Lecture Notes on Industry Dynamics II

March 2009

Klette and Kortum 2004

- Very ambitious framework: reconcile firm-level studies of R&D, productivity, patenting, and growth with endogenous growth theory.
- Two separately developed literature:
  - R&D, productivity, and patenting. (Griliches 1998). Not answered: sources of heterogeneity in innovative effort across firms; why some firms prosper while doing little or no R&D; what is a good measure of innovative output;
  - Firm growth, entry, exit, and size distribution. (Long literature: Lucas, Jovanovic, Hopenhayn, Sutton, etc.). Not answered: sources of heterogeneity among firms; what explains Gibrat's law as well as Zipf's law?

- An integrated treatment by KK:
    - poisson process for a firm's innovation: arrival rate depends on current R&D and knowledge stock. (Patent and R&D, i.e. Kortum 1997)
    - optimal R&D investment rule: growth rate independent of size. (Gibrat's Law)
    - stochastic process for innovation: size distribution of firms. (Zipf's Law)
- key ingredient: innovating and multiproduct firms (Penrose (1959))
- industry equilibrium:
    - firms' innovative successes come at the expense of competitors (Betrand competition on quality, i.e. Aghion and Howitt (1992))
    - allows for simultaneous entry and exit, with a skewed firm size distribution (Hopenhayn (1992))

Model: Innovating Firm

- profit flow of each good is $0 < \bar{\pi} < 1$, heterogeneity of π will be introduced later. Firm's state is summarized by a scalar, the number of goods n is products.
- firm's R&D determines the Poisson rate I at which its next product innovation arrives. Most recently innovator of any good takes over the market for that good. KEY ASSUMPTION: not directed innovative effort-the good draws from U[0,1].
- Firms don't innovate on a good it is currently producing. However, it faces the possibility that being overtaken by other firms for its current good. Poisson hazard: μ.
- At the background: quality ladder model of Grossman and Helpman. (Check why incumbents never want to innovate!)

Model: Innovation Technology

- $I = G(R, n)$, R is R&D input and n proxies knowledge capital. $G(.)$ strictly increasing and concave in R (decreasing returns), strictly increasing in n. Another important ASSUMPTION: homgomenous of degree one in both inputs. (neutralizes the effect of firm size on innovation)

- Write in terms of cost function: $R = C(I, n) = nc(I/n)$, then firm's Bellman equation is:

$$rV(n) = max_I\{\bar{\pi}n - C(I, n) + I[V(n+1) - V(n)] \\ -\mu n[V(n) - V(n-1)]\}$$

- Let $\lambda = I(n)/n$ be firm's innovation intensity and $v = V(n)/n$. F.O.C implies $c'(\lambda) = v$ or $c'(0) > v$ and $\lambda = 0$. Further $(r + \mu - \lambda)v = \bar{\pi} - c(\lambda)$, which implies innovation intensity is increasing in $\bar{\pi}$, decreasing in μ, and decreasing in an upward shift of c'.(Check notes for the F.O.C.!)

Model: Firm's Life Cycle

- $p_n(t; n_0)$: prob that a firm size n at date t given that it was size n_0 at date 0. Then $\dot{p}_n(t; n_0)$ follows that:

$$\dot{p}_n(t; n_0) = (n-1)\lambda p_{n-1}(t; n_0) + (n+1)\mu p_{n+1}(t; n_0) - n(\lambda+\mu)p_n(t; n_0), n \geq 1$$

- $\dot{p}_n(t; n_0) = \mu p_1(t; n_0)$ if $n = 0$, i.e. exit is an absorbing state.
- Solving the differential equation system using probability-generating function techniques could lead us to several nice implications about firm growth that links back to Dunne, Roberts, and Samuelson (1989).

Model Implications: Firm Growth and Failure

- Simplify that $n_0 = 1$, the authors prove that $p_0(t;1) = \frac{\mu[1-e^{-(\mu-\lambda)t}]}{\mu-\lambda e^{-(\mu-\lambda)t}}$. (Not that $lim_t p_0(t;1) = 1$, so a firm always exit).
- $p_n(t;1) = [1 - p_0(t;1)][1-\gamma(t)]\gamma_t^{n-1}$, for $n = 1, 2, 3, ...$, where $\gamma(t) = \lambda p_0(t;1)/\mu$. Intuition: think about the evolution of an entire firm as summing the evolution of independent divisions, each as a firm starting with a single product, so $p_n(t;1) = p_{n-1}(t;1)\gamma(t), n = 2, 3, ...,$ $\gamma(t)$ captures the relative strength of creation and destruction.
- Thus, conditioinal on survival, the size of the firm at date t follows a simple geometric distribution $\frac{p_n(t;1)}{1-p_0(t;1)} = [1-\gamma(t)]\gamma_t^{n-1}, n = 1, 2, ...$
- In addition, since $\gamma(t)$ is growing overtime, the distribution grows stochastically larger overtime (More weight on larger n).

Model Implications: Firm Growth and Failure, cont.

- Another natural implication is that a firm of size n exits within t periods is $p_0(t;1)^n$. (Think about it as n firms of 1 products). Larger firms have a lower hazard of exiting.
- Let $G_t = ((N_t - N_0)/N_0$. Then $E[G_t|N_0 = n_0] = e^{-(\mu-\lambda)t} - 1$. So the Gibrat's Law holds in the sense that the expected growth is independent of size.
- Expected growth conditional on survival is $E[G_t|N_t > 0, N_0 = n_0] = \frac{e^{-(\mu-\lambda)t}}{1-[p_0(t;1)]^{n_0}} - 1$, i.e. it is a decreasing function of initial size.

Generating Heterogeneous Research Intensity

- How to capture considerable cross-sectional variability in research intensity: size is NOT a good predictor.
- A quite mechanical way: introduce different profit types in π, i.e. some firm's ALL products are more profitable. Also, type π firm has a research cost function $C_\pi(I, n) = (\pi/\bar{\pi})C(I, n)$, i.e. it is more costly to develop larger innovations.
- This setting keeps λ the same across all types. But value per product $v_\pi = (\pi/\bar{\pi})v$ and R&D intensity $C_\pi(I, n)/n = (\pi/\bar{\pi})c(\lambda)$ carries the heterogeneity in π.

Industry Behavior

- Denote the measure of firms with n products at date t by $M_n(t)$. Then total measure of firms is $M(t) = \sum_n M_n(t)$. Further $\sum_n nM_n(t) = 1$.
- Take as whole, industry incumbents innovate at rate $\sum_n M_n(t)n\lambda = \lambda$.
- Entry: a mass of potential entrants invest at rate F for a Poisson hazard 1 of entering with single product. Free entry condition $F = E[v_\pi] = E[(\pi/\bar{\pi})v] = v$, the rate of innovation by entrants is η. The rate of innovation by entrants are pinned down by $\eta = (\bar{\pi} - c(\lambda))/F - r$.
- Size distribution: with $\mu = \eta + \lambda$, for $n \geq 2$

$$n \geq 2 : \dot{M}_n(t) = (n-1)\lambda M_{n-1}(t) + (n+1)\mu M_{n+1}(t) - n(\lambda + \mu)M_n(t)$$
$$n = 1 : \dot{M}_n(t) = \eta + 2\mu M_2(t) - (\lambda + \mu)M_1(t)$$

The authors prove that the resulting steady state distribution with entry/exit is logrithm.

Lentz and Mortensen

- Estimate an equilibrium model of productivity growth through innovation
- Question: what is the role of resource reallocation in the growth process?
- Borrows heavily from Klette and Kortum (2004).
- Background: Bailey,Campbell,and Hulten (1992) Decomposition of aggregate productivity growth. Within firm growth v.s. reallocation (or entry/exit).

Preference and technology

- $U_t = \int_t lnC_s e^{-r(s-t)} ds$ and nomial household expenditure $E_t = P_t C_t$. We normalize $E_t = E$ for all t, thus P_t declines at the rate that consumption grows.
- Aggregate CES production function:

$$C_t = [\int_{j=0}^{1} Z(j)(A_t(j)X_t(j))^{(\sigma-1)/\sigma} dj]^{\sigma/(\sigma-1)}$$

  $Z(j)$ represents the expenditure share.
- Each input j's productivity is determined by $A_t(j) = \prod_{i=1}^{J_t(j)} q_i(j)$, where $J_t(j)$ is the number of innovation steps made for j up to time t.

- Suppose the quality of other intermediate goods increase at an average rate g, for good j (whose latest innovation happens at t), its demand at $t+a$ is $x_{t+a}(j) = z_{t+a}(j)/p_{t+a}(j)$, where $z_{t+a}(j) = zZ_t(j)^{\sigma}(P_t(j)/P_t)^{1-\sigma}e^{g(1-\sigma)a}$. When a firm innovates, it doesn't know which j is directed, so demand shifter $z_{t+a}(j)$ has random realization $\tilde{z}$ and $z = E(\tilde{z})$.
- All quality-adjusted prices $P_t(j) = p_t(j)/A_t(j)$

Value of firm

- Pricing for good j follows "limit-pricing", i.e. markup $m(q(j),\sigma) = \sigma/(\sigma-1)$ if $q > \sigma/(\sigma-1)$ and $m(q(j),\sigma) = q(j)$ otherwise.
- Thus the profit is:

$$\begin{aligned}\Pi_{t+a}(j) = (p(j) - w - \kappa)z_{t+a}(j)/p_j = \pi(q(j),\sigma)z_{t+a}(j) \\ = (1 - m(q(j),\sigma)^{-1})z_{t+a}(j)\end{aligned}$$

- Following KK, total number of products supplied by a firm is k, new products are generated by R&D at an arrival rate γk, with an cost $wc(\gamma)k$. Also there is a common destruction rate δ that one product is replaced by a more productive version by another firm.
- Introduce additional heterogeneity: firms differ in their innovation steps q when born. A τ type innovates at step q_τ, which draws from $F_\tau(.)$.

- A firm's state is characterized by $(k, \tilde{q}^k, \tilde{z}^k)$.
- A value of type τ firm has a closed form solution, and define ν_τ as type τ conditional expected value of a product. It is straightforward to show that $w\hat{c}'(\gamma_\tau) = \nu_\tau$, where $\hat{c}(\gamma) = c(\gamma)/z$.
- Firm entry: let there is a constant measure of m potential entrants, the aggregate entry rate is $\eta = m\gamma_0$. We will need free-entry condition that $w\hat{c}'(\eta/m) = \sum_\tau \nu_\tau \phi_\tau$.
- The τ specific steady state distribution of k is again well-defined, the total number of products produced by each type τ firm is defined as K_τ.
- Finally, we have $\delta = \eta + \sum_\tau K_\tau \gamma_\tau$.

Steady State Equilibrium:

- A triple (w, δ, g) together with entry rate $\eta = m\gamma_0$, creation rate γ_τ, and a steady state size K_τ for each type that satisfy (1) optimal choice of creation rate for both entrants and incumbents (two equations) (2) balanced entry/exit of mass into different type τ (3) destruction rate is the sum of entry rate and the aggregate create rates of incumbents (4) growth rate in consumption $g = \delta E[ln(q)]$ (5) labor market clearing.

Steady State Equilibrium Solution given model parameters.

- w is estimated from the data, so leave out (5) labor market clearing
- Look for a mapping $(\Psi', \delta', g') = \Upsilon(\Psi, \delta, g)$, where $\nu_\tau = \frac{\bar{\pi}_\tau}{r+\delta-g(1-\sigma)} + \Psi_\tau$.
- Solve γ_τ and η using (1), then $\Psi'_\tau = \frac{\gamma_\tau \nu_\tau - w\hat{c}(\gamma_\tau)}{r+\delta'}$
- Update K_τ, δ', g' using (2) (3) (4)
- Iterate until reaching a fixed point.

Estimation and Simulation

- Parameters:

$$\omega = (r\&d(c_0, c_1), capital(\kappa), demand(\sigma, \beta_Z s), innovation - step \\ (\beta_q, \xi_1, \xi_2, \xi_3), type(\phi_1, \phi_2), measurement(\sigma_Y, \sigma_W))$$

- Moments: Mean, Dispersion, and Correlation of Y, W, and N of 4,9872 Danish firms in 1992 and 1997.
- Using Simulated Method of Moments technique to estimate model parameters. KEY: Identification with unobeserved heterogeneity.(Which is quite hard to get from their writing).