Financial Frictions and Firm Dynamics

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Integrate the financial behavior of firms (eg. Fazzari etc 88, Gilchrist and Himmelberg 95) with firm dynamics

- small and younger firms: fewer dividend, more debt, invest more
- small firms: higher q, more sensitive to cash flow (after controlling profitability)
- "age dependence" and "size dependence"

Key result: embed financial frictions with **Hopenhayn (92)** can account for these.

- persistent productivity shocks
- size jointly determined by productivity and equity
- equity and debt are not perfectly substitutes: costly default and new share issuing

Frictionless Economy

- revenue function: $y = (z + \epsilon)F(k)$ cost: $\phi k = [\delta + w(l/k)]$
- shocks $z \in Z = (z_0, ..., z_N)$ and follows a FOMP $\Gamma(z'/z)$, ϵ i.i.d mean zero.
- $F(.)$ is strictly increasing, strictly concave (span-of-control or monopolistic competition) and continuously differentiable.
- exogenous exit $z_0 = 1$ and $\Gamma(z_0/z_0) = 1$.
- Modigliani-Miller theorem applies, $1/\beta - 1 = r$.
- Incumbent: $Max_k[\int_\epsilon (z + \epsilon)F(k)f(d\epsilon) - (r + \phi)k]$
- Free-entry implies: $V(z_N) = \kappa$.

Two key assumptions

- cost λ per unit of new share, e.g re-investing profit is less costly
- firm can default with a cost of ξ, borrowing cost higher in equilibrium. Default doesn't lead to exit, but a re-negotiation of debt.
- A consequent assumption $1/\beta - 1 > r > 0$ ensures equity is in compact set $[e_{min}, e_{max}]$ (i.e. firms don't save too much)
- Firm's net worth $\pi(e, b, z+\epsilon) = (z+\epsilon)F(e+b) + (1-\phi)(e+b) - (1+\tilde{r})b$, default decision happens when ϵ is realized and next period $z^{'}$ observed.
- Define $\epsilon^*(z, e, b, z^{'})$ be threshold below which firms default and renegotiate debt to $e^*(z^{'})$, which is endogenously determined in equilibriium.

The transition of net worth $q(e, b, z + \epsilon, z^{'})$

- is $e^*(z^{'})$ if $\epsilon < \epsilon^*(z, e, b, z^{'})$.
- is $e^*(z^{'}) + (\epsilon - \epsilon^*(z, e, b, z^{'}))F(e + b)$ if $\epsilon > \epsilon^*(z, e, b, z^{'})$.

Financial intermediary set $\tilde{r}$ such that:

$$(1 + r)b =$$

$$(1 + \tilde{r})b \int_{\epsilon^*} f(d\epsilon) + \int^{\epsilon^*} [(1 - \phi)(e + b) + (z + \epsilon)F(e + b) - \xi]f(d\epsilon)$$

We can further eliminate $\tilde{r}$ using the fact that:

$$e^*(z') = (z + \epsilon^*)F(e + b) + (1 - \phi)(e + b) - (1 + \tilde{r})b$$

Firm's Problem

Timing

- firm observe (z, e) and make optimal debt decision
- (ϵ, z') are realized and make default decision and determine new net worth $q(e, b, z+\epsilon, z')$.
- finally, issue new shares/pay dividends and determine e', based on z' and new net worth $q(e, b, z+\epsilon, z')$.

Finally the firm's value is
$\Omega(z, e) = max_b\{\beta \sum_{z'} \int_{\epsilon^*(z,e,b,z')} \hat{\Omega}(z', q(e, b, z+\epsilon, z'))\Gamma(z'/z)f(d\epsilon)\}$
given

- financail intermediary's decison (pins down $\epsilon^*(z, e, b, z')$)
- $\hat{\Omega}(z', e^*(z')) = 0$ (pins down $e^*(z')$)
- equity decision is optimal: issue new shares if lower than $\underline{e}(z')$ or distribute dividend if higher than $\overline{e}(z')$.

Free-entry

- $\Omega(z_N, \underline{e}(z_N)) = \kappa + (1+\lambda)\underline{e}(z_N)$
- The authors prove an invariate measure of $\prod_{i=1}^{N} [\underline{e}(z_i), \overline{e}(z_i)]$.

Financial behaviors of firms with i.i.d shocks (i.e. $z \in \{z_0, z_1\}$).

- small (equity) firms take on more debt (decreasing returns in profit vs. marginal increase in volatility), and a higher prob of default
- small firms issue more shares and pay fewer dividends. (because they have higher profitability).

However, their case of full model with persistent shocks and financial frictions are not empirically interesting. (**Jonvanovic 82** can easily generate these patterns!)