Lecture Notes on Productivity II

April 2009

Diewert: Exact and Superlative Index Numbers

Several key ideas in Index Numbers: approximation and optimization

- an aggregator function f is "**flexible**" if it can provide a second order approximation to an arbitrary twice differntiable linearly homogenous function
- define a quantity index $Q(p^0, p^r; x^0, x^r)$. if $\frac{f(x^r)}{f(x^0)} = Q(p^0, p^r; x^0, x^r)$ for any period $r = 1, 2, ..R$ then we say Q is "**exact**" for f.
  - for instance, $\prod_{i=1}^{N}(x_i^1/x_i^0)^{s_i}$ is exact for Cobb-Douglas aggregator function, where $s_i = p_i^0 x_i^0 / p^0 x^0$
  - a quantity index Q is "**superlative**" if it is exact for an f which is flexible.
- Focus on a homogeneous translog function, which is "flexible": $lnf(x) = \alpha_0 + \sum_{j=1}^{N} \alpha_i lnx_n + 1/2 \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} lnx_i lnx_j$, where $\sum_{n=1}^{N} \alpha_n = 1, \gamma_{ij} = \gamma_{ji}$ and $\sum_{i}^{N} \gamma_{ij} = 1$ for $j = 1, 2, ..., N$. (Christensen,Jorgenson, and Lau (1971)).

Main result from Diewert (1976)

- assume that $x^r > 0_N$ is a solution to the aggregator maximization problem $max_x\{f(x) : p^r x = p^r x^r, x \geq 0_N\}$, and f is the **homogeneous translog function**.
- using first order conditions for the maximization problem and Qruadratic Approximation Lemma, it can be shown that: $ln[f(x^1)/f(x^0)] = \sum_{n=1}^{N} \frac{1}{2}[s_n^1 + s_n^0] ln[x_n^1/x_n^0]$, where $s_n^r = p_n^r x_n^r / p_n x_n$ for period r.
    - It is immediate that $Q_0(p^0, p^1; x^0, x^1) = \prod_{n=1}^{N} [x_n^1/x_n^0]^{1/2[s_n^1+s_n^0]}$ is "**superlative**" quantity index.
    - The above maximization problem can be interpreted naturally as producer's problem, where s_n^r is the nth share of cost in period r.

Productivity Measurement by Jorgensen and Griliches (1972)

- how to allow for multiple output? A producer is producing M outputs $y = (y_1, ...y_M)$. Define unit revenue function for each price vector $w \geq 0_M$ by $r(w) = max_y\{wy : g(y) \leq 1, y \geq 0_M\}$, where $g(y)$ is the minimum amount of aggregate input required to produce y.

- assume g is a translog function as we defined, and $y^r > 0_M$ is a solution to the aggregate input minimization probelm $min_y\{g(y) : w^r y = w^r y^r\}$, then similarly by the first order condition with linear homogeneity of g, we have $g(y^1)/g(y^0) = \prod_{m=1}^{M}[y_m^1/y_m^0]^{1/2[s_m^1+s_m^0]}$, where s_m^r is the mth share of revenue in period r.

- finally, production possibility set can be represented as outputs y and x such that $g(y) = f(x)$ and technical progress is "neutral" that $g(y^0) = f(x^0)$ and $g(y^1) = (1+\tau)f(x^1)$. So

$$1+\tau = \prod_{m=1}^{M}[y_m^1/y_m^0]^{1/2[s_m^1+s_m^0]} / \prod_{n=1}^{N}[x_n^1/x_n^0]^{1/2[s_n^1+s_n^0]}$$

Solow's Residual: A Summary by Hall

Solow's orginal exposition is close to index number, but assume exact production function

- key assumptions: constant returns to sacle, perfect competition
- definition of technical progress:

$$\Delta q_t - \alpha_t \Delta n_t - (1-\alpha_t)\Delta k_t = \theta_t$$
$$\alpha_t = \frac{wN_t}{P_t Q_t}$$

- by definition: θ_t is uncorrelated with all other variables. But this usually fails.
- potential relaxation of assumptions: crs, pc, and their interaction with value-added measure.

Hall's modification

- assume long-run cost minization, with CRS
- under a first-order taylor expansion of $Q = F(K, N)$

$$\frac{\Delta Q}{Q} \approx \frac{\partial F}{\partial K}\frac{\Delta K}{K}\frac{K}{Q} + \frac{\partial F}{\partial N}\frac{\Delta N}{N}\frac{N}{Q}$$

$$\Delta q \approx \frac{\partial F}{\partial K}\frac{K}{Q}\Delta k + \frac{\partial F}{\partial N}\frac{N}{Q}\Delta n$$

- by profit maximization:
- by cost minimization: $\frac{\partial F}{\partial K} = (1-\alpha)\frac{Q}{K}$ and $\frac{\partial F}{\partial N} = \alpha\frac{Q}{N}$.
- it implies that in the presence of market power + crs, solow residual measure based on "cost shares" $\alpha = \frac{wN}{rK+wN}$ is valid.
- but it's almost always infeasible, because we rarely observe r.

Hall's modification

- if we still with CRS, we simply estimate the market power
- by definition and CRS: market power $\mu = \frac{P}{x}$, where x is the marginal (average) cost.
- by cost minimization $\frac{\partial F}{\partial N} = \frac{w}{x}$

$$
\begin{aligned}
\frac{\Delta Q}{Q} - \frac{\Delta K}{K} &\approx \frac{wN}{xQ}\left(\frac{\Delta N}{N} - \frac{\Delta K}{K}\right) \\
&= \mu \frac{wN}{PQ}\left(\frac{\Delta N}{N} - \frac{\Delta K}{K}\right)
\end{aligned}
$$

- this builds the foundation of empirical equation to estimate markup μ
- Klette (1999) is a micro-level study that futher exploits this idea.

Foster,Haltiwanger,Syverson

reallocation mechanism as a source of aggregate productivity movement

- tons of studies on productivity-survival link, in reality, selection is on profitability.
- prices typically unobserved, typical studies measure establishment output as revenue divided by common industry-level deflator.
- idiosyncratic demand shocks/market power or production efficiency?
- how to tell them apart: get better data, focus on physically homogeneous products, avoid large quality variations.
- examples: ready-mixed concrete, raw cane sugar, white pan bread, etc.

a theoretical motivation

- following Melitz and Ottaviano (2005), producer $i \in I$, consumer's utility is:

$$U = y + \int_i (\alpha + \delta_i) q_i di - 0.5\eta(\int_i q_i di)^2 - 0.5\gamma \int_i q_i^2 di$$

- the technology is $q_i = \omega_i x_i$, so marginal cost if w_i/ω_i.
- profit-maximization price gives

$$p_i - \bar{p} = 0.5(\delta_i - \bar{\delta}) + 0.5(w_i/\omega_i - \overline{w/}\omega)$$

  so higher price could be (1) higher demand (2) higher cost (lower efficiency)

- the producer's operating profit is $\pi_i = 1/4\gamma(\phi_i - \phi*)^2$, where $\phi_i = \delta_i - w_i/\omega_i$ and $\phi*$ is the critical value of operating.

related productivity measures

- physical productivity: $TFPQ_i = q_i / x_i = \omega_i$
- revenue productivity:

$$TFPR_i = p_i \omega_i = 0.5[\frac{\gamma\alpha}{\eta N + \gamma} + \frac{\eta N}{\eta N + \gamma}(\bar{p} - \bar{\delta})]\omega_i + 0.5\delta_i \omega_i + 0.5 w_i$$

- two measures are positively correlated, but $TFPR_i$ is confounded by δ_i and w_i.

data and measurement issues

- census of manufactures 1977-1997, built on seven-digit SIC, reported value and physical units of shipment. these are used to construct prices.
- $tfp_{it} = y_{it} - \alpha_l l_{it} - \alpha_k k_{it} - \alpha_m m_{it} - \alpha_e e_{it}$, use industries' average cost shares over the sample to calculate input elasticities.
- different y_{it} implies different measures: *TFPQ* uses physical output data, while *TFPR* uses revenue measure, deflated by revenue-weighted geometric mean price across all plants in same industry.
- interesting patterns: *TFPQ* is more dispersed than *TFPR*. There is negative correlation between price and physical productivity.

selection dynamics

- measuring the idiosyncratic demand elements:

$$lnq_{it} = \alpha_0 + \alpha_1 lnp_{it} + \sum \alpha_t year_t + \alpha_2 ln(income_{mt}) + \eta_{it}$$

- evolution of key distributions:
  - entrants have significantly higher physical TFP than incumbents
  - this advantage is clouded in revenue productivity because of lower demand shocks
  - plant's prices rise with plant age
- selection on productivity or profitability:
  - even controlling for plant size, one standard deviation increase in demand shocks accounts for a decrease in the likelihood by 5 percentage point
  - this is three times larger than physical productivity

industry productivity growth decomposition

- BHC/FHK

$$\Delta TFP_t = \sum_{i\in C}\theta_{it-1}\Delta tfp_{it} + \sum_{i\in C}(tfp_{it-1} - TFP_{t-1})\Delta\theta_{it} + \sum_{i\in C}\Delta tfp_{it}\Delta\theta_{it}$$
$$+ \sum_{i\in N}\theta_{it}(tfp_{it} - TFP_{t-1}) - \sum_{i\in X}\theta_{it-1}(tfp_{it-1} - TFP_{t-1})$$

- GR

$$\Delta TFP_t = \sum_{i\in C}\bar{\theta}_i\Delta tfp_{it} + \sum_{i\in C}(t\bar{f}p_i - T\bar{F}P)\Delta\theta_{it}$$
$$+ \sum_{i\in N}\theta_{it}(tfp_{it} - T\bar{F}P) - \sum_{i\in X}(tfp_{it-1} - T\bar{F}P)$$

- Not surprisingly, TFPR growth has a much smaller entry component than TFPQ. (0.9 v.s. 1.35) misallocation of resources across firms

- *TFPR* should be equated across firms in the absence of distortions, differences across firms can be used to recover a measure of firm-level distortion
- Compare China and India v.s. U.S., using Chinese Industrial Survey, Indian Annual Survey of Industries and the U.S. Census of Manufacturing
- simple model of monopolistic competition with heterogeneous firms
    - aggregate industry output $Y_s = (\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$
    - production function is cobb-douglas $Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}$

distortions

- the profit is $\pi_{si} = (1 - \tau_{Ysi}) P_{si} Y_{si} - wL_{si} - (1 + \tau_{Ksi}) RK_{si}$, and profit maximization problem gives:

$$P_{si} = \frac{\sigma}{\sigma - 1} \left(\frac{w}{1 - \alpha_s}\right)^{1-\alpha_s} \left(\frac{R(1 + \tau_{Ksi})}{\alpha_s}\right)^{\alpha_s} \frac{1}{A_{si}(1 - \tau_{Ysi})}$$

- the marginal revenue product of labor is $\frac{w}{1-\tau_{Ysi}} \propto \frac{P_{si} Y_{si}}{L_{si}}$, similarly the marginal revenue product of capital is $\frac{R(1+\tau_{Ksi})}{1-\tau_{Ysi}} \propto \frac{P_{si} Y_{si}}{K_{si}}$.
- measure distortions using the distinctions between TFPR and TFPQ:

$$TFPQ_{si} = A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} (wL_{si})^{1-\alpha_s}}$$

$$TFPR_{si} = P_{si} A_{si} = \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} (wL_{si})^{1-\alpha_s}} \propto \frac{(1 + \tau_{Ksi})^{\alpha_s}}{1 - \tau_{Ysi}}$$

Thus higher TFPR within an industry is a sign that the plant confronts capital and output barriers and make it smaller than optimal

- very dramatic result, how general it is...if TFPQ and TFPR are jointly log-normal, then

$$lnTFP_s = \frac{1}{M_s}\sum_{i=1}^{M_s} lnA_{si} + \frac{\sigma-1}{2}[var(lnA_{si}) - var(lnTFPR_{si}) - 2cov(lnA_{si}, lnTFPR_{si})]$$

- parametrization: $R = 0.10, \sigma = 3$, α_s use CM/ASM data. Then

$$1 + \tau_{Ksi} = \frac{\alpha_s}{1-\alpha_s}\frac{wL_{si}}{RK_{si}}$$

$$1 - \tau_{Ysi} = \frac{\sigma}{\sigma-1}\frac{wL_{si}}{(1-\alpha_s)P_{si}Y_{si}}$$

$$A_{si} = \kappa_s \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s}(wL_{si})^{1-\alpha_s}}$$