Lecture Notes on the Estimation of Production Function

Arpil 2008

The basic criticism:

- think about a Cobb-Douglas production function $y = \alpha z + \beta x + u$, where z is fixed input (say, capital) and x is variable input. u represents all other disturbances: left out factors, efficiency, functional form discrepancies, measurement error...
- Marschak and Andrews: simple profit maximization model (price normalized to 1) gives: $y = x + w + v - ln(\beta)$, v is discrepancies from assumed conditions like perfect competition, perfect foresight, etc.
- $x = (\alpha z - (w + v) + u)/(1 - \beta)$, if x is chosen approximately optimally, it is a function of u.
- repsonses to this problem: micro-level studies–instruments (Arellano and Bond), proxy (Olley and Pakes, etc); macro-level studies–(Solow, Hall, etc)

The Anatomy of Error: let $u_{it} = a_{it} + e_{it} + \epsilon_{it}$

- a and e are ultimately known by producers, while not econometricians. ϵ is measurement error.
- a is known to affect current choice of x, while e reveals itself only later on. (delayed transmission). so e is serially uncorrelated, but (a, z, x) maybe affected by its past value.
- Now $y - e - \epsilon_y = x - \epsilon_x + w + v - ln(\beta)$.
- Thus $(x + w - \epsilon_x) - (y - e - \epsilon_y) = ln(\beta) - v$.

Panel Data Response

- $y_{it} = \alpha z_{it} + \beta x_{it} + a_i + \lambda_t + e_{it}$, ignoring measurement error for the time being.
- Within estimator: $y_{it} - y_i = \alpha(z_{it} - z_i) + \beta(x_{it} - x_i) + (e_{it} - e_i)$. In practice, often unsatisfactory results: low/insignificant capital coefficient, unreasonably low returns to scale.
- Chamberlain (1982): within estimator requires no "delayed transmission", i.e. e_{it} must be pure error. This leads to first differencing and GMM estimation.
  $y_{it} - y_{it-1} = \alpha(z_{it} - z_{it-1}) + \beta(x_{it} - x_{it-1}) + \xi_{it} + (e_{it} - e_{it-1})$, where $\xi_{it} = a_{it} - a_{it-1}$.
- Lagged x and z as instruments, however, usually are quite poor and possess little resolving power.

Empirical Reality

- let $u_{it} = a_{it} + \lambda_{it} + e_{it} + \epsilon_{it}$, where $a_{it} = \xi_{it} + a_{it-1}$ and $\lambda_{it} = \lambda_t + g_i t$.
- the first differences gives $du_{it} = \xi_{it} + d\lambda + g_i + de_{it} + d\epsilon_{it}$, if xi_{it} is still "transimitted", need valid instruments.
- another "within" transformation will eliminate g_i, the individual growth rate.
- measurement error concerns here: (TO BE ADDED)

Alternative Approach: Olley Pakes

- A simple model with investment and exogenous productivity change
- assume that $u_{it} = a_{it} + \epsilon_{it}$, and a_{it} follows a FOMP, notice that this encompasses the fixed effects, however, not allowing for BOTH.
- $k_{it} = (1 - \delta)k_{it-1} + i_{it-1}$ and single period profit is $\pi(z_{it}, a_{it}, \Delta_t) - c(i_{it}, \Delta_t)$.
- the dynamic problem (as in Ericson and Pakes) generates two decision rules: exit $\chi_{it} = 1$ if $a_{it} \geq \bar{a}(k_{it})$, investment $i_{it} = i(k_{it}, a_{it}, \Delta_t)$. An important STRICT monotonicity condition.
- invert to generate $a_{it} = h_t(k_{it}, i_{it})$. The scalar unobservable assumption is important here. So:

$$y_{it} = \beta x_{it} + \phi_t(k_{it}, i_{it}) + \epsilon_{it}$$

where $\phi_{it} = \alpha k_{it} + h_t(k_{it}, i_{it})$.

- The second stage: make use of the assumption of FOMP, so $a_{it} = E[a_{it}|a_{it-1}] + \xi_{it} = g(a_{it-1}) + \xi_{it}$
- use the first stage result to back out $\hat{\phi}_{it}$, we have:

$$\hat{\phi}_{it} = \alpha k_{it} + g(\hat{\phi}_{it-1} - \alpha k_{it-1}) + \xi_{it} + \epsilon_{it}$$

- Wooldridge (2004) also proposed to combine both stages into a single set of moments and estimate in one step.
- Endogenous selection in the second stage: $E[y_{it} - \beta x_{it}|\chi_{it} = 1, I_{it-1}] = \alpha k_{it} + E[a_{it}|I_{it-1}, \chi_{it} = 1]$. We know that $E[a_{it}|I_{it-1}, \chi_{it} = 1] = E[a_{it}|I_{it-1}, a_{it} \geq \bar{a}(k_{it})] = g(a_{it-1}, \bar{a}(k_{it}))$
- Let $P_{it} = P(\chi_{it} = 1|a_{it-1}, \bar{a}(k_{it}))$ (Propensity Score), the second stage regression can be modified as:

$$\hat{\phi}_{it} = \alpha k_{it} + \tilde{g}(\hat{\phi}_{it-1} - \alpha k_{it-1}, P_{it}) + \xi_{it} + \epsilon_{it}$$

Comparison between dynamic panel and OP

- The flexibility of the process of a_{it}, linear v.s. nonlinear
- no selection v.s. selection
- possible fixed effects in addition to serially correlated process
- assumptions regarding input demand equation

Extensions to OP Method: Levinsohn and Petrin

- OP is applied to US telecommunication industry, which is investment intensive. However, LP noticed that there are a lot of zero investments in the micro-data of other industries and countries.
- Focus on intermediate inputs (electricity, fuels, and materials), these are rarely zero.
- $m_{it} = m_t(k_{it}, a_{it})$, the first stage is then simply $y_{it} = \beta x_{it} + \phi_t(m_{it}, k_{it}) + \epsilon_{it}$.
- The rest of the steps follows from OP, LP has a built-in STATA routine.

Ackerberg, Caves, and Frazer

- key assumptions in OP/LP: strict monotonicity; scalar unobservable; timing–k_{it} is decided exactly or prior to time period $t-1$, how about x_{it}?
- collinearity problem depends on the timing of x_{it}.
- LP: x_{it} is chosen together with m_{it}, then there is identificatioin problem.
  - $x_{it} = f_t(a_{it}, k_{it}) = h_t(m_{it}, k_{it})$, so β is not identified
  - firm-specific input prices, dynamic labor don't solve this problem, as long as both x_{it} and m_{it}'s decisions both depend on them.
  - modify timing: m_{it} is decided before/after x_{it}. "Before" introduces endogeneity problem back to x_{it}, "after" makes m_{it} directly depends on ~~l_{it}~~.
  - measurement error/optimization error: if in m_{it} violates "scalar unobservable". The only feasible case is that there is significant optimization error in x_{it}, but no error in m_{it}.

ACF's Remedy

- give up trying to estimate β in the first stage and let x_{it} chosen before m_{it}. so $y_{it} = \phi_t(k_{it}, \cancel{l_{it}}, m_{it}) + \epsilon_{it}$, where $\phi_{it} = \alpha k_{it} + \beta x_{it} + h_t(k_{it}, \cancel{l_{it}}, m_{it})$
- let $a_{it}(\alpha, \beta) = \hat{phi}_{it} - \alpha k_{it} - \beta l_{it}$, nonparametrically regress $a_{it}(\alpha, \beta)$ on $a_{it-1}(\alpha, \beta)$ and get the residual ξ_{it}.
- finally making use of the moment conditions that $\xi_{it}(\alpha, \beta)$ is othogonal to k_{it} and x_{it-1}.
- can also easily accomodate Wooldridge's one step GMM method.