

My hint for Exercise F.3 is not correct. Instead, let us define the binary relation \oplus on $(0, 1)$ by

$$x \oplus y := f^{-1}(f(x) + f(y))$$

where f is some invertible real map on $(0, 1)$ such that $f(x) + f(1 - x) = 0$ for each x in $(0, 1)$. It is not difficult to check that $((0, 1), \oplus)$ is an Abelian group (whose additive identity is $\frac{1}{2}$) such that the inverse of x is $1 - x$ for each x in $(0, 1)$. It remains to pick an f with the desired properties. There are many choices in this regard. Here is one:

$$f(x) := \begin{cases} 2 - \frac{1}{x}, & \text{if } 0 < x \leq \frac{1}{2} \\ \frac{1}{1-x} - 2, & \text{otherwise.} \end{cases}$$

(This solution is communicated to me by Kuntal Banerjee.)