SCRAP is common to all firms.

- Profits for the entrant:

$$u_i = -SUNK_i - (\beta_0 + \beta_1 INV_i^e + \beta_2 (INV_i^e)^2)$$

INV^e is the initial capacity level for the entrant

Transition Process for the State Variables

Single element of the state vector (one firm's capacity):

$$\Pr(s_i'|s_i) = \quad I(s_i > 0)(1 - \Pr(exit_i|s))\Pr(s_i'|s, INV_i) + \\ I(s_i = 0)(\Pr(entry_i|s))\Pr(s_i'|s, INV_i^e)$$

First line is the evolution of firm i capacity when the firm is an incumbent. Second line is the evolution of the firm's capacity when it is a potential entrant.
Will treat the last terms as deterministic: $s_i' = s_i + INV_i$.

Can estimate the transition process for the state vector if we have estimates of $\Pr(exit_i|s), \Pr(entry_i|s)$, and $INV_i(s)$

Estimation Strategy

- Stage 1
  - Estimate SR profit function and market demand.
  - Estimate "policy functions" for exit, entry, investment (actions as functions of the state vector): $\Pr(exit_i|s), \Pr(entry_i|s)$, and $INV_i(s)$
- Stage 2
  - Estimate the transition process for the state variables $\Pr(s'|s)$ and compute value functions
  - Estimate the dynamic parameters (costs of adjustment for capacity, scrap value, dist of entry costs) by using the equilibrium conditions for entry, exit, investment.

Empirical Model- Stage 1

- Market demand - estimate with market level data, use factor prices as IV.

$$\ln Q_{mt} = \alpha_0 + \alpha_1 \log P_{mt} + \sum_m \alpha_m D_m + u_{mt}$$

- Firm production costs (MC, CAPCOST, υ in each of the two regulatory regimes)$-$ only has data on output.
- Use f.o.c for output choice (marginal revenue=marginal cost) and solve for q^*_{it}. Minimize deviations between q_{it} and q^*_{it}
- Construct ω_{it} as the residual in the f.o.c. (shock to firm's marginal cost)
- Policy functions - depend on the state vector of capacities. Describe firm's actual (optimal) action for each state. (They do not tell you how the firm's actions are related to the structural parameters).
- Exit: probit model of plant exit. Explanatory variables- firm's capacity, total rival capacity, regime dummy, ω_{it}

Policy functions (continued)

- Entry: probit model of plant entry. Explanatory variables- total rival capacity, regime dummy. (must be assuming every plant not in operation is a potential entrant)
- Investment levels (data has lumpy adjustment): firm has target capacity that depends on the state variables

$$TARGET_{it} = \alpha_4 Capacity_{it} + \alpha_5 (Rivalcapacity_{it}) + \alpha_6 \omega_{it} + e_{it}$$

but only adjusts actual capacity when it deviates substantially from the target (hits bands around the target)

$$BAND_{it} = TARGET_{it} \pm \exp(\alpha_7 Capacity_{it} + \alpha_8 (Rivalcapacity_{it}) + \alpha_9 \omega_{it} + u_{it})$$

When the plant changes its capacity he observes $TARGET_{it}$ = new capacity level and $BAND_{it}$ defines inactions.

Why Estimate the Policy Functions?

Allows him to simulate state vector forward in time $S_{1,}S_{2,}...S_T$ from a given starting point $S_{0.}$. For example:

- If firm i is not active in year t then $s_{it} = 0$. Draw a random number from U(0,1) and compare with estimated $Prob(entry|S_t)$. If draw is high, add the firm and calculate INV_{it}^e using the policy function. Update the state vector for the next period.
- If firm i is active then $s_{it} > 0$. Compare a random U(0,1) draw with $Prob(exit|S_t)$. If they stay in, calculate INV_{it}. Update the state vector.(how is ω_{it} handled?)
- These future states are the ones that are consistent with optimal decisions by the firms.
- Also, can perturb the entry, exit, investment policies (i.e. add random noise inside the exit probit) and generates different path for future states.
- This can be repeated many times to generate many paths for the state variables. The future paths will not be consistent with the optimal decisions by the firms.

Stage 2: Estimate the *dynamic parameters* - adjustment cost parameters for investment $\alpha = (\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1, \gamma_2, \mathit{SCRAP})$, and distribution of entry costs

Basic insight - compare the present value of the future payoffs for the different sets of future states. The present value must be larger when we use the *true* policy functions rather than the *perturbed* policy functions.

Objective for estimation is to choose the dynamic parameters to minimize the possibility of observing higher present value from the perturbed policy functions.

- Present value of future payoffs is $W_i(S_0; \sigma_i^*, \sigma_{-i}, \alpha) = E_0 \sum_{t=0}^{T} \delta^t u$ which depends on both π_{it} (now data) and the dynamic parameters (α) and choices $(INV, exit, enter)$. σ_i^* is the true (optimal) policy functions for firm i and σ_{-i} are the true policy functions for its rivals.
- Given the functional forms for u_{it}, $W_i(S_0; \sigma_i^*, \sigma_{-1,}\alpha) = W_i(S_0; \sigma_i^*, \sigma_{-1})\alpha$ is linear in the dynamic parameters.
- $W_i(S_0; \sigma_i', \sigma_{-1})\alpha$ is the PV of future payoffs using the perturbed payoff functions σ_i'
- Profits from deviating from the optimal policy are: $g(x, \alpha) = (W_i(S_0; \sigma_i', \sigma_{-1}) - W_i(S_0; \sigma_i^*, \sigma_{-1}))\alpha$
- Pick a set n of perturbed payoff functions and then choose α to minimize the sample objective function:

$$Q(\alpha) = \frac{1}{n} \sum^{n} I(g(x_j, \alpha) > 0) g(x_j, \alpha)^2$$

Estimate the distribution of sunk entry costs.

Calculate the present value of profits from being an entrant - depends on the payoffs from being an incumbent and private entry cost:

$$\begin{aligned} & V_{it}^{E}(S_t, SUNK_i) \\ = \quad & -SUNK_i + \max_{INV_i^e} \left\{ -(\beta_0 + \beta_1 INV_i^e + \beta_2 (INV_i^e)^2) + \delta EV(s'/s) \right\} \\ = \quad & -SUNK_i + EV_{it}^{E} \end{aligned}$$

Everything is known or can be calculated except $SUNK_i$ which is treated as an iid draw from an entry cost distribution G

Match observed entry rate with the probability of entry predicted by the model: $Prob(entry/S_t) = Prob(SUNK_i < EV_{it}^{E})$

Selected Results

- Elastic demand; $\epsilon = -2.95$
- MC of production and CAPCOST do not change across regulatory regimes
- Investment policy function - fits data well (current productivity shock is not significant)
- Exit probit - rival capacity raises exit, regime dummy lowers exit in regulated period
- Entry probit-only regime is significant. It is negative.
- Investment adjustment costs - fixed costs (intercept) are large implying lumpy adjustment. Scrap value is similar in two regimes
- Sunk entry costs - mean sunk cost for entrants increases 25% over time. \$46 million to \$58/million
- Welfare implications: Rise in entry costs reduces entry, strengthens market power of existing firms. Higher prices, lower market output. Consumers lose the most.