An *sS* Model with Adverse Selection

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We present a model of the market for a used durable in which agents face fixed costs of adjustment, the magnitude of which depends on the degree of adverse selection in the secondary market. We find that, unlike typical models, the *sS* bands in our model *contract* as the variance of the shock increases. We also analyze a dynamic version of the model in which agents are allowed to make decisions that are conditional on the age of the durable. We find that, as the durable ages, the lemons problem tends to decline in importance, and the *sS* bands contract.

I. Introduction

Consumers purchase durables infrequently, and for this reason many economists believe that fixed adjustment costs are an important feature of the market for consumer durables. Among the fixed costs most often cited are "lemons costs."[1] These costs arise because adverse selection in the secondary market reduces the price of a durable and therefore discourages trade (Akerlof 1970). A distinguishing feature of the lemons cost is that its size is endogenous. The size of the cost depends on the

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[1] See, e.g., Bar-Ilan and Blinder (1992), Caballero (1994), and Eberly (1994). Abel et al. (1996) use lemons effects to motivate investment inertia.

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distribution of quality in the secondary market, and this distribution depends on the sales decisions of consumers. In spite of the obvious endogeneity of such costs, models of durables typically assume that the costs of trade are exogenous.

The model that we present is a standard *sS* model of car ownership, except that the adjustment cost arises endogenously from adverse selection in the secondary market. In the model, consumers care about both the quality of the car they own and how well the car fits their current needs. Only the owner of a car knows its true quality. Fit is captured by a match parameter. Consumers may sell cars either because the quality is low or because the car is not a good match. Because the quality of cars on the secondary market tends to be low, holders of relatively good cars face a cost to selling their car and purchasing a new one. This is the only adjustment cost in the model. The relative proportions of consumers selling on the basis of quality and fit will influence the size of this cost. The more high-quality cars there are on the secondary market, the lower the adjustment cost is.

The optimal adjustment policy is a state-contingent *sS* policy. Contingent on the age and the quality of the car, consumers continue to hold the car if the match parameter lies within some range about the optimum. The limits of this range of inaction, the consumers' *sS* bands, tend to be wider, the higher the quality of the auto. This reflects the adverse selection in the secondary market. Owners of high-quality autos have higher adjustment costs than owners of low-quality autos.

In Section III, we study the interaction between adverse selection and adjustment in a model in which cars last for only two periods. This setting illustrates the strategic complementarity between individual adjustment decisions. If some agents sell their cars in order to improve their match, then the quality of cars in the secondary market improves, the adjustment costs fall, and the incentive to trade increases. This feedback between sales decisions and the adjustment cost leads to the possibility of multiple equilibria. In some equilibria the *sS* bands are wide, adjustment by owners of high-quality cars is rare, and the lemons problem is severe. In others the *sS* bands are narrow, adjustment is common, and the lemons problem is mild.

Our first comparative static result concerns the effect of an increase in the variance of the shock to the match parameter. In *sS* models with exogenous fixed costs, an increase in this variance would lead agents to widen their bands and to adjust more often. In effect, they divide the cost of the increased variance between larger deviations from the optimal state and more frequent payment of the adjustment cost. In our model, however, there is an additional effect. More frequent adjustment means that more agents are selling good cars. Average quality in the secondary market improves and the adjustment cost falls. Hence the bands may narrow instead.

We also consider an increase in the variance of the unobserved quality of new cars. In contrast to the variance of the match parameter, increases in the variance of quality exacerbate the adverse selection problem. This increases the *sS* bands and reduces trade. We can therefore divide heterogeneity into two types: "good" heterogeneity, which induces trade and reduces adjustment costs, and "bad" heterogeneity, which causes adverse selection and increases the cost of adjustment. The kinds of shocks traditionally considered in the *sS* literature, such as shocks to tastes and income, fall in the former category.

In Section IV, we extend the life of cars. This allows us to observe what happens to the adverse selection problem as the durable ages. We find that the *sS* bands tend to narrow with age, so that adverse selection is initially severe but lessens with time. This effect comes from two sources. First, because the match between an owner and a car deteriorates over time, the incentive to adjust becomes greater as the car ages. Since there is more incentive to sell a good old car than a good new car, the distribution of quality in the secondary market tends to improve with age. This improvement causes the adjustment cost to fall and the *sS* bands to narrow as the car ages.

The second reason that the bands may narrow with age is that, if trading history is observable, then holding a car may serve as a signal that the car is high-quality. Since agents with good cars face higher adjustment costs and have wider *sS* bands, cars that remain unsold are more likely to be high-quality. Cars that are sold early are more likely to be lemons. This may explain the desirability of publicizing "original owner" in advertisements.

In both cases, the prospect that the market may improve with time creates a further incentive for agents to wait and sell at a later date. Agents know that as quality in the secondary market improves, so will the price that they can receive for their car. This effect also widens the bands for newer cars.

Together these effects can explain why the lemons problem might be more severe for new cars. New cars lose as much as 20 percent of their value as soon as they are driven off the lot (Stiglitz 1997, p. 433). There is understandably considerable reluctance to sell a new car. At the same time, few people appear to have similar trouble selling four- or five-year-old cars.

While we cast our model in terms of the used car market, the analysis has applications for any area in which fixed adjustment costs and adverse selection may interact, such as the market for equities, investment, or labor.

II. Related Literature

Our paper relates two literatures: *sS* adjustment and adverse selection. It is somewhat surprising that these literatures have not been brought together earlier. Each attempts to explain market inertia, and the market for consumer durables features prominently in both.

The *sS* model was developed by Arrow, Harris, and Marschak (1951) in the context of inventories and extended to the consumption of a durable good by Grossman and Laroque (1990). Although there is by now a large literature on durable adjustment, most of these models do not contain any equilibrium interactions (see, e.g., Bertola and Caballero 1990; Bar-Ilan and Blinder 1992; Caballero 1993; Eberly 1994; Carroll and Dunn 1997; Caplin and Leahy 1999; Leahy and Zeira 1999; Adda and Cooper 2000).

Papers that do allow for equilibrium interactions focus on the determination of the price of the commodity, not the adjustment cost. Among these papers, Stolyarov (2002) is the most closely related to our work.[2] The author considers an environment in which cars depreciate over time and trade is motivated by heterogeneity in the taste for quality. He shows which qualities are produced new and which are traded on secondary markets, and he solves for the prices that clear secondary markets. He does not, however, incorporate adverse selection. The costs of adjustment in his model are exogenous.

A novel feature of our model relative to most of the adverse selection literature is that the durable good may last for more than two periods.[3] The most closely related paper is Hendel and Lizzeri (1999). The authors construct a dynamic model with adverse selection in the used car market. They also motivate trade by assuming that agents differ in their taste for quality. They find that some agents refrain from selling high-quality used cars because of the cost imposed by adverse selection in the secondary market. They also demonstrate the possibility of multiple equilibria. There are several differences between our approach and theirs. First, we focus on the comparative statics of the *sS* bands. We constructed our model to differentiate the *sS* features of the model from the adverse selection features. Hendel and Lizzeri do not analyze the comparative statics of their adjustment thresholds, and their shock confounds changes in the motivation for trade with changes in adverse selection. Second, in Hendel and Lizzeri's paper, cars last for only two periods. Consequently, they cannot analyze the evolution of the *sS* thresholds over time.

Eisfeldt (2004) constructs a model of an equity market that has aspects

[2] Caplin and Leahy (1999) and Leahy and Zeira (1999) endogenize prices in *sS* models.

[3] A recent paper by Hendel, Lizzeri, and Siniscalchi (2003) explores longer-lived durables in a different setting.

of both *sS* behavior and adverse selection. Agents issue equity for one of two reasons: they know that the project is bad or they need the money. Higher aggregate productivity causes agents to increase the size of their investments, which has the effect of increasing the variance of their income. As this variance increases, more agents sell claims to high-quality projects, which reduces the adverse selection problem and improves the efficiency of the equity market. The fundamental properties of this mechanism are very similar to the mechanism at work in our static model of Section III. Eisfeldt, however, does not consider multiperiod projects and so cannot analyze how the threshold for issuing equity evolves over time.

The evidence of the importance of adverse selection in markets for used cars is mixed. Lacko (1986) and Genesove (1993) find evidence for adverse selection among older cars. Lacko compares the quality of cars purchased from family and friends with the quality of those purchased from newspaper ads. Genesove analyzes prices from dealer auctions. Bond (1982), on the other hand, compares the maintenance costs of trucks that are sold on the secondary market to the costs of those that are not. He finds no evidence of adverse selection. Hendel and Lizzeri (1999) find that the cross-sectional correlation of price and trade volume is more consistent with a model in which depreciation motivates trade rather than adverse selection. Adverse selection has been found to be important in other markets. Rosenman and Wilson (1991) find evidence of adverse selection in the wholesale market for fruit, and Chezum and Wimmer (1997) find evidence for adverse selection in the market for thoroughbred horses.

Two empirical results bear directly on our theoretical results. First, Genesove (1993) finds, in his study of auto auctions, that one-owner cars sell for roughly 9 percent more than cars with multiple owners. To our knowledge, ours is the first paper to rationalize this effect. Second, Stolyarov (2002) finds that trade volume is very low among cars that are less than two to three years old. It peaks at about four or five years and then levels off at a moderate level. Our model will capture this increase in trade volume over time.

Our theoretical results also have several empirical implications. One concerns the estimation of depreciation from market prices. If the adverse selection problem is initially severe, then the initial fall in price reflects a combination of depreciation and adverse selection. Physical depreciation is therefore overestimated initially. If adverse selection becomes less severe as cars age, then subsequent changes in price reflect both depreciation and the easing of adverse selection. Physical depreciation is therefore underestimated in later years.

A second empirical implication deals with the estimation of the effect of an increase in the variance of the shocks on the width of the adjustment triggers in an *sS* model. Eberly (1994) regresses the size of *sS* bands on the variance of an individual's income and finds a mildly positive coefficient on income variance. Our model provides a reason that her estimates may be biased toward zero. If individual income variance is correlated with the variance of shocks to the market, her estimates mix the traditional *sS* effect and the thick market effect that arises from adverse selection. To see this, suppose that there is a group of agents with high income variance and a group of agents with low income variance and that there are two distinct markets for cars (which may be differentiated by price, space, or time). If the two income variance types are randomly distributed across the two markets, then the econometrician will observe that in each market agents with high income variance will have wider bands. If agents with high income variance trade exclusively in one market, then there will tend to be more trade in that market and the adverse selection effect will temper the effect of variance on the adjustment bands. In this case, the estimate of the traditional *sS* effect would be biased downward. The finding of a mildly positive effect of income variance on the width of the bands may therefore mean either that the traditional *sS* effect is small or that there is a correlation across agents between income variance and the markets in which they participate.

III. The Model

Time is discrete and is indexed by $t \in \{0, 1, \ldots\}$. There is a continuum of infinitely lived consumers indexed by $i \in [0, 1]$, each of whom inelastically demands a single automobile. Consumers care about both the quality of the car they own and how well it meets their needs. Cars come in two types: good and bad. Consumers derive greater utility from good cars. Needs are reflected by a match parameter, z_{it}, which summarizes all other motivations for trade besides car quality, including, among other things, tastes, income, and demographics. A value of $z = 0$ reflects a perfect match, and the absolute value of z reflects the degree to which the car and its owner are mismatched. Each period t, each agent i receives utility

$$U_{it} = x_{it} - z_{it}^2 - \varphi_{it}.$$

Here x is a random variable taking the value one if the quality of the car is good and $\phi \in (0, 1)$ if the quality is bad, and φ_{it} represents net spending on cars. Consumers discount future payoffs by β.

Cars last for two periods. After two periods they depreciate completely and the owner must purchase another. Owners of one-year-old cars may also purchase another car, but to do so they must sell the car that they possess. In any period there are two markets in operation: a market for new cars and a market for (one-year-old) used cars.

New cars are supplied by dealers. We do not model the dealers' problem in detail. The only properties that we need are the price of a new car and the probability that a new car is good-quality.[4] Let p_0 denote the exogenous price of a new car, and let $\pi \in (0, 1)$ denote the exogenous probability that a new car is good-quality. It will also be useful to let $q_0 = \pi + (1 - \pi)\phi$ denote the expected quality of a new car.

The quality of any particular car, whether new or used, is the private information of the owner. This gives rise to the adverse selection problem that makes adjustment in this model interesting.

When consumers purchase a car, they choose one that is a perfect match. Thus a new or used car purchased in period t will have a match parameter $z_{it} = 0$. As time passes, the consumers' needs may change as their income, tastes, and family situation change. As a result, the match parameter may change. We assume that if a consumer bought a new car in period $t - 1$, the match parameter z_{it} is a random variable, whose distribution is described by a distribution function F on $\mathbb{R}$. We shall make assumptions on F as necessary in order to ensure uniqueness, continuity, or differentiability of a solution. At this point we assume only that the z_{it} are independent across time and across agents and that $\int_{-z}^{z} dF < 1$ for all finite $z > 0$. This last assumption ensures that some agents receive shocks to their match that are so bad that they adjust under any circumstances.

In period 0, one-half of the consumers begin with one-year-old cars. The rest begin without cars. The timing of moves in each period is as follows. At the beginning of period t, holders of one-year-old cars observe the match parameter z_{it}. Next the markets for new and used cars open simultaneously. After trade is completed, consumers observe the quality of the car they own (if they do not already know it) and realize utility U_{it}.

Holders of one-year-old cars choose whether to sell their used cars, and purchasers of cars decide between new and used cars. We look for a stationary symmetric Nash equilibrium as a solution to our model.

A. *Solution*

We solve the model under the assumption that there is positive demand for both new and used cars. This need not be the case. If p_0 is too high, consumers will not willingly purchase new cars. If p_0 is too low, consumers may prefer to scrap their used cars and then purchase new ones.

[4] Given the inelastic demand for autos, the price of new cars will be determined in equilibrium by the marginal cost of production.

At the end of the subsection we present a lemma that provides sufficient conditions for all markets to be active.[5]

Since all consumers purchasing cars are alike and since there is an active secondary market for used cars, consumers who are making adjustments must be indifferent in equilibrium between purchasing a new car and a used car. We use this fact to solve for the optimal adjustment strategies. We solve the model recursively. First, we solve for the optimal adjustment policy of the holder of a one-year-old car. This determines the average quality of cars in the used car market. We then solve for the price of used cars.

Let $V_1(x, z)$ denote the value of an optimal policy for an agent who enters the period holding a one-year-old car of quality x and a match z. The agent decides whether to keep the car or to purchase another car. Since in equilibrium the agent is indifferent between purchasing a new car and a used car, we may assume for the purpose of determining the optimal policy that the agent decides to purchase another one-year-old car. The value function becomes

$$V_1(x, z) = \max\{x - z^2 + \beta V_0,\ Q + \beta V_0\},$$

where $Q \in [\phi, 1]$ is the average quality of cars in the secondary market and V_0 is the value of purchasing a new car. The first term inside the braces is the value of holding on to the car. The second is the expected payoff from selling the car and buying another used car. Note that the price of used cars does not appear in this second term, since the agent both buys and sells a used car. Regardless of the consumer's decision, the car dies after one period, and the consumer is forced to purchase another car. For simplicity we assume that this is a new car (recall that the consumer will be indifferent). Since the continuation payoff is independent of the current choice, the consumer faces what is essentially a static decision: buy or hold depending on the current period's payoff.

Since $Q \in [\phi, 1]$, it follows immediately that holders of lemons always choose to adjust and holders of good used cars adjust if $|z| \geq Z$, where Z is the equilibrium cutoff:[6]

$$Z = \sqrt{1 - Q}. \tag{1}$$

Since every agent with a lemon adjusts but only some agents with good cars adjust, we know that the expected quality on the secondary market is less than the expected quality of new cars, $Q \leq q_0$. It follows that $Z > 0$. If F places positive probability on the neighborhood $(-Z, Z)$, then in equilibrium there will be a positive measure of agents who choose not to trade.

[5] See House and Leahy (2000) for an analysis of the model with scrapping.

[6] For simplicity we assume that agents adjust when indifferent.

In equilibrium, the quality of cars in the secondary market depends on the number of agents holding good used cars who decide to adjust; that is, Q depends on Z and F. Let λ_F denote the proportion of cars in the secondary market that are good-quality. Then

$$\lambda_F = \frac{(1 - \int_{-Z}^{Z} dF)\pi}{1 - \pi\int_{-Z}^{Z} dF} = \frac{2\pi F(-Z)}{(1-\pi) + 2\pi F(-Z)} \tag{2}$$

and

$$Q = \lambda_F + (1 - \lambda_F)\phi. \tag{3}$$

In order to prove that an equilibrium exists, it is useful to construct the mapping $T_F: [\phi, 1] \to [\phi, 1]$ as follows. Given $Q \in [\phi, 1]$, equations (1) and (2) pin down Z and λ_F recursively. Then, given λ_F, equation (3) defines $Q' \in [\phi, 1]$. We set $T_F(Q) = Q'$. The equilibrium level of Q is then a fixed point of T_F. Equilibrium levels of Z and λ_F follow from equations (1) and (2).

Existence of an equilibrium Q follows from the monotonicity of T_F and Tarski's fixed-point theorem. All proofs are contained in the Appendix.

PROPOSITION 1. The function T_F is nondecreasing and upper semicontinuous and has the fixed-point property.

Given Z, λ_F, and Q, the price in the secondhand market is determined by arbitrage between the new and used car markets:

$$V_0 = Q - p_1 + \beta V_0,$$

where p_1 is the price of a used car. When we solve for the price of used cars,

$$p_1 = Q - (1-\beta)V_0. \tag{4}$$

Predictably, the price of used cars is increasing in the quality and decreasing in the value of purchasing new cars.

Finally, we can solve for the value of purchasing a new car:

$$V_0 = \pi\left(1 + \beta\left\{2F(-Z)Q + [1 - 2F(-Z)]\left[1 - \frac{1}{1-2F(-Z)}\int_{-Z}^{Z} z^2 dF\right]\right\}\right) + (1-\pi)(\phi + \beta Q) + \beta^2 V_0 - p_0. \tag{5}$$

A new car purchased in period t is good with probability π. With probability $2F(-Z)$ the match worsens in period $t+1$ to the point that the agent sells the car. Given indifference between purchasing a new and a used car, we assume in this case that the agent purchases another used car. With probability $1 - 2F(-Z)$ the agent holds on to the car in $t+1$ and receives the conditional expectation of $x - z^2$. The new car is bad with probability $1 - \pi$, in which case the agent receives ϕ in period t and sells the car in period $t+1$. Whether the new car is good or bad, the price of the car in period t is p_0 and the agent purchases a new car in period $t+2$.

Together equations (1)–(5) determine Z, λ_F, Q, V_0, and p_1.

Up to this point, there is nothing to ensure that V_0 and p_1 are positive. It remains to present conditions under which consumers willingly purchase new cars and the used car market is active. Lemma 1 presents these conditions and shows that there are parameterizations of the model that satisfy them.

LEMMA 1. The following statements are true: (1) $p_0 \leq q_0 + \beta\phi$ is a sufficient condition for $V_0 > 0$, (2) $p_0 \geq (q_0 - \phi)(1 + \beta)$ is sufficient for $p_1 > 0$, and (3) $\phi \geq \frac{1}{2}$ is sufficient for there to exist a p_0 that satisfies both of these conditions.

The first condition ensures that V_0 is positive. It states that the p_0 is not so high that consumers do not wish to purchase cars. The initial price must be below $q_0 + \beta\phi$, which is the expected value of holding the average new car for one period and then trading it for a bad used car. The optimal strategy can do no worse than this. The second condition ensures that p_1 is positive and rules out scrapping. It relates the price of a new car p_0 to the maximum possible loss in quality from visiting the used car market, $q_0 - \phi$. It follows from equation (4) and the fact that

$$V_0 \leq \frac{q_0}{1-\beta} - \frac{p_0}{1-\beta^2}. \tag{6}$$

This condition says that an agent can expect to do no better than hold the average car with a perfect match.

This completes the solution to the model.

B. *Discussion*

The first thing to notice is that, in a lemons model, different agents face different incentives when contemplating adjustment. These incentives depend on the quality of their car. Owners of higher-quality cars face greater costs of adjustment. This contrasts with the fixed costs of adjustment normally imposed by *sS* models.

We can think of $1 - Q$ as the cost of adjustment faced by holders of good cars. The lower Q is, the greater the cost of adjustment and the wider the range of inaction, $(-Z, Z)$. Since $Q \geq \phi$, holders of lemons actually receive an adjustment subsidy. This is why they all adjust.

1. A Useful Graphical Analysis

We can depict the equilibria of the model as the intersections of two curves in the (Z, Q) plane. The first curve gives the quality on the secondary market that results from any choice of Z. When equations (2) and (3) are combined,

$$Q(Z) = \frac{2\pi F(-Z)}{(1-\pi) - 2\pi F(-Z)} + \frac{1-\pi}{(1-\pi) - 2\pi F(-Z)}\phi.$$

The weights on one and ϕ reflect the proportion of good and bad cars on the secondary market. For this reason we refer to $Q(Z)$ as the *distribution curve*. Note that $Q(Z)$ is monotonically decreasing in Z since the proportion of good cars is decreasing in Z. When $Z = 0$, all cars are traded, so $Q(0) = q_0$. When $Z = \infty$, no good cars are traded, so $Q(\infty) = \phi$.

The second curve gives the optimal choice of Z given Q:

$$Z(Q) = \sqrt{1-Q}.$$

We call this the *reaction curve*. The function $Z(Q)$ is monotonically decreasing in Q, with $Z(0) = 1$ and $Z(1) = 0$.

Figure 1 depicts these two curves. For simplicity we have placed Z^2 rather than Z on the x-axis. This makes the reaction curve linear. Both curves are decreasing in Z. When $Z = 0$, the distribution curve lies below the reaction curve. As $Z \to \infty$, the distribution curve eventually lies above the reaction curve. All equilibrium Z must lie in the interval $[\sqrt{1-q_0}, \sqrt{1-\phi}]$.

2. Multiple Equilibria

Recall that the fixed-point mapping T_F was nondecreasing. The monotonicity of T_F is a reflection of the positive feedback in adjustment. If agents believe that the quality of cars in the secondary market has improved, then the range of inaction narrows and quality in the secondary market improves. This strategic complementarity opens the possibility for multiple equilibria, a possibility that is confirmed by the following example.

EXAMPLE 1. Suppose that F describes a discrete probability distribution on $\{-\hat{z}, \hat{z}\}$, where $\hat{z} \in [\sqrt{1-q_0}, \sqrt{1-\phi})$. Then if agents believe that the average quality in the secondary market is ϕ, no holder of a good car adjusts and the expected quality in the secondary market is ϕ; if agents believe that the average quality in the secondary market is q_0, all holders of good cars adjust and the expected quality is q_0.

We can understand this multiplicity in the context of figure 1. The choice of F in example 1 implies that the distribution curve $Q(Z)$ is

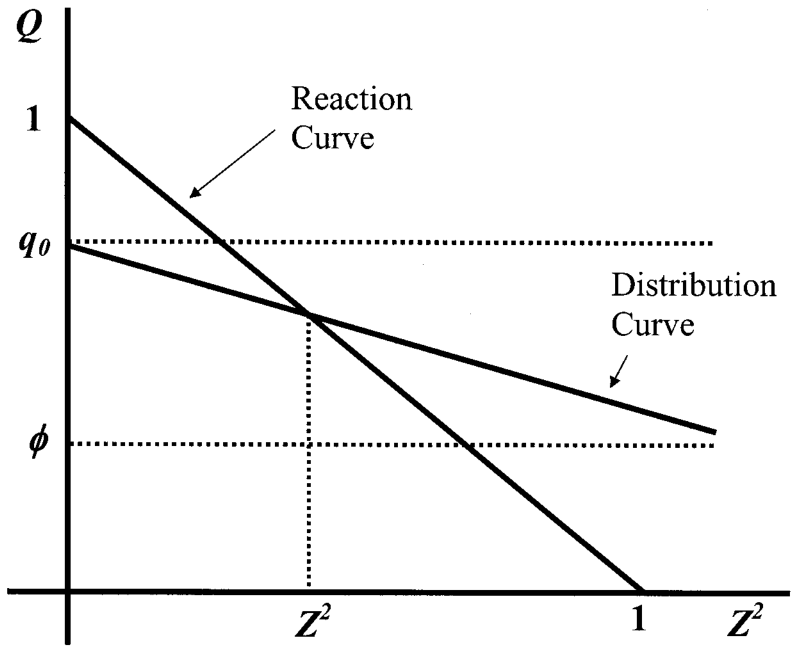

FIG. 1

equal to q_0 for all $Z \leq \hat{z}$ and equal to ϕ for all $Z > \hat{z}$. The distribution curve therefore intersects the reaction curve at two points: $Z = \{1 - q_0, 1 - \phi\}$.[7]

Ruling out such multiplicity simplifies the comparative statics. Assumption 1 presents a sufficient condition for a unique equilibrium.

ASSUMPTION 1. The distribution F has a density $f(z)$. The density $f(z)$ is symmetric about zero and single-peaked with

$$f(0) < \frac{1}{\sqrt{1-\phi}}\left(\frac{1-\pi}{\pi}\right).$$

Figure 1 can help to clarify the assumption. In order to rule out multiple equilibria, we need the distribution curve to be flat. There are two ways to do this. First, as the cutoff, Z, increases, some people with good cars who were adjusting now hold their cars; the bound on $f(0)$ ensures that this number is small so that the change in Q is small. Second, the bound on $f(0)$ is weaker for low π or high ϕ. The reason

[7] Recall that we have assumed that agents adjust when indifferent. If we allowed agents to randomize when indifferent, then all qualities between q_0 and ϕ would be possible at $\hat{z}$ and there would be a third intersection at $\hat{z}$.

is that the distribution curve must lie between ϕ and q_0. Reductions in π or increases in ϕ both narrow this interval, forcing the distribution curve to be flat regardless of the distribution F.

PROPOSITION 2. Given assumption 1, the equilibrium is unique.

3. Comparative Statics

We wish to analyze the effect of an increase in the dispersion of the shock on the size of the *sS* bands. Before presenting this result, we must first clarify what we mean by an "increase in dispersion." We borrow our definition of dispersion from Bickel and Lehmann (1979) and Sargent (1987, pp. 64–65).

DEFINITION 1. Consider two Borel probability measures μ and ν on $\mathbb{R}$. Suppose that both $E_\mu[x]$ and $E_\nu[x]$ are well defined and finite and that $E_\mu[x] = E_\nu[x] \equiv \bar{x}$. Then ν has *greater dispersion* than μ iff, for all x_1 and x_2 such that $x_1 \leq \bar{x} \leq x_2$, we have

$$\int_{-\infty}^{x_1} d\nu \geq \int_{-\infty}^{x_1} d\mu, \quad \int_{x_2}^{+\infty} d\nu \geq \int_{x_2}^{+\infty} d\mu.$$

If these inequalities are strict, then ν has strictly *greater dispersion* than μ.

This is the natural definition of dispersion in the context of *sS* adjustment, where the main issue that we care about is whether or not a shock takes the agent outside the *sS* bands. Intuitively, an increase in dispersion requires that the probability that the agent adjusts increases *regardless* of how the bands (about the mean) are defined. Notice that if ν is more disperse than μ, then it is also a mean-preserving spread of μ.[8]

With this definition in hand, we present the main result of this section.

PROPOSITION 3. If the equilibrium is unique, a (strict) increase in the dispersion of the shock to match quality ϵ leads to a (strict) reduction in the adjustment trigger Z and a (strict) increase in the quality of used cars Q.

Again figure 1 provides intuition. An increase in dispersion shifts the distribution curve in figure 1 up. Given any range of inaction $(-Z, Z)$, an increase in dispersion increases the probability that holders of good cars adjust. This adjustment leads to an increase in the average quality of cars in the secondary market. Since the reaction curve does not shift, Z falls and Q rises.[9]

[8] The converse does not necessarily hold.

[9] It is easy to see what happens if there are multiple equilibria. The distribution curve still rises. At all equilibria at which the distribution curve cuts from below (above), Z falls (rises) and Q rises (falls). In particular, Z falls at the two extreme equilibria.

This effect is the opposite of what we usually observe in *sS* models. Normally, an increase in the variance of the underlying shock increases the option value to waiting and causes the range of inaction to widen. Since holders of used cars in our model have a one-period horizon, the adjustment decision is essentially static. The standard effect therefore does not appear. In a model with more dynamics, such as the model of the next section, both effects would be present. In that case, we would not be able to sign the effect of an increase in the dispersion of the shock on the width of the *sS* bands.

4. Good Heterogeneity and Bad Heterogeneity

We have shown that an increase in the dispersion of matches reduces the width of the *sS* bands. There is another source of heterogeneity in our model, namely car quality. We can ask how an increase in dispersion of car quality affects the bands.

To model an increase in the dispersion of quality without affecting the mean quality, we let ϕ_h denote the quality of a good car (which was previously fixed at one) and ϕ_l denote the quality of a bad car (which was previously denoted by ϕ). All other aspects of the model are the same.

We consider an experiment in which we increase ϕ_h and reduce ϕ_l such that the average quality stays the same:

$$\pi\Delta\phi_h + (1 - \pi)\Delta\phi_l = 0.$$

Following the logic of subsection *A*, the cutoff is given by

$$Z = \sqrt{\phi_h - Q}.$$

The increase in dispersion has a direct effect on Z through ϕ_h and an indirect effect through Q. Both effects work to increase Z. Since the proportion of good cars on the secondary market is less than π, the increase in dispersion reduces Q for any fixed Z.

PROPOSITION 4. If the equilibrium is unique, a (strict) increase in the dispersion of car quality x leads to a (strict) increase in the adjustment trigger Z and a (strict) reduction in the quality of used cars Q.

Heterogeneity plays two roles in our model. Heterogeneity in incomes, tastes, or demographics motivates trade. Therefore, increases in this type of heterogeneity cause the *sS* bands to contract. Heterogeneity in quality reduces trade because of the adverse selection problem. Consequently, increases in this type of heterogeneity cause the bands to widen.[10]

IV. Dynamics

We now extend the life of the cars so that we can study how the *sS* bands evolve as cars age. We assume that cars last for three periods instead of just two periods. At times we shall find it convenient to make comparisons between the model in which cars last for three periods and the previous model in which cars lasted for only two periods. We refer to the previous model as the "two-period" model and to the models of this section as "three-period" models. We consider two information structures. In the first, the trading history is not observable, so that buyers cannot distinguish between two-year-old cars that have had two owners and those that have had only one owner. This version highlights the effect that deteriorating match quality has on the time profile of the adjustment triggers. In the second information structure, trading history is observable. This version illustrates the role of trade history in signaling quality and the effect that this signaling has on the evolution of the adjustment triggers over time.

A. *Unobservable Trading Histories*

We retain the basic structure of Section II, making alterations to allow for three-period-lived cars. First, if consumers cannot observe the history of ownership of the cars they buy, there will be three markets: a market for new cars, a market for one-year-old cars, and a market for two-year-old cars.

Second, we amend the matching function. As before, $z = 0$ during the first period in which an agent owns a car whether or not the car is new or used. In the second period of ownership, $z = \epsilon_1$, where ϵ_1 is distributed on $\mathbb{R}$ according to the distribution F. In the third period of ownership the fit is given by $z = \epsilon_1 + \epsilon_2$, where ϵ_2 is an additional independent draw from F. Note that without trading, matches will tend to deteriorate over time.

To simplify the analysis, we assume that F has a density f that is symmetric about zero and everywhere positive on $\mathbb{R}$. The fact that F has a density will allow us to use Brouwer's fixed-point theorem to prove existence. Symmetry ensures that upward and downward adjustments are the same. Positivity ensures that some agents receive shocks so large

[10] The only shock in Hendel and Lizzeri (1999) is a shock to car quality. Therefore, if they were to consider an increase in dispersion, it would reduce trade. We do not include depreciation in the model. Depreciation could encourage trade if it were observable or discourage trade if it were unobservable.

that they adjust, and thus supply in all three markets is positive. As we did before, we assume that there is a positive demand for new and used cars. At the end of the subsection we present a lemma that presents sufficient conditions for all markets to be active.

Agents choose whether or not to adjust, and when they adjust they choose what type of car to buy. The optimal adjustment strategies are described by threshold levels of z contingent on the age and quality of the car that the agent possesses. We look for a competitive equilibrium in which (1) the prices p_1 and p_2 clear the market for one- and two-year-old cars, respectively; (2) agents choose their thresholds optimally given prices and the expected qualities in the market for one- and two-year-old cars, q_1 and q_2; and (3) expectations are rational in that q_1 and q_2 are the average qualities of cars in the market for one- and two-year-old cars.

As in the two-period model, we fix the price of new cars at p_0 and the expected quality of new cars at q_0.

1. Solution

We begin by characterizing the adjustment triggers as a function of the quality of cars in the used car market. Consider first the decision of the agent who enters period t with a two-year-old car. Let V_2 denote the value of this agent's optimal policy. This value depends on the quality of the car, which is known to be $x \in \{\phi, 1\}$, and on the match parameter z. Given these state variables, the agent chooses whether or not to sell the car and, if the choice is to sell, what type of car to purchase. As in the two-period model, buyers will be indifferent between what type of car they buy in equilibrium. Without loss of generality, we equate the value of adjusting with the value of purchasing a two-year-old car. The value of an optimal policy is therefore

$$V_2(x, z) = \max\{x - z^2 + \beta V_0,\ q_2 + \beta V_0\}, \tag{7}$$

where V_0 is the value of purchasing a new car. The interpretation is essentially the same as in the two-period model. The first term represents the value of holding on to the current car, and the second term represents the value of adjusting.

Next consider the decision of an agent who enters period t with a one-year-old car. The quality of the car is known to be $x \in \{\phi, 1\}$, and the match parameter is z. In this case, the value of an optimal policy is

$$V_1(x, z) = \max\{x - z^2 + \beta E_\epsilon[V_2(x, z + \epsilon)],\ q_1 + \beta E_{(x',\epsilon)}[V_2(x', \epsilon)]\}, \tag{8}$$

where E_ω represents the mathematical expectation with respect to the distribution of the random variable ω. Again the first term is the value of holding and the second term is the value of adjusting (this time to another one-year-old car). In each case, the agent must form expectations concerning the evolution of the match. If the agent chooses to trade for another car, the agent must also form expectations over the quality of that car x'. Quality x' is drawn from the equilibrium distribution of qualities in the market for one-year-old cars.

Since some holders of good cars receive terrible matches and adjust, the expected quality on the secondary market is above that of a lemon. Therefore, all holders of lemons adjust in each period.

LEMMA 2. In any equilibrium, all agents with lemons adjust in every period regardless of the age of their car.

Let $\bar{z}_1$ and $\bar{z}_2$ denote the adjustment thresholds for holders of good one- and two-year-old cars. Equations (7) and (8) imply that an agent sells a good one-year-old car if $|z| \geq \bar{z}_1$, where

$$\bar{z}_1 = \sqrt{1 - q_1 + \beta E_{(x',\epsilon)}[V_2(1, \bar{z}_1 + \epsilon) - V_2(x', \epsilon)]}, \tag{9}$$

and a good two-year-old car if $|z| \geq \bar{z}_2$, where

$$\bar{z}_2 = \sqrt{1 - q_2}. \tag{10}$$

Equations (9) and (10) determine the adjustment thresholds as functions of q_1 and q_2. We now analyze the determination of these qualities. Consider first the market for one-year-old cars. Given $\bar{z}_1$, the mass of agents with high-quality cars who adjust is $2\pi F(-\bar{z}_1)$. Hence the proportion of good cars is

$$\lambda_1 = \frac{2\pi F(-\bar{z}_1)}{(1-\pi) + 2\pi F(-\bar{z}_1)} \leq \pi, \tag{11}$$

and the expected quality is

$$q_1 = \lambda_1 + (1 - \lambda_1)\phi. \tag{12}$$

The situation is slightly more complex in the market for two-year-old used cars. Given $\bar{z}_1$, the distribution of matches for two-year-old cars is

$$G_{\bar{z}_1}(z) = F(z)[2F(-\bar{z}_1)] + \int_{-\bar{z}_1}^{\bar{z}_1} F(z - \epsilon) f(\epsilon) d\epsilon, \tag{13}$$

with the associated density

$$g_{\bar{z}_1}(z) = 2f(z)F(-\bar{z}_1) + \int_{-\bar{z}_1}^{\bar{z}_1} f(z - \epsilon) f(\epsilon) d\epsilon. \tag{14}$$

The first term in (13) represents the z's of the agents who have held their cars for one period. Their match parameter is described by a single draw from F. The second term in (13) represents the z's of agents who have held their cars for two periods. Their match is represented by the sum of two draws from F, $\epsilon_1 + \epsilon_2$. As these agents chose not to sell their cars last period, ϵ_1 lies between $-\bar{z}_1$ and $\bar{z}_1$. The density of these realizations is $f(\epsilon_1)$. For $\epsilon_1 + \epsilon_2 \leq z$, it must be the case that $\epsilon_2 \leq z - \epsilon_1$. This gives the second term.

Given $\bar{z}_2$ and $G_{\bar{z}_1}(z)$, the mass of agents with two-year-old good cars is $2\pi G_{\bar{z}_1}(-\bar{z}_2)$. Hence the proportion of good cars in the two-year-old market is

$$\lambda_2 = \frac{2\pi G_{\bar{z}_1}(-\bar{z}_2)}{(1-\pi) + 2\pi G_{\bar{z}_1}(-\bar{z}_2)} \leq \pi, \tag{15}$$

and the expected quality is

$$q_2 = \lambda_2 + (1-\lambda_2)\phi. \tag{16}$$

In order to prove existence and analyze the properties of equilibrium, it is useful to construct the mapping $T: [\phi, 1] \times [\phi, 1] \rightarrow [\phi, 1] \times [\phi, 1]$ as follows. Given q_1, we can use (12) to define λ_1. Given q_1, q_2, and λ_1, we can solve (7), (9), and (10) implicitly for the adjustment triggers $\bar{z}_1$ and $\bar{z}_2$. Then given $\bar{z}_1$ and $\bar{z}_2$, equations (11) and (15) pin down λ'_1 and λ'_2. Finally, given λ'_1 and λ'_2, equations (12) and (16) define q'_1, $q'_2 \in [\phi, 1] \times [\phi, 1]$. We set $T(q_1, q_2) = (q'_1, q'_2)$. The equilibrium values of q_1 and q_2 arise as fixed points of T.

Existence of an equilibrium (q_1, q_2) follows from the continuity of T and Brouwer's fixed-point theorem.

PROPOSITION 5. There exists an equilibrium in the model with unobservable trading histories.

Given the equilibrium qualities and adjustment triggers, we can solve for the equilibrium prices. We first calculate the value of purchasing a new car. The value V_0 must satisfy

$$V_0 = q_0 + \beta E[V_1(x, \epsilon)] - p_0. \tag{17}$$

Note that, given q_1 and q_2, the existence of a solution V_0 to (17) follows from standard dynamic programming arguments.

The utility from purchasing in the new car market must be the same as that from purchasing in the two-year-old car market:

$$V_0 = q_2 - p_2 + \beta V_0.$$

This pins down p_2:

$$p_2 = q_2 - (1-\beta)V_0. \tag{18}$$

Similarly, the utility from purchasing in the new car market must be the same as from purchasing a one-year-old car:

$$V_0 = E[x + \beta(\max\{V_0 + p_2,\ x - \epsilon^2 + \beta V_0\})] - p_1.$$

This pins down p_1:

$$p_1 = E[x + \beta(\max\{V_0 + p_2,\ x - \epsilon^2 + \beta V_0\})] - V_0. \tag{19}$$

It remains to place assumptions on the parameters that ensure that V_0, p_1, and p_2 are all positive. The following lemma serves this purpose. The intuition for these conditions is similar to the intuition for lemma 1.

LEMMA 3. The following statements are true: (1) $p_0 \le q_0 + \beta\phi + \beta^2\phi$ is a sufficient condition for $V_0 > 0$; (2) $p_0 \ge (q_0 - \phi)(1 + \beta + \beta^2)$ is sufficient for $p_1 > 0$ and $p_2 > 0$; and (3) if $\phi \ge \frac{1}{2}$, then there exists a p_0 that satisfies both of these conditions.

2. Evolution of the Bands

In this subsection we analyze the equilibrium cutoffs $\bar{z}_1$ and $\bar{z}_2$. We find it useful to compare $\bar{z}_1$ and $\bar{z}_2$ to the cutoff Z in the two-period model. To simplify this comparison we shall assume that the parameters satisfy assumption 1, so that Z is unique.

Consider first the relationship between $\bar{z}_2$ and Z. Given any $\bar{z}_1$, $\bar{z}_2$ and q_2 are determined by a distribution curve and a reaction curve similar to the two curves in figure 1. In fact, the reaction curve is equation (10), which is the same as the reaction curve in the two-period model. The distribution curve is given by combining equations (15) and (16):

$$q_2 = \frac{2\pi G_{\bar{z}_1}(-\bar{z}_2)}{(1-\pi) + 2\pi G_{\bar{z}_1}(-\bar{z}_2)} + \left[1 - \frac{2\pi G_{\bar{z}_1}(-\bar{z}_2)}{(1-\pi) + 2\pi G_{\bar{z}_1}(-\bar{z}_2)}\right]\phi.$$

This is the same distribution curve as in the two-period model with the exception that $G_{\bar{z}_1}$ replaces F. Lemma 4 relates $G_{\bar{z}_1}$ to F. It shows that $G_{\bar{z}_1}$ is more disperse than F for any $\bar{z}_1$.

LEMMA 4. For any $\bar{z}_1 > 0$, the distribution $G_{\bar{z}_1}$ is weakly more disperse than F. Moreover, $g(0) \le f(0)$, so that if f satisfies assumption 1, then so does $g_{\bar{z}_1}$.

Since $G_{\bar{z}_1}$ satisfies assumption 1, the equilibrium $\bar{z}_2$ is unique. The fact that $G_{\bar{z}_1}$ is more disperse than F implies that the distribution curve that determines $\bar{z}_2$ is bounded below by the distribution curve that determined Z. This discussion establishes the following proposition.

PROPOSITION 6. Let f, π, ϕ, and β be given, and let f satisfy assumption 1. Then $\bar{z}_2 \le Z$ and $q_2 \ge Q$.

Proposition 6 says that because matches are worse after two periods of shocks than after only one, there will be a greater incentive to trade. The increased trade causes increased quality.

Now consider the relationship between $\bar{z}_1$ and Z. Again we can think in terms of a distribution curve and a reaction curve. The distribution curve is the same as in the two-period model. The reaction curve is now given by

$$\bar{z}_1 = \sqrt{1 - q_1 + \beta E_{(x',\epsilon)}[V_2(1, \bar{z}_1 + \epsilon) - V_2(x', \epsilon)]}.$$

If it were not for the term $E_{(x',\epsilon)}[V_2(1, \bar{z}_1 + \epsilon) - V_2(x', \epsilon)]$, the reaction curve would also be the same as in the two-period model. Unfortunately, the sign of this term is ambiguous. The term represents the trade-off between entering the last period with a good car and entering the last period with a good match. The following proposition presents a sufficient condition under which this term is positive. If this condition is satisfied, then the reaction curve associated with $\bar{z}_1$ is bounded below by the reaction curve in the two-period model. It follows that in this case $\bar{z}_1 \geq Z$.

PROPOSITION 7. Let f and ϕ be given and let

$$\pi < \min\left\{\pi^*(f, \phi), \frac{1}{f(0)\sqrt{1-\phi}+1}\right\},$$

where $\pi^*(f, \phi)$ is

$$\pi^*(f, \phi) = 1 - \sqrt{1 - \min\left\{\frac{1}{1-\phi}\int_0^{2\sqrt{1-\phi}} [2(\sqrt{1-\phi})\epsilon - \epsilon^2]f(\epsilon)d\epsilon,\ 1\right\}}$$
$$> 0.$$

$$\pi^*(f, \phi) = 1 - \sqrt{1 - \min\left\{\frac{1}{1-\phi}\int_0^{2\sqrt{1-\phi}} [2(\sqrt{1-\phi})\epsilon - \epsilon^2]f(\epsilon)d\epsilon,\ 1\right\}}$$
$$> 0.$$

Then $\bar{z}_1 \geq Z \geq \bar{z}_2$ and $q_1 \leq Q \leq q_2$.

Proposition 7 places a bound on π. Low π increases the value of having a good car by reducing the probability of finding another good car on the secondary market. This raises $EV_2(1, \bar{z}_1 + \epsilon)$ relative to $EV_2(x', \epsilon)$.

While it may seem natural that $\bar{z}_1 \geq Z \geq \bar{z}_2$, this is not a necessary outcome of this model. We now present an example in which $\bar{z}_1 < \bar{z}_2 = Z$.

EXAMPLE 2. Suppose that F has mean zero, mass .5 at zero, and mass .5 distributed uniformly over the range $[-10, 10]$. Suppose also that $\pi = \phi = .5$ and $\beta = .95$. The unique equilibrium is given by $\bar{z}_1 = .5548$ and $Z = \bar{z}_2 = .5830$.

There are two notable aspects to this example. First, $Z = \bar{z}_2$. The reason for this is that while lemma 4 says that $G_{\bar{z}_1}$ is more disperse than F, it does not say that it is everywhere strictly more disperse than F. The mass at zero and the uniform distribution on $[-10, 10]$ imply that $G_{\bar{z}_1}(z)$ and $F(z)$ are equal on $[\bar{z}_1 - 10, -\bar{z}_1] \cup [\bar{z}_1, 10 - \bar{z}_1]$. This range includes $\bar{z}_2$.

The other notable feature is that $\bar{z}_1 < \bar{z}_2$. The wide uniform component of F and the mass point at zero also figure in this result.[11] The wide uniform component ensures that there is a significant amount of adjustment. This raises the probability of finding a good car on the secondary market. The mass of .5 at zero implies that there is substantial persistence to the matches. Both of these factors raise $V_2(x', \epsilon)$ relative to $V_2(1, \bar{z}_1 + \epsilon)$.

B. *Observable Trading Histories*

If the trading history of a car is observable to the buyers, there will be four markets: new cars, one-year-old used cars, two-year-old cars with original owners, and two-year-old cars with new owners. As before, we use subscripts to distinguish a car's age. Because there are two markets for two-year-old cars, we use the superscript "orig" to indicate the original owner market and the superscript "new" to indicate the new owner market. The prices in each market are denoted $\{p_0, p_1, p_2^{\text{orig}}, p_2^{\text{new}}\}$. To distinguish the cutoffs in this model from the cutoffs in the previous models, we denote the cutoff matches for each market with an $\bar{s}$.

We solve the model backward, beginning with two-year-old cars. The reasoning is similar to that of the two-period model. Given $j \in \{\text{orig}, \text{new}\}$, the value functions are

$$V_2^j(x, z) = \max\{x - z^2 + \beta V_0,\ q_2^j + \beta V_0\}. \tag{20}$$

The first term in the maximum gives the value of holding on to the current car, and the second term gives the value of adjusting. Any agents holding a two-year-old lemon will choose to adjust, since they cannot trade for a worse car, and they will improve their match. The adjustment triggers for holders of good cars are given by

$$\bar{s}_2^j = \sqrt{1 - q_2^j}. \tag{21}$$

[11] Danziger (1999) has used this distributional assumption to simplify the aggregation of a dynamic sS model.

Working backward gives us the value of owning a one-year-old car of quality x with current shock z as

$$V_1(x, z) = \max\{x - z^2 + \beta E_\epsilon[V_2^{\text{orig}}(x, z + \epsilon)], \; q_1 + \beta E_{x',\epsilon}[V_2^{\text{new}}(x', \epsilon)]\}.$$

Again the first term gives the value of holding, and the second term gives the value of adjusting. It is no longer the case that agents with one-year-old lemons will necessarily adjust. It may be more profitable to enter the second period with a two-year-old car that is firsthand. We therefore must calculate triggers for both types of car. The trigger for high-quality car owners is now

$$\bar{s}_1 = \sqrt{1 - q_1 + \beta[E_\epsilon V_2^{\text{orig}}(1, \bar{s}_1 + \epsilon) - E_{x',\epsilon} V_2^{\text{new}}(x', \epsilon)]}. \tag{22}$$

Let $\hat{s}_1$ denote the adjustment trigger for holders of one-year-old lemons. Then we have

$$\hat{s}_1 = \sqrt{\max\{0, \phi - q_1 + \beta[E_\epsilon V_2^{\text{orig}}(\phi, \hat{s}_1 + \epsilon) - E_{x',\epsilon} V_2^{\text{new}}(x', \epsilon)]\}}.$$

Because agents with two-year-old lemons always adjust, $E_\epsilon V_2^{\text{orig}}(\phi, \hat{s}_1 + \epsilon) = q_2^{\text{orig}} + \beta V_0$, and

$$\hat{s}_1 = \sqrt{\max\{0, \phi - q_1 + \beta[q_2^{\text{orig}} + \beta V_0 - E_{x',\epsilon} V_2^{\text{new}}(x', \epsilon)]\}}. \tag{23}$$

The value of $\hat{s}_1$ is greater than zero when the value of suffering with a bad car today and selling it as an original owner tomorrow, $\phi + \beta q_2^{\text{orig}} + \beta^2 V_0$, exceeds the value of selling a lemon.

Given these adjustment triggers, we now characterize the quality of cars traded on the secondhand markets. We consider the three markets in turn. Consider first the market for one-year-old cars. Given $\bar{s}_1$ and $\hat{s}_1$, the total mass of agents with high-quality cars who adjust is $2\pi F(-\bar{s}_1)$, and the total mass of agents with low-quality cars who adjust is $2\pi F(-\hat{s}_1)$. Hence the proportion of good cars in the market is

$$\lambda_1 = \frac{\pi F(-\bar{s}_1)}{(1 - \pi)F(-\hat{s}_1) + \pi F(-\bar{s}_1)}, \tag{24}$$

and the expected quality is

$$q_1 = \lambda_1 + (1 - \lambda_1)\phi. \tag{25}$$

Consider now the market for two-year-old cars with new owners. Given $\bar{s}_2^{\text{new}}$, the fraction of agents with high-quality cars who adjust is $2F(-\bar{s}_2^{\text{new}})$. As agents with good cars make up a fraction λ_1 of new car owners and since the remaining $1 - \lambda_1$ who own bad cars all adjust, the proportion of good cars in this market is

$$\lambda_2^{\text{new}} = \frac{2F(-\bar{s}_2^{\text{new}})\lambda_1}{(1 - \lambda_1) + 2\lambda_1 F(-\bar{s}_2^{\text{new}})}. \tag{26}$$

Consider finally the market for two-year-old cars with original owners. Originally, a fraction π of agents receive good-quality cars and a fraction $1-\pi$ receive bad-quality cars. Of the agents who receive good-quality cars, only those owners for whom $\epsilon_1 \in [-\bar{s}_1, \bar{s}_1]$ retain their cars when they are one year old. Given $\bar{s}_1$, we can calculate the measure of matches for original owners of good two-year-old cars. This measure is

$$H_{\bar{s}_1}(z) = \pi \int_{-\bar{s}_1}^{\bar{s}_1} F(z-\epsilon_1) f(\epsilon_1) d\epsilon_1. \tag{27}$$

Note that the integration is over ϵ_1 and that the mass of these agents is equal to $\pi[1-2F(-\bar{s}_1)]$. The number of these agents who adjust is $2H_{\bar{s}_1}(-\bar{s}_2^{\text{orig}})$. Of the agents who originally receive bad cars, a fraction $1-2F(-\hat{s}_1)$ retain their cars when they are one year old. All these agents adjust in the next period. The total number of original owners of bad two-year-old cars who put their cars on the market is therefore $(1-\pi)[1-2F(-\hat{s}_1)]$. It follows that the fraction of good cars in this market is

$$\lambda_2^{\text{orig}} = \frac{2H_{\bar{s}_1}(-\bar{s}_2^{\text{orig}})}{(1-\pi)[1-2F(-\hat{s}_1)] + 2H_{\bar{s}_1}(-\bar{s}_2^{\text{orig}})}. \tag{28}$$

We can now calculate the average quality in each second-year market:

$$q_2^j = \lambda_2^j + (1-\lambda_2^j)\phi. \tag{29}$$

Finally, the value of purchasing a new car, V_0, must satisfy

$$V_0 = q_0 + \beta E[V_1(x, z)] - p_0.$$

The prices in the second period are

$$p_2^{\text{orig}} = q_2^{\text{orig}} - (1-\beta)V_0$$

and

$$p_2^{\text{new}} = q_2^{\text{new}} - (1-\beta)V_0.$$

Again, the utility from purchasing in the new car market satisfies

$$V_0 = E[x + \beta(\max\{V_0 + p_2^{\text{new}},\ x - \epsilon^2 + \beta V_0\})] - p_1.$$

This pins down p_1:

$$p_1 = E[x + \beta(\max\{V_0 + p_2^{\text{new}},\ x - \epsilon^2 + \beta V_0\})] - V_0.$$

Existence of an equilibrium is established in a manner similar to that in the previous section.

Proposition 8. There exists an equilibrium in the model with observable trading histories.

It remains to present conditions that ensure that V_0 and all prices are positive. Lemma 5 presents sufficient conditions for all markets to be active. We omit the proof since it is essentially identical to that of lemma 3.

LEMMA 5. The following statements are true: (1) $p_0 \leq q_0 + \beta\phi + \beta^2\phi$ is a sufficient condition for $V_0 > 0$; (2) $p_0 \geq (q_0 - \phi)(1 + \beta + \beta^2)$ is sufficient for p_1, p_2^{new}, and p_2^{orig} all positive; and (3) $\phi \geq \frac{1}{2}$ is sufficient for there to exist a p_0 that satisfies both of these conditions.

1. Properties of Equilibrium

The following proposition states the main results of this section.

PROPOSITION 9. Let f, π, ϕ, and β be given and let f satisfy assumption 1. Then every equilibrium in the observable case satisfies $\hat{s}_1 < \bar{s}_1$, $\bar{s}_2^{\text{orig}} \leq Z$, and $Z \leq \{\bar{s}_2^{\text{new}}, \bar{s}_1\}$. Moreover, there exist parameterizations for which $\hat{s}_1 > 0$.

The intuition underlying this proposition is natural. First, people with one-year-old lemons have a greater incentive to adjust than people with one-year-old good cars. Thus $\hat{s}_1 < \bar{s}_1$. Because people with lemons are more likely to trade, the proportion of good cars in the new car market is less than π and the proportion of good cars in the original owner market is greater than π. For this reason, turnover will be a signal of car quality: original owner cars will tend to be higher-quality than new owner cars. This signaling imposes an adjustment cost on holders of lemons. It is now possible that holders of lemons who have good matches will refrain from trade in order to take advantage of the higher-quality cars in the original owner market.

Consider now the new owner market. The equilibrium is characterized by the familiar two curves. The reaction curve is the same as in the two-period model. The distribution curve is given by

$$q_2^{\text{new}} = \frac{2F(-\bar{s}_2^{\text{new}})\lambda_1}{(1-\lambda_1) + \lambda_1 2F(-\bar{s}_2^{\text{new}})} + \frac{1-\lambda_1}{(1-\lambda_1) + \lambda_1 2F(-\bar{s}_2^{\text{new}})}\phi.$$

This is the same as the distribution curve in the two-period model except that λ_1 replaces π. Since $\lambda_1 < \pi$, this curve lies everywhere below the distribution curve in the two-period model. Hence, $Z \leq \bar{s}_2^{\text{new}}$.

Now compare the original owner market to the two-period model. Again, the reaction curves are the same. The distribution curve is given by

$$q_2^{\text{orig}} = \frac{2H_{\bar{s}_1}(-\bar{s}_2^{\text{orig}}) + \phi(1-\pi)[1-2F(-\hat{s}_1)]}{(1-\pi)[1-2F(-\hat{s}_1)] + 2H_{\bar{s}_1}(-\bar{s}_2^{\text{orig}})}.$$

There are two differences between this curve and the distribution curve in the two-period model. First, H_{s_1} takes the place of πF. The same arguments that showed that $G_{\bar{z}_1}$ is more disperse than F imply that H_{s_1} is more disperse than F. That is, $H_{s_1}(-z) \geq \pi F(-z)$ (for $z > 0$). Second, $1 - 2F(-z) \leq 1$ for $z > 0$. Both of these differences shift the distribution curve up relative to the distribution curve in the two-period model. Hence, $\bar{s}_2^{\text{orig}} \leq Z$.

Finally, consider the first-period market. The distribution curve is the same as in the two-period model. To see that the reaction curve shifts down, we must consider two possibilities: either some agents with lemons choose to hold their cars or all holders of lemons adjust. If they all adjust, then there are no lemons with original owners and the quality in the original owner market is one. It follows that $V_2^{\text{orig}}(1, \bar{s}_1 + \epsilon) = 1 + \beta V_0$ since any agent in this market can trade and receive a perfect match. It follows that $E_\epsilon V_2^{\text{orig}}(1, \bar{s}_1 + \epsilon) - E_{x',\epsilon} V_2^{\text{new}}(x', \epsilon) > 0$, so the reaction curve shifts up and $\bar{s}_1 > Z$. If instead some agents with lemons decide to hold on to their cars, the reason must be that the original owner market is much better than the new owner market. Again $E_\epsilon V_2^{\text{orig}}(1, \bar{s}_1 + \epsilon) - E_{x',\epsilon} V_2^{\text{new}}(x', \epsilon) > 0$. One can show that, in this case, $\bar{s}_1 > \sqrt{1 - \phi} > \hat{s}_1 > Z$.

V. Efficiency

In this section we discuss the efficiency properties of our model and compare the equilibria to the equilibrium under perfect information.

If car quality were observable, then all agents would trade every period in order to guarantee themselves a perfect match. The secondary market price would adjust so that agents would be indifferent between buying good and bad used cars. The ex ante value of an optimal policy in this case is

$$V^* = \frac{q_0}{1 - \beta} - \frac{p_0}{1 - \beta^n}.$$

In our model, with unobservable quality, inefficiency arises as a result of a lack of trade. The equilibrium is not informationally constrained Pareto optimal, since a social planner could achieve the first-best simply by forcing all agents to trade. In each period, people would get a car of average quality and a perfect match. The value of such an arrangement would be V^*.

Another way to implement the first-best solution would be to lease a car. Agents would rent a car for one period under the condition that they trade that car in at the end of the period. In practice, however, standard leasing contracts tend not to look like this. They frequently contain a clause that allows the owner to purchase the leased car at a predetermined price. As long as the price is not so high that agents choose never to keep their car, the adverse selection problem will return. Some agents will keep good cars in spite of imperfect matches. This raises the question of whether the optimal leasing contract is renegotiation proof. Under the optimal leasing contract, an agent trades in his car in return for a car of average quality (recall that quality is unobservable by all agents other than the owner). An agent with a good car and with a match $z < \sqrt{1 - q_0}$ has an incentive to make a side payment to the leasing company in order to keep the car. The leasing company has an incentive to accept the side payment. What form a renegotiation-proof equilibrium with leasing takes is an interesting question that is beyond the scope of this paper.[12]

In the model with unobservable trading history, the only misallocation relative to perfect information was that some agents with good cars failed to trade. In the model with observable history, there is an additional misallocation in the one-year-old market. Some of the agents with lemons decide not to trade. The presence of signaling introduces an additional opportunity cost to adjustment for all agents.[13] In the second period, however, there may be a gain in efficiency. As agents learn which cars are good and which are bad, the adverse selection problem is reduced and trade incidence increases.

VI. Conclusion

The used car market features so prominently in both the literature on adverse selection and the literature on *sS* adjustment that it is surprising that existing models of this market have not, to this date, incorporated both features. We presented a model in which *sS* adjustment arose from an adverse selection problem. The presence of adverse selection creates a complementarity between agents' adjustment decisions. This has several implications for the nature of the equilibrium *sS* policies. As the variance of the shock process increases, more agents adjust and the *sS* bands shrink. As the car ages, matches deteriorate, more adjustment takes place, and the *sS* bands tend to shrink.

Appendix

Proofs of the Propositions

[12] Hendel and Lizzeri (2002) study an adverse selection model with leasing.

[13] In contrast, Hendel et al. (2003) find that observable trading histories improve efficiency. In their model, agents differ in their taste for quality. Trading history provides information about quality and helps improve the match between agents and their durable.

Proof of Lemma 1

To prove statement 1, note that $V_0 \geq q_0 + \beta\phi + \beta^2 V_0 - p_0$. The optimal strategy can do no worse than the value of purchasing a new car, holding it for one period, and then trading it for a bad used car. The statement follows immediately from this condition.

To prove statement 2, note that equations (4) and (6) imply that

$$p_1 \geq Q - q_0 + \frac{p_0}{1+\beta}.$$

The statement follows immediately from the fact that $Q \geq \phi$.

Statement 3 follows immediately from the observation that

$$(q_0 + \beta\phi) - (q_0 - \phi)(1+\beta) = \phi + \beta(2\phi - q_0)$$

and the fact that $q_0 \leq 1$. Q.E.D.

Proof of Lemma 2

Consider first holders of two-year-old lemons. They choose either $q_2 + \beta V_0$ if they adjust now or $\phi - z^2 + \beta V_0$ if they hold on to the car. Since $q_2 \geq \phi$, they adjust. Now consider holders of one-year-old lemons. They receive $q_1 + \beta E_{(x,\epsilon)}[V_2(x, \epsilon)]$ if they adjust and $\phi - z^2 + \beta E_\epsilon[V_2(\phi, z+\epsilon)]$ if they hold. Since holders of two-year-old lemons adjust, $V_2(\phi, z+\epsilon) = q_2 + \beta V_0$. It follows from (7) that $E_{(x,\epsilon)}[V_2(x, \epsilon)] \geq q_2 + \beta V_0$. Since $q_2 \geq \phi$, they adjust. Q.E.D.

Proof of Lemma 3

To prove statement 1, note that $V_0 \geq q_0 + \beta\phi + \beta^2\phi + \beta^3 V_0 - p_0$. The optimal strategy can do no worse than purchasing a new car, trading this car for a bad one-year-old used car in the next period, and then trading that car for a bad two-year-old used car in the next period before buying a new car again. The statement follows immediately from this condition.

To prove that condition 2 implies $p_2 > 0$, first note that together equations (17), (8), and (7) imply that

$$V_0 < \frac{q_0}{1-\beta} - \frac{p_0}{1-\beta^3}. \tag{A1}$$

Given the deterioration of the match quality, it is not possible to do better than purchasing the average car every three periods. This condition, combined with equation (18), implies

$$p_2 \geq q_2 - q_0 + \frac{p_0}{1+\beta+\beta^2}.$$

The statement follows immediately from the fact that $q_2 \geq \phi$.

To prove that condition 2 also implies $p_1 > 0$, note that equation (19), $q_1 > \phi$, and $q_2 > \phi$ jointly imply $p_1 \geq \phi - (1-\beta)V_0$. The statement follows immediately from (A1).

Statement 3 follows immediately from the observation that

$$(q_0 + \beta\phi + \beta^2\phi) - (q_0 - \phi)(1+\beta+\beta^2) = \phi + (\beta+\beta^2)(2\phi - q_0)$$

and the fact that $q_0 \leq 1$. Q.E.D.

Proof of Lemma 4

Given that $G_{\bar{z}_1}$ and F are symmetric about zero, it is sufficient to show that, for $z \leq 0$, $G_{\bar{z}_1}(z) \geq F(z)$.

Let $z < 0$. If we divide through by $F(z)$, (13) becomes

$$\frac{G_{\bar{z}_1}(z)}{F(z)} = 2F(-\bar{z}_1) + \int_{-\bar{z}_1}^{\bar{z}_1} \frac{F(z-\epsilon_1)}{F(z)} f(\epsilon_1) d\epsilon_1.$$

We can rewrite the integral on the left-hand side as

$$\int_{-\bar{z}_1}^{\bar{z}_1} \frac{F(z-\epsilon_1)}{F(z)} f(\epsilon_1) d\epsilon_1 = \int_0^{\bar{z}_1} \frac{F(z+\epsilon_1) + F(z-\epsilon_1)}{F(z)} f(\epsilon_1) d\epsilon_1.$$

We now argue that, for $\epsilon \in [0, \bar{z}_1]$ and $z < 0$,

$$\tfrac{1}{2}[F(z+\epsilon_1) + F(z-\epsilon_1)] \geq F(z).$$

If $z + \epsilon_1 < 0$, the result follows immediately from our assumptions on f, which imply that F is convex over the interval $(-\infty, 0]$.

The case in which $z + \epsilon_1 > 0$ is more complex since F is convex at $z - \epsilon_1$ and concave at $z + \epsilon_1$. Consider any z such that $-\bar{z}_1 < z \leq 0$ and any ϵ_1 for which $z + \epsilon_1 > 0$, and consider the point $-z + \epsilon_1 > z + \epsilon_1$. Define the line $L(x)$ as follows:

$$L(x) = \left[\frac{-x - z + \epsilon_1}{2(-z+\epsilon_1)}\right] F(z-\epsilon_1) + \left[\frac{x - z + \epsilon_1}{2(-z+\epsilon_1)}\right] F(-z+\epsilon_1).$$

Because f is symmetric, $L(0) = \frac{1}{2}$. Because F is concave on $[0, \infty)$, any $x \in [0, -z + \epsilon_1]$ will have $F(x) \geq L(x)$. Because F is convex on $(-\infty, 0]$, any $x \in [z - \epsilon_1, 0]$ will have $F(x) \leq L(x)$. Take $x = z + \epsilon_1$ so that $0 < x < -z + \epsilon_1$. Then $F(z + \epsilon_1) \geq L(z + \epsilon_1)$. Now define the line $l(x)$ as

$$l(x) = \left(\frac{-x + z + \epsilon_1}{2\epsilon_1}\right) F(z-\epsilon_1) + \left(\frac{x - z + \epsilon_1}{2\epsilon_1}\right) F(z+\epsilon_1).$$

Note that $L(z-\epsilon_1) = l(z-\epsilon_1) = F(z-\epsilon_1)$ and $L(z+\epsilon_1) \leq F(z+\epsilon_1) = l(z+\epsilon_1)$. As a result, $l(x) \geq L(x)$ for any $x \in [z - \epsilon_1, \infty]$. We now have

$$\tfrac{1}{2}[F(z+\epsilon_1) + F(z-\epsilon_1)] = l(z) \geq L(z) \geq F(z).$$

Since $F(z+\epsilon_1) + F(z-\epsilon_1) \geq 2F(z)$ for any $z \leq 0$, we have

$$\int_{-\bar{z}_1}^{\bar{z}_1} \frac{F(z-\epsilon_1)}{F(z)} f(\epsilon_1) d\epsilon_1 \geq \int_0^{\bar{z}_1} \frac{2F(z)}{F(z)} f(\epsilon_1) d\epsilon_1 = 2[F(\bar{z}_1) - F(0)].$$

Using the fact that $F(0) = \frac{1}{2}$, we have

$$\frac{G_{\bar{z}_1}(z)}{F(z)} \geq 2F(-\bar{z}_1) + 2[1 - F(-\bar{z}_1) - \tfrac{1}{2}] = 1$$

so that $G_{\bar{z}_1}(z) \geq F(z)$ for any $z \leq 0$. Thus $G_{\bar{z}_1}$ is more disperse than F. Q.E.D.

Proof of Proposition 1

Consider Q, $Q' \in [\phi, 1]$ with $Q < Q'$. It follows from (1) that $Z(Q) > Z(Q')$. It follows from (2) that $\lambda_F(Z(Q)) \leq \lambda_F(Z(Q'))$; the inequality is strict if F places positive probability on the set $[-Z(Q'), -Z(Q)) \cup (Z(Q), Z(Q')]$. It follows from (3) that $T_F(Q) \leq T_F(Q')$. Therefore, $T_F(Q)$ is nondecreasing in Q. We conclude that a fixed point exists by Tarski's fixed-point theorem. Upper semicontinuity follows from our assumption that, when indifferent, agents adjust. Q.E.D.

Proof of Proposition 2

Combining equations (1), (2), and (3), we construct the mapping $Z \rightarrow Z'$ defined by

$$Z' = \sqrt{\frac{(1-\pi)(1-\phi)}{1-\pi+2\pi F(-Z)}}.$$

Any equilibrium cutoff is a fixed point of this mapping. Consider

$$\frac{dZ'}{dZ} = \sqrt{\frac{(1-\pi)(1-\phi)}{1-\pi+2\pi F(-Z)}}\,\frac{\pi f(-Z)}{1-\pi+2\pi F(-Z)}$$

$$\leq \frac{\pi}{1-\pi} f(-Z)\sqrt{1-\phi}.$$

Assumption 1 implies $dZ'/dZ < 1$, which proves the proposition. Q.E.D.

Proof of Proposition 3

Consider two densities f and g that have the same mean. Suppose that f is more disperse than g in the sense of definition 1. Consider $Q \in [\phi, 1]$. Let $Z(Q)$ be defined by (1). Since f is more disperse than g,

$$\int_{-Z(Q)}^{Z(Q)} g(z)dz \geq \int_{-Z(Q)}^{Z(Q)} f(z)dz,$$

which implies that $\lambda_g(Z(Q)) \leq \lambda_f(Z(Q))$. It follows that $T_g(Q) \leq T_f(Q)$ for all Q. Let Q^* be the fixed point under g. Since $T_f(Q^*) \geq T_g(Q^*) = Q^*$ and since T_g is nondecreasing, the unique fixed point under f must lie in the interval $[Q^*, 1)$. Note that if f is strictly more disperse so that

$$\int_{-x}^{x} g(z)dz > \int_{-x}^{x} f(z)dz$$

for any x, then the unique fixed point under f must be strictly greater than Q^*. Q.E.D.

Proof of Proposition 4

We define the mapping $T_F(Q;\ \phi_h, \phi_l)$ as in Section III*A* with the obvious modifications. We write $T_F(\cdot;\ \phi_h, \phi_l)$ to indicate that this mapping depends on the parameters ϕ_h and ϕ_l. As before, this mapping is nondecreasing and upper semicontinuous and has the fixed-point property. Moreover, we assume that there is a unique point at which $T_F(Q;\ \phi_h,\ \phi_l) = Q$. Consider $\phi_h^1 < \phi_h^2$ and $\phi_l^1 > \phi_l^2$ such that $\pi\phi_h^1 + (1-\pi)\phi_l^1 = \pi\phi_h^2 + (1-\pi)\phi_l^2$.

We first show that $T_F(Q;\ \phi_h^1,\ \phi_l^1) > T_F(Q;\ \phi_h^2,\ \phi_l^2)$. Fix Q. Let $Z(Q,\ \phi_h) = \sqrt{\phi_h - Q}$. Then $Z(Q,\ \phi_h^1) < Z(Q,\ \phi_h^2)$ implies that $\lambda_F(Z(Q,\ \phi_h^1)) > \lambda_F(Z(Q,\ \phi_h^2))$. This together with the fact that $\lambda_F(Z(Q,\ \phi_h^2)) < \pi$ and $\pi\phi_h^1 + (1-\pi)\phi_l^1 = \pi\phi_h^2 + (1-\pi)\phi_l^2$ establishes $T_F(Q;\ \phi_h^1,\ \phi_l^1) > T_F(Q;\ \phi_h^2,\ \phi_l^2)$.

Let Q^1 be the unique fixed point of $T_F(Q;\ \phi_h^1,\ \phi_l^1)$ and Q^2 the unique fixed point under $T_F(Q;\ \phi_h^2,\ \phi_l^2)$. Since $T_F(Q;\ \phi_h^1,\ \phi_l^1) > T_F(Q;\ \phi_h^2,\ \phi_l^2)$ and $T_F(Q;\ \phi_h^1,\ \phi_l^1)$ is nondecreasing, $Q^2 < Q^1$. It follows that $Z(Q^2,\ \phi_h^2) > Z(Q^1,\ \phi_h^1)$. Q.E.D.

Proof of Proposition 5

First we define a mapping $T: [\phi,\ 1] \times [\phi,\ 1] \rightarrow [\phi,\ 1] \times [\phi,\ 1]$ as follows.

Let q_1 and q_2 be given. Then, from equation (10) we have $\bar{z}_2 = \sqrt{1 - q_2}$, and from equation (12) we have $\lambda_1 = (q_1 - \phi)/(1-\phi)$. Now consider

$$\bar{z}_1 = \sqrt{1 - q_1 + \beta E_{(x',\epsilon)}[V_2(1,\ \bar{z}_1 + \epsilon) - V_2(x',\ \epsilon)]}.$$

It follows from the definition of V_2 (eq. [7]) that

$$E_{(x',\epsilon)}[V_2(1,\ \bar{z}_1 + \epsilon) - V_2(x',\ \epsilon)] =$$
$$E_\epsilon[\max\{1 - (\bar{z}_1 + \epsilon)^2,\ q_2\} - \lambda_1 \max\{1 - \epsilon^2,\ q_2\} - (1 - \lambda_1)\max\{\phi - \epsilon^2,\ q_2\}].$$

Therefore, $\bar{z}_1$, if it exists, is implicitly defined by the following equation:

$$\bar{z}_1^2 = 1 - q_1 + \beta E_\epsilon[\max\{1 - (\bar{z}_1 + \epsilon)^2,\ q_2\} - \lambda_1 \max\{1 - \epsilon^2,\ q_2\} - (1 - \lambda_1)\max\{\phi - \epsilon^2,\ q_2\}].$$

That there exists $\bar{z}_1 > 0$ that satisfies this equation follows from three observations. First, given q_1, q_2, and λ_1, both sides of this equality are continuous in $\bar{z}_1$. Second, if $\bar{z}_1 = 0$, the right-hand side is greater than the left-hand side. Third, if $\bar{z}_1 = \infty$, the left-hand side is equal to infinity, whereas the right-hand side is finite. That $\bar{z}_1$ is unique follows from the observation that the left-hand side is strictly increasing in $\bar{z}_1$, whereas the right-hand side is weakly decreasing in $\bar{z}_1$.

Given $\bar{z}_1$ and $\bar{z}_2$, equations (11) and (15) pin down λ_1' and λ_2'. Given λ_1' and λ_2', equations (12) and (16) define $q_1',\ q_2' \in [\phi,\ 1] \times [\phi,\ 1]$. We set $T(q_1,\ q_2) = (q_1',\ q_2')$. The equilibrium values of q_1 and q_2 arise as fixed points of T.

The existence of an equilibrium follows from the continuity of T. To see that T is continuous, first note that $\bar{z}_1$ is continuous in q_2 and q_1; $\bar{z}_2$ is a continuous function of q_2 by definition. The fact that F has a density means that λ_1' and λ_2' are continuous in $\bar{z}_1$ and $\bar{z}_2$. Finally, it follows from (12) and (16) that q_1' and q_2' are continuous in λ_1' and λ_2'. Existence follows from Brouwer's fixed-point theorem. Q.E.D.

Proof of Proposition 7

Consider a mapping $T: [0, \infty] \rightarrow [0, \infty]$ defined as follows. Given $z \in [0, \infty]$, define the following:

$$q_1 = \frac{2\pi F(-z)}{1 - \pi + 2\pi F(-z)}, \qquad \lambda_1 = \frac{q_1 - \phi}{1 - \phi},$$

$$q_2 = \frac{2\pi G_z(-z_2)}{1 - \pi + 2\pi G_z(-z_2)}, \quad z_2 = \sqrt{1 - q_2}.$$

(Note that G_z satisfies assumption 1 as long as F does, so for any z there is a unique $z_2(z)$, $q_2(z)$.) Define $T(z) = z'$ as

$$z' = \sqrt{1 - q_1(z) + \beta E[\max\{1 - (z + \epsilon)^2, q_2(z)\} - \xi(z)]},$$

where

$$\xi(z) = \lambda_1(z) \max\{1 - \epsilon^2, q_2(z)\} + [1 - \lambda_1(z)]q_2(z).$$

Equilibria are fixed points of T.

Propositions 1 and 2 imply $\sqrt{1 - q_1(z)} > z$ for all $z < Z$. If, in addition, $E[\max\{1 - (z + \epsilon)^2, q_2(z)\} - \xi(z)] > 0$ for all $z < Z$, then the equilibrium $\bar{z}_1 > Z$. To show this, first note that

$$\xi(z) < \pi + (1 - \pi)q_0 = [1 - (1 - \pi)^2] + (1 - \pi)^2\phi.$$

Second, it is easy to show that $E[\max\{1 - (z + \epsilon)^2, \phi\}]$ is decreasing in z for $z > 0$. So, since $Z \leq \sqrt{1 - \phi} \equiv z^{\max}$, we know that, for any $z < Z$,

$$E[\max\{1 - (z + \epsilon)^2, \phi\}] > E[\max\{1 - (z^{\max} + \epsilon)^2, \phi\}]$$
$$= \phi + \int_0^{2z^{\max}} [2(z^{\max})\epsilon - \epsilon^2] f(\epsilon) d\epsilon.$$

Combining two inequalities yields

$$E[\max\{1 - (z + \epsilon)^2, q_2(z)\} - \lambda_1(z) \max\{1 - \epsilon^2, q_2(z)\} - [1 - \lambda_1(z)]q_2(z)]$$
$$> \phi + \int_0^{2z^{\max}} [2(z^{\max})\epsilon - \epsilon^2] f(\epsilon) d\epsilon - [1 - (1 - \pi)^2] - [(1 - \pi)^2]\phi$$
$$= \int_0^{2z^{\max}} [2(z^{\max})\epsilon - \epsilon^2] f(\epsilon) d\epsilon - [1 - (1 - \pi)^2](1 - \phi).$$

Let π^* be defined as in the statement of the proposition. Then if $\pi < \pi^*$,

$$\int_0^{2z^{\max}} [2(z^{\max})\epsilon - \epsilon^2] f(\epsilon) d\epsilon - [1 - (1 - \pi)^2](1 - \phi) > 0.$$

Using proposition 6, we conclude that any equilibrium has the property that $\bar{z}_1 > Z > \bar{z}_2$ and $q_1 < Q < q_2$. Q.E.D.

Proof of Proposition 8

The first step is to define the mapping $T: [\phi, 1] \times [\phi, 1] \times [\phi, 1] \to [\phi, 1] \times [\phi, 1] \times [\phi, 1]$ such that $T(q_1, q_2^{\text{orig}}, q_2^{\text{new}}) = (q_1', (q_2^{\text{orig}})', (q_2^{\text{new}})')$.

Equations (21) and (25) imply $\bar{s}_2^j = \sqrt{1 - q_2^j}$ and $\lambda_1 = (q_1 - \phi)/(1 - \phi)$. Now consider

$$\bar{s}_1 = \sqrt{1 - q_1 + \beta E_{(x',\epsilon)}[V_2^{\text{orig}}(1,\ \bar{s}_1 + \epsilon) - V_2^{\text{new}}(x',\ \epsilon)]}.$$

Manipulation of the definition of V_2^{orig} and V_2^{new} (eq. [20]) yields

$$E_{(x',\epsilon)}[V_2^{\text{orig}}(1,\ \bar{s}_1 + \epsilon) - V_2^{\text{new}}(x',\ \epsilon)] =$$

$$E_\epsilon[\max\{1 - (\bar{s}_1 + \epsilon)^2,\ q_2^{\text{orig}}\} - \lambda_1 \max\{1 - \epsilon^2,\ q_2^{\text{new}}\} - (1 - \lambda_1)q_2^{\text{new}}].$$

Therefore, $\bar{s}_1$ is implicitly defined by

$$\bar{s}_1^2 = 1 - q_1 + \beta E_\epsilon[\max\{1 - (\bar{s}_1 + \epsilon)^2,\ q_2^{\text{orig}}\} - \lambda_1 \max\{1 - \epsilon^2,\ q_2^{\text{new}}\} - (1 - \lambda_1)q_2^{\text{new}}].$$

That there exists $\bar{s}_1 > 0$ that satisfies this equation follows from three observations. First, given q_1, q_2^{orig}, q_2^{new}, and λ_1, both sides of this equality are continuous in $\bar{s}_1$. Second, if $\bar{s}_1 = 0$, the right-hand side is greater than the left-hand side. Third, as $\bar{s}_1 \to \infty$, the left-hand side approaches infinity, whereas the right-hand side is finite. The uniqueness of $\bar{s}_1$ follows from the observation that the left-hand side is strictly increasing in $\bar{s}_1$, whereas the right-hand side is weakly decreasing in $\bar{s}_1$.

Finally, equation (23) and the definition V_2^{new} give

$$\hat{s}_1 = \sqrt{\max\{0,\ \phi - q_1 + \beta E_\epsilon[q_2^{\text{orig}} - \lambda_1 \max\{1 - \epsilon^2,\ q_2^{\text{new}}\} - (1 - \lambda_1)q_2^{\text{new}}]\}}. \qquad \text{(A2)}$$

With $\bar{s}_1$, $\hat{s}_1$, $\bar{s}_2^{\text{orig}}$, and $\bar{s}_2^{\text{new}}$, equations (24), (26), (27), and (28) pin down λ_1', $(\lambda_2^{\text{new}})'$, and $(\lambda_2^{\text{orig}})'$. Finally given λ_1', $(\lambda_2^{\text{new}})'$, and $(\lambda_2^{\text{orig}})'$, equations (25) and (29) define $(q_1', (q_2^{\text{orig}})', (q_2^{\text{new}})') \in [\phi,\ 1] \times [\phi,\ 1] \times [\phi,\ 1]$. We set $T(q_1,\ q_2^{\text{orig}},\ q_2^{\text{new}}) = (q_1',\ (q_2^{\text{orig}})',\ (q_2^{\text{new}})')$. The equilibrium values of q_1, q_2^{orig}, and q_2^{new} arise as fixed points of T.

The existence of an equilibrium follows from the continuity of T and Brouwer's fixed-point theorem. The argument for continuity is similar to that made in proposition 5 and is therefore omitted. Q.E.D.

Proof of Proposition 9

We begin by showing that $\bar{s}_1 \geq Z$. There are two cases. First, if $\hat{s}_1 = 0$, then all agents with lemons adjust. Consider the determination of $\bar{s}_1$ using the apparatus of figure 1. The distributional curve is the same as in the two-period model. The reaction curve, however, shifts up. To see this, note that since there are no original owners with lemons, all original owners get to adjust regardless of their match. Thus $V_2^{\text{orig}}(1, \bar{s}_1 + \epsilon) = 1 + \beta V_0$ and $E_{(x',\epsilon)}[V_2^{\text{new}}(x', \epsilon)] < 1 + \beta V_0$. This implies that

$$\bar{s}_1 = \sqrt{1 - q_1 + \beta E_{(x',\epsilon)}[V_2^{\text{orig}}(1,\ \bar{s}_1 + \epsilon) - V_2^{\text{new}}(x',\ \epsilon)]} > \sqrt{1 - q_1}.$$

It follows that $\bar{s}_1 > Z$.

In the second case, $\hat{s}_1 > 0$. If $\hat{s}_1 > 0$, equation (23) implies that the value of holding a lemon with a perfect match is strictly better than the value of trading, so that $\beta[E_{(x',\epsilon)}V_2^{\text{new}}(x',\ \epsilon)] < \phi - q_1 + \beta q_2^{\text{orig}} + \beta^2 V_0$. Agents with good cars can do no worse than if they were forced to trade, so that $\beta E_\epsilon[V_2^{\text{orig}}(1,\ \bar{s}_1 + \epsilon)] > \beta q_2^{\text{orig}} + \beta^2 V_0$. Combining these two expressions, we get

$$\beta E_{(x',\epsilon)}[V_2^{\text{orig}}(1,\ \bar{s}_1 + \epsilon) - V_2^{\text{new}}(x',\ \epsilon)] > q_1 - \phi > 0.$$

It follows from equation (22) that $\bar{s}_1 > \sqrt{1-\phi} > Z$.

We now show that $\bar{s}_2^{\text{new}} > Z$. Consider again figure 1. The reaction curve is the same as in the two-period model. But the distribution curve shifts down because in any equilibrium, $\hat{s}_1 < \bar{s}_1$, so that the fraction of agents with lemons who adjust is greater than the fraction of agents with good cars who adjust. This implies that the proportion of good cars in the new owner market is less than π. Given any first-period cutoffs ($\bar{s}_1 > \hat{s}_1$), the equilibrium cutoff ($\bar{s}_2^{\text{new}}$) and quality (q_2^{new}) in the new owner market are determined just as they would be in a two-period model with a lower π (note that uniqueness is ensured since π is lower). Note also that, in equilibria in which $\hat{s}_1 > 0$, we have the additional restriction that $Z < \bar{s}_2^{\text{new}} < \sqrt{1-\phi} < \bar{s}_1$.

Now we show that $\bar{s}_2^{\text{orig}} < Z$. Again, the reaction curve is the same as in the model in which cars last for only two periods. Now, however, the distributional curve shifts up since there is a greater fraction of agents with good cars ($\hat{s}_1 < \bar{s}_1$) and their matches (given by $H_{\bar{s}_1}(z)$) are more disperse in the sense of definition 1. Thus the equilibrium must have $\bar{s}_2^{\text{orig}} < Z$.

Finally, we show that if $\pi < (1-\pi)^2\beta$, then $\hat{s}_1 > 0$. The proof is done by contradiction. Suppose that $\hat{s}_1 = 0$. Then $q_2^{\text{orig}} = 1$. This implies that

$$\phi - q_1 + \beta E_\epsilon[q_2^{\text{orig}} - \lambda_1 \max\{1-\epsilon^2,\ q_2^{\text{new}}\} - (1-\lambda_1)q_2^{\text{new}}] \geq$$
$$\phi - q_0 + \beta[1 - \pi - (1-\pi)q_0] > 0,$$

where the last inequality follows from $\pi < (1-\pi)^2\beta$. The expression for $\hat{s}_1$ in (A2) implies $\hat{s}_1 > 0$. This completes the proof. Q.E.D.

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