How important is discrete adjustment in aggregate fluctuations?

Andrew Caplin John Leahy*

June 1, 2002

Abstract

Individual actions such as the purchase of a machine or the changing or a price are often discrete and infrequent. Macroeconomic models, however, often ignore this intermittent behavior. In this paper, we present conditions under which a representative agent model, which ignores discrete adjustment on the individual level, is observationally equivalent to a model that takes discrete adjustment into account. We also provide a mapping between the preference and technology parameters of the two models and present conditions under which this mapping is an identity.

1 Introduction

The papers in this volume tend to fall into two camps: those that take a microeconomic perspective on growth and productivity and those that take a more aggregate perspective. This paper contributes to the on-going effort in macroeconomics to link the two perspectives.

One of the factors that makes this link difficult is that adjustment at the level of the individual or firm is often discrete, whereas adjustment at the macroeconomic level is more smooth and continuous. This is especially true of the decisions that contribute to growth and productivity. Individual decisions, such the decision build a new factory, the decision to adopt a new technology, or the decision to enter a new market, are all are decisions that carry large fixed costs at the microeconomic level. Individuals and firms therefore tend to take these actions infrequently. Other decisions, such as the decision to buy a new car or to change a price also share this characteristic. Few people, for example, change the car that they drive every day in response to the current value of their stock portfolio or their current utility from driving. Rather they let their car depreciate over time and occasionally upgrade to a new one that is consistent with their current tastes and wealth.

In spite of the discreteness of many microeconomic decisions, the standard approach to modelling in macroeconomics is to ignore all of this discrete behavior and assume that all firms are represented by a single representative firm that makes all of the investment decisions or that all consumers are represented by a single representative consumer that makes all of the consumption decisions. The decisions, which look so infrequent and discontinuous when viewed from the perspective of the individual firm or consumer, become quite smooth and continuous when viewed from the perspective of these representative agents. These representative agents typically care only about the total stock of capital or durable goods or the average level or technology or prices. They make minor adjustments to these variables in every period in order to equate the relevant marginal costs and marginal benefits of adjustment. While this abstraction provides a tractable microeconomic foundation for the modelling of aggregate investment or durable demand, it is clearly the wrong microeconomic foundation. The question is whether this makes any difference.

Whereas this paper is more theoretical and abstract than most of the other papers in this volume, the issue of how to go from realistic microeconomic analysis to realistic macroeconomic analysis is an important one for all researchers interested in growth and productivity. There is a trade-off between realism and complexity in macroeconomic modelling. We want models with dependable microfoundations, models that reflect the influence of factors such as discrete and infrequent adjustment. We also want simple and useful models of the economy as a whole, models like the representative agent model. The message of this paper is that this trade-off is not as costly as one might think. In spite of the importance of discrete adjustment at the microeconomic level, representative agent models can capture aggregate dynamics fairly accurately. The catch is that the representative agent must be parameterized to represent the market and not any given individual. This means that standard macroeconomic analysis need only be altered slightly in order to incorporate microeconomic discreteness at the microeconomic level.

In recent years there has developed a large body of research which appears to indicate the opposite. This literature suggests that microeconomic frictions might have large macroeconomic consequences.[1] The potential for discrete adjustment to matter lies in the potential for the distribution of durable goods, capital, prices, or technology to vary over time. For concreteness consider the capital stock. The capital stock of a representative agent is simply the capital stock. In a discrete adjustment model, a given aggregate stock of capital may be consistent with many distributions of capital across firms. A typical firm will allow its capital holdings to drift away from its optimal level, adjusting only when it hits some adjustment trigger. The distribution of capital relative to these adjustment triggers will affect current productivity and influence future investment. The greater the misallocation of capital, the more inefficient the economy and the greater the need for subsequent adjustment. If there are relatively many agents near an upward adjustment trigger then investment will tend to be high in the future. If there are relatively many near a downward adjustment trigger then disinvestment is possible. In this way the distributional dynamics can add an additional source of aggregate fluctuation as misalignment rises and falls over time. This added noise complicates both forecasts and the interpretation of aggregate statistics.

In spite of this flurry of recent research, the importance of discrete adjustment in macroeconomics is still an unsettled question. Bar-Ilan and Blinder simply claim that "one implication of [discrete adjustment] at the microeconomic level is that aggregate data cannot be generated by a representative agent." They base this claim on the fact that their model has margins of adjustment not present in the representative agent model, namely the number of agents adjusting and the size of individual adjustment. They do not, however, compare the dynamics of their model to the dynamics of a representative agent model. Caballero and co-authors report statistically significant effects of discrete adjustment, but they do not show that these effects are economically significant, nor do their models endogenize prices. On the other hand, Caplin and Spulber present a model in which discrete behavior aggregates to a representative agent, and Thomas argues that equilibrium feedback may smooth out the effects of discrete adjustment. Moreover, Adda and Cooper (2000b) argue empirically that most aggregate fluctuations in durable goods markets are associated with fluctuations in price rather than the distribution of holdings.

Investigations of the role of discrete adjustment have been hampered by the difficulty of constructing equilibrium models that can be easily compared to their smooth representative agent counterpart. Given the importance of the distributional dynamics, the dimension of the state space quickly becomes unmanageable. The literature tends to deal with this problem in one of two ways. Much of the literature simply assumes that prices are exogenous to agents' actions. This severs the links among agents, so that the decision problem of each agent can be studied in isolation. Aggregation simply involves integrating across agents' actions. Other papers reduce the dimensionality of the problem by making assumptions on the allowable distributions. For example, Caplin and Leahy [1997] restrict attention to distributions that are uniform in relative prices, and Dotsey, King and Wolman [1999] assume that the support contains a bounded number of points.

In this paper, we use a more realistic approximation to compare the aggregate dynamics of a discrete adjustment model to that of a representative agent model with continuous adjustment.[2] The approximation was developed by Caplin and Leahy [2002a] in the context of durable goods. The idea behind the approximation is that if there is enough time between an agent's purchases then individual heterogeneity will smooth the echoes of previous cycles. Consider a market in which agents with holdings of a durable below some trigger "little s" rebuild their stocks to some level "big S". High demand today then creates a lump in the cross-sectional distribution of holdings at big S. If there is no individual heterogeneity then this lump passes through the (S,s) bands as holdings depreciate and produces an echo in demand when it reaches little s. This echo creates a link between the market today and the market in the far future. Breaking this link greatly simplifies the analysis. In this paper we break this link by assuming that the durable goods holdings of different agents depreciate at different rates. This heterogeneity tends to disperse the lump and reduce the echo.

It is important to note that, in assuming pervasive microeconomic heterogeneity, we are taking to heart one of the principle conclusions of the accumulating microeconomic literature on discrete adjustment. It is a common finding in this literature that the variance of the idiosyncratic shocks faced by an individual or firm is many times greater than that of aggregate shocks.[3] This heterogeneity tends to weaken the correlation of adjustment across firms. From a theoretical perspective, it is not important that we put this heterogeneity in the depreciation rate. Any form of heterogeneity will do the trick. We could just as well have assumed that tastes, income, wealth, or demographic variables were heterogeneous. The advantage of our approximation is that it produces a comparatively simple equilibrium model that can be solved analytically and compared to the representative agent model.

It may seem that by smoothing the echoes we are eliminating the distributional dynamics that make the discrete adjustment model distinctive. This is only partially the true. Whereas we rule out fluctuations in the density of holdings at the purchase trigger, we still allow the distribution to shift with movements in "big S" and "little s". We would argue that most of the distributional dynamics that people associate with the business cycle are in fact shifts in these thresholds. When the stock market crashes and people feel less wealthy, they tend to hold on to their cars a bit longer and then purchase less expensive cars. These decisions are well captured by shifts in the adjustment trigger and target. They are not directly related to the density of holdings.[4] Our model is consistent with the observation that fluctuations in aggregate investment activity are driven by to a large extent by variation in the number of firms making large investments.[5]

We present the representative agent model and the Caplin-Leahy approximation to the discrete choice model in the next section and compare them in Section 3. It turns out that the representative agent model and the Caplin-Leahy approximation are observationally equivalent. Each implies that the first difference in sales follows an ARIMA(1,1). In principle, this means that one could construct a mapping between the two models: choose realistic microeconomic parameters that characterize the discrete adjustment model and then find a representative agent model that yields similar dynamics. We construct such a mapping and analyze some of its properties. First, we consider the special case in which the supply curve is perfectly elastic and find that in this case the mapping between the parameters of the two models is the identity mapping; the models are equivalent.[6] We then show that the mapping is non-trivial when there is a price response to high demand. In particular, the depreciation rate and the real interest rate for the representative agent model need to be adjusted in order match the dynamics of the discrete adjustment model.

To get a sense of the importance of the differences that arise, we use data from the U.S. automobile industry to calibrate the Caplin-Leahy model. The model fits the data well with a depreciation rate of 31% per annum, which compares favorably to the estimate of 33% reported by Jorgenson and Sullivan [1981]. We then use the mapping to find the corresponding representative agent model. The representative agent model that mimics the dynamics of the Caplin-Leahy model has a depreciation rate of 27% and a real interest rate of 12%. While these differences may appear large, it turns out that the market dynamics are relatively insensitive to these two parameters. Therefore when the two models are calibrated with the same parameters their dynamics do not differ greatly.

We conclude that in the case of the U.S. automobile industry not much is lost by ignoring discrete adjustment at the microeconomic level and instead modeling demand according to the continuous adjustment of a representative agent. In more general settings, care needs to be taken in parameterizing the representative agent model. Parameters that appear reasonable on a microeconomic level may not be appropriate for a representative agent who proxies for a group of consumers facing adjustment costs. This distinction may be especially important when conducting policy experiments, since in this case the representative agent model that mimics the discrete choice model might change with the change in policy regime.

We conclude the paper with some observations on when our approximation should hold and when it may not.

2 Two Models

In this section we present log linearized versions of the representative agent model and Caplin and Leahy's approximation of the (S,s) model.

2.1 Representative Agent

We consider the problem of a representative agent who derives utility from a stock of a durable good K_t. Utility is separable between durable an non-durable consumption.[7] Utility from durables takes a constant elasticity form, $U(K) = K^{\alpha}/\alpha$. The durable depreciates at a rate Δ. The price of the durable is p_t and the marginal utility of wealth is λ_t.

The consumer maximizes the present value of utility less the cost of new purchases:

$$\max_{\{K_t\}} \sum \beta^t \left(U(K_t) - p_t\lambda_t[K_t - (1-\Delta)K_{t-1}]\right)$$

The first order condition for this problem is to set the marginal utility from the durable equal to a form of Jorgenson's user cost:

$$U'(K_t) = p_t\lambda_t - (1-\Delta)\beta E_t p_{t+1}\lambda_{t+1}.$$

We close our description of the market with assumptions on price and the marginal utility of wealth. Let $Q_t = K_t - (1-\Delta)K_{t-1}$ denote purchases of the durable in period t. We assume that price is equal to marginal cost, and that marginal cost is a function of purchases and a cost shock

$$p_t = Q_t^{\gamma} c_t.$$

We assume that both shocks, c_t and λ_t, follow random walks.[8]

2.2 The Caplin-Leahy Model

Since many of the parameters, such as α and β, have the same meaning in the two models we will reuse them. If it becomes important to distinguish between the parameters of one model or the other we will use subscripts or superscripts such as α_{rep} or q^{cl}.

Consider a continuum of consumers indexed by $i \in [0, 1]$ who derive utility from their holdings of a durable. Like the representative agent model we assume that each agent receives utility $U(K_{it}) = K_i^{\alpha}/\alpha$, that the price of a unit of the durable is p_t, and that the marginal utility of wealth is λ_t.[9] We make two changes to the individual's problem. First, when individuals alter their holdings of the durable good they must pay a fixed cost equal to a fraction c of their current holding of the durable. This cost generates intermittent adjustment. Agents will wait until the gain from adjustment justifies incurring the fixed cost.

Second, in order to spread purchases of the durable over time we introduce heterogeneity in the form of random depreciation. We assume that each period each agent's durable depreciates by an amount Δ_{it} which is i.i.d. with a mean equal to Δ.

Let $V(K_t, \omega_t)$ denote the value of an optimal policy for a consumer holding a durable of size K_t given that the state of the market, to be discussed in detail below, is ω_t. This problem may be written as

$$\begin{aligned} V(K_{it}, \omega_t) &= \max_{\{T_j, S_{T_j}\}} E_t \sum_{s=t}^{\infty} \beta^{s-t} U(K_{is}) \\ &\quad - \sum_{j=1}^{\infty} \beta^{T_j - t} p(\omega_{T_j}) \lambda(\omega_{T_j}) \left[S_{T_j} + (1-c)(1-\Delta_{it}) K_{T_j - 1} \right], \end{aligned} \tag{1}$$

where,

$$K_{is} = \begin{cases} K_{it} & \text{if } s = t \text{ and } T_1 > t; \\ S_{T_j} & \text{if } s = T_j; \\ (1-\Delta_{it}) K_{is-1} & \text{otherwise}; \end{cases}$$

Here E_t is the mathematical expectation conditional on date t information. The first summation represents the utility that the agent receives from the durable. β is the consumer's discount rate, and $U(K_s)$ is the utility from holding a durable of size K_s. The second summation represents the cost of successive purchases of the durable. T_j is a random time representing the date of the jth purchase. On these dates the consumer sells a fraction $1-c$ of her current holdings of the durable and purchases S_{T_j} new units of the durable good. Both purchase and sale take place at a price p_{T_j}. λ_{T_j} is the marginal utility of wealth and translates the purchase price into utility terms. Between purchase dates the durable depreciates by an amount Δ_{it}.

If the depreciation rate is great enough and the cost of adjustment is high enough, then it will be rare for agents to reduce their holdings of the durable.[10] Adjustment will be one sided. Given the state of the market ω_t, there will be a purchase target $S(\omega_t)$ and a purchase trigger $s(\omega_t) < S(\omega_t)$ such that all agents with holdings less than $s(\omega_t)$ adjust their holdings to $S(\omega_t)$.

We close the model in the same manner as the representative agent model. We assume that price is equal to marginal cost and that marginal cost depends on total sales and a cost shock:

$$p_t = Q_t^{\gamma} c_t.$$

In this case total sales are equal to the product of the number of purchases and the size of each purchase.

Solving for equilibrium in such a setting is made difficult by the fact that included in the state vector ω_t is the entire distribution of durable goods holdings across agents. The number of agents with small holdings matters because this will influence demand and hence price. The rest of the distribution helps to predict the evolution of this lower tail. The Caplin-Leahy model makes two assumptions that simplify these dynamics. Both assumptions are motivated by the idea that when the time between purchases is sufficiently long then the present will exert very little impact on the future. The first assumption is that the value of a new durable is exogenous to the current state of the market, and can therefore be expressed as $V(K)$. The idea is that current market influences will die out before the next purchase is made. The second assumption is that the density of holdings in the neighborhood of the purchase trigger is log uniform. This assumption requires that there sufficient time between purchases that the heterogeneity in depreciation smooths out the lumps in the distribution that may occur if a large number of agents purchase the durable at one time. The precise conditions necessary to support these assumptions are discussed in Caplin and Leahy .[2002a]. The assumption that heterogeneity smooths away the echoes of past shocks removes some of the distributional dynamics associated with discrete adjustment models. It is important to note, however, that an important source of distributional dynamics remains, namely movement in the adjustment trigger s_t. When s_t lies below its steady state level there will be "pent up demand", and when s_t lies above its steady state level demand will be below average for some time. Simulations of the model calibrated to the U.S. automobile market indicate that the assumptions hold remarkably well (Caplin and Leahy .[2002a]).

Given these assumptions the first order conditions for an optimal policy are

$$
\begin{aligned}
V'(S_t) &= p_t \lambda_t \\
V(S_t) - (S_t - (1-c)s_t)\, p_t \lambda_t &= \frac{s_t^\alpha}{\alpha} \\
&\quad + \beta E_t \left[V(S_{t+1}) - (S_{t+1} - (1-c)(1-\Delta)s_t)\, p_{t+1} \lambda_{t+1} \right]
\end{aligned}
$$

The first equation states that the optimal target is determined by equating the value of the marginal purchase to the cost. Note that the adjustment cost does not appear here since it is sunk once the agent decides to purchase. Note also that the determination of the optimal purchase size S_t is essentially a static decision, in much the same way that non-durable consumption is a static decision. This is a consequence of the unpredictability of the future. The second equation states that the optimal trigger is determined by indifference between buying today and buying tomorrow. Note here that we have replaced Δ_{it+1} with its mean.

V will inherit certain homogeneity properties from the constant elasticity utility function and the proportional depreciation rate. It will be useful to normalize V by the level of holdings of the durable good that would occur in steady state in the absence of frictions. Let κ_t denote this level of holdings. Caplin and Leahy (2002) show that V will be homogenous of degree α in κ_t

$$V(S_t) = v(S_t/\kappa_t)\kappa_t^{\alpha}$$

Note from our analysis of the representative agent model we know that:

$$\kappa_t^{\alpha-1-\gamma} = (1-\beta(1-\Delta))\Delta c_t \lambda_t.$$

Finally, the number of purchases is determined by depreciation and the evolution of the purchase trigger. With the assumption that the distribution of holdings is log uniform, the number of purchases becomes

$$n_t = \mu(\ln s_t - \ln s_{t-1} + \delta)$$

where μ is the density of holdings and $\delta = -\ln(1-\Delta) \sim \Delta$. Sales are therefore

$$Q_t = \mu S_t(\ln s_t - \ln s_{t-1} + \delta).$$

The evolution of cost and the evolution of marginal utility are as before. This completes the presentation of the model.

2.3 Linearization

Our interest is in the first-order differences between the two models. We therefore log linearize the dynamics. Appendix A presents the details of the derivation. Here we present the results.

The representative agent model is defined by the following system of equations

$$(\alpha - 1)[1 - (1-\Delta)\beta]\hat{k}_t = (\hat{p}_t + \hat{\lambda}_t) - \beta(1-\Delta)E_t(\hat{p}_{t+1} + \hat{\lambda}_{t+1}) \tag{2}$$

$$\Delta\hat{q}_t = \hat{k}_t - (1-\Delta)\hat{k}_{t-1} \tag{3}$$

$$\hat{p}_t = \gamma\hat{q}_t + \hat{c}_t \tag{4}$$

$$\hat{c}_t = \hat{c}_{t-1} + \eta_{ct} \tag{5}$$

$$\hat{\lambda}_t = \hat{\lambda}_{t-1} + \eta_{\lambda t} \tag{6}$$

There are three endogenous variables, $\hat{k}_t$, $\hat{p}_t$, and $\hat{q}_t$, and two state variables, $\hat{c}_t$ and $\hat{\lambda}_t$. All variables are in log deviations from their steady state values. Equation (2) is the first order condition for the optimal holding of the durable good. Equation (3) defines sales as a function of the change in durable holdings. Equation (4) defines marginal cost and Equations (5) and (6) define the evolution of the exogenous variables.

Appendix A shows that these equations may be combined to yield a second order difference equation in $\hat{k}_t$ which has a solution of the form

$$\hat{k}_t = x_{rep}\hat{k}_{t-1} + y_{rep}\hat{e}_t$$

where $x_{rep} \in [0, 1]$ and $y_{rep} < 0$.

The Caplin-Leahy model is defined by the following system of equations

$$(\alpha - 1)\hat{S}_t = \hat{p}_t + \hat{\lambda}_t \tag{7}$$

$$\begin{aligned} \left[s^\alpha - (1-\beta(1-\Delta))(1-c)sp\lambda\right]\hat{s}_t = & \\ & -p\lambda\left(S-(1-c)s\right)\left(\hat{p}_t + \hat{\lambda}_t\right) \\ & +p\lambda\left(S-(1-c)(1-\Delta)s\right)\beta E_t\left(\hat{p}_{t+1} - \hat{\lambda}_{t+1}\right) \\ & +\left[\alpha v\kappa^\alpha - p\lambda S\right]\left(\hat{\kappa}_t - E_t\beta\hat{\kappa}_{t+1}\right) \end{aligned} \tag{8}$$

$$\hat{q}_t = \hat{S}_t + \frac{1}{\delta}(\hat{s}_t - \hat{s}_{t-1}) \tag{9}$$

$$(\alpha - 1 - \gamma)\hat{\kappa}_t = \hat{\lambda}_t + \hat{c}_t \tag{10}$$

$$\hat{p}_t = \gamma\hat{q}_t + \hat{c}_t \tag{11}$$

$$\hat{c}_t = \hat{c}_{t-1} + \eta_{ct} \tag{12}$$

$$\hat{\lambda}_t = \hat{\lambda}_{t-1} + \eta_{\lambda t} \tag{13}$$

There are four endogenous variables, $\hat{S}_t$, $\hat{s}_t$, $\hat{p}_t$, and $\hat{q}_t$, and two state variables, $\hat{c}_t$ and $\hat{\lambda}_t$ ($\hat{\kappa}_t$ is a function of these). As before, all variables are in log deviations from their steady state values. Equation (7) is the first order condition for purchase target. Equation (8) is the first order condition for purchase trigger. Equation (9) defines sales as a function of the target and the change in the purchase trigger. Equation (10) defines the frictionless steady-state holdings. Equations (11), (12), and (13) are the same as their representative agent counterparts.

Appendix A shows that these equations may be combined to yield the following second order difference equation in $\hat{s}_t$, which has the form

$$\hat{s}_t = x_{cl}\hat{s}_{t-1} + y_{cl}\hat{e}_t \tag{14}$$

where $x_{cl} \in [0, 1]$ and $y_{cl} < 0$.

At this point we note several differences between the two models. First, purchases in the Caplin-Leahy model depend separately on the number of individual purchases and the size of each individual purchase. Second, there is no role for the aggregate stock of durables as in the representative agent model. Only the agents who make purchases affect sales. Third, whereas purchases in the representative agent model depend on the current price and the price next period, the purchase target in the Caplin-Leahy model depends on the current price and the price in the distant future as reflected in the steady-state target. Finally, there is no role for the adjustment cost in the representative agent model.

There are also similarities. Most notably that both $\hat{s}_t$ and $\hat{k}_t$ follow second-order difference equations.

3 A Comparison

In this section, we compare the dynamic properties of the two models. We begin by solving for the dynamics of sales in each case. The dynamics of price follows form the supply curve.

The stock of durables in the representative agent model evolves according to:

$$\hat{k}_t = x_{rep}\hat{k}_{t-1} + y_{rep}\hat{e}_t$$

Hence sales equal:

$$\begin{aligned}\hat{q}_t^{rep} &= \frac{1}{\Delta}\hat{k}_t - \frac{1-\Delta}{\Delta}\hat{k}_{t-1} \\ &= x_{rep}\hat{q}_{t-1}^{rep} + y_{rep}\left(\frac{1}{\Delta}\left(\hat{\lambda}_t + \hat{c}_t\right) - \frac{1-\Delta}{\Delta}\left(\hat{\lambda}_{t-1} + \hat{c}_{t-1}\right)\right)\end{aligned}$$

Sales in the Caplin-Leahy model evolve according to:

$$\hat{q}_t = \hat{S}_t + \frac{1}{\delta}(\hat{s}_t - \hat{s}_{t-1})$$

substituting for $\hat{S}_t$ and $\hat{s}_t$ yields:

$$\begin{aligned}\hat{q}_t^{cl} &= \frac{1-\alpha}{1-\alpha+\gamma}\frac{1}{\delta}(\hat{s}_t - \hat{s}_{t-1}) - \frac{1}{1-\alpha+\gamma}\hat{e}_t \\ &= x_{cl}\hat{q}_t^{cl} + \left[\frac{1-\alpha}{1-\alpha+\gamma}\frac{1}{\delta}y_{cl} - \frac{1}{1-\alpha+\gamma}\right]\hat{e}_t - \left[\frac{1-\alpha}{1-\alpha+\gamma}\frac{1}{\delta}y_{cl} - \frac{x_{cl}}{1-\alpha+\gamma}\right]\hat{e}_{t-1}\end{aligned}$$

In each case, the first difference of sales follows an ARIMA(1,1). There is therefore a sense in which the two models are observationally equivalent. This equivalence is remarkable since it relates a model with discrete adjustment at the microeconomic level to a representative agent model with *no* adjustment costs.

We can think about how to parameterize the representative agent model to mimic the dynamics of the Caplin-Leahy model. For this we need to match the coefficients on $\hat{q}_t$, $\hat{e}_t$, and $\hat{e}_{t-1}$. This requires:

$$\begin{aligned}
x_{rep} &= x_{cl} \\
\frac{y_{rep}}{\Delta} &= \frac{1-\alpha}{1-\alpha+\gamma}\frac{1}{\delta}y_{cl} - \frac{1}{1-\alpha+\gamma} \\
y_{rep}\frac{1-\Delta}{\Delta} &= \frac{1-\alpha}{1-\alpha+\gamma}\frac{1}{\delta}y_{cl} - \frac{x_{cl}}{1-\alpha+\gamma}
\end{aligned}$$

or

$$\begin{aligned}
x_{rep} &= x_{cl} \\
y_{rep} &= \frac{x_{cl}-1}{1-\alpha_{cl}+\gamma} \qquad (15) \\
\Delta &= \frac{x_{cl}-1}{\frac{1-\alpha}{\delta}y_{cl}-1}
\end{aligned}$$

In principle, a mapping between the parameters of the two models can be constructed as follows. Given any parameterization of the Caplin-Leahy model (α_{cl}, β_{cl}, δ_{cl}, and c), solve for x_{rep}, y_{rep}, and Δ_{rep} using (15). Given x_{rep}, y_{rep}, and Δ, derive α_{rep}, β_{rep} and K using the definitions of x_{rep} and y_{rep} and the steady state relationship

$$K_{rep}^{\alpha-1} = (1-(1-\Delta_{rep})\beta_{rep})p\lambda.$$

How do the two models differ? We attempt to answer this question in two ways. First, we consider a simple situation in which the supply curve is perfectly elastic and the equations simplify greatly. Second, we match the parameters of the two models to data from the market for new cars in the United States, and ask whether and how much they differ in this case.

3.1 A simple case

We begin with a situation in which the mapping is simple. If $\gamma = 0$, then it can be shown that $x_{cl} = 0$ and $y_{cl} = \frac{1}{\alpha_{cl}-1}$. This implies that $x_{rep} = 0$, $y_{rep} = \frac{1}{\alpha_{rep}-1}$, and $\Delta_{rep} = \frac{\delta_{cl}}{1+\delta_{cl}}$. Hence, $\alpha_{rep} = \alpha_{cl}$ and Δ_{rep} is equal to Δ_{cl} to a first order. Note that in this case, β_{rep} and K_{rep} do not affect the dynamics of the representative agent model and c does not affect the dynamics of the Caplin-Leahy model. In sum, identical parameterizations of the two models yield identical dynamics. In this case, the two models are identical.

Proposition 1 *If $\gamma = 0$, then the response of the Caplin-Leahy model to a shock is identical to the response of a similarly parameterized representative agent model.*

This result is similar to the neutrality result of Caplin and Spulber [1985] in that a heterogeneous agent model with fixed costs delivers dynamics similar to a representative agent model without frictions. The intuition is straight forward. In the absence of a price response, a shock in the Caplin-Leahy model causes a once and for all shift in both $\hat{S}_t$ and $\hat{s}_t$ by an amount $\frac{1}{\alpha-1}$. The intuition is the same as for the permanent income hypothesis: a shift in price or marginal utility causes a proportional shift in policy. Total purchases, $\hat{q}_t$, depend on $\hat{S}$ and the first difference of $\hat{s}_t$. $\hat{S}$ rises permanently, but $\Delta\hat{s}_t$ rises only for one period. the result is that total purchases follow an MA(1). Since $\Delta\hat{s}_t$ receives a weight $1/\delta$ in $\hat{q}_t$, the lagged MA coefficient is approximately $1 - \Delta$ as in Mankiw's [1982] representative agent model.

4 Evidence form the U.S. auto market

In the general case in which $\gamma > 0$, this exact mapping between the two models fails to hold. This can easily be seen from the fact that the adjustment cost c enters the equations that determine x_{cl} and y_{cl}. This cost plays no role in the representative agent model.

In order to see how important these differences may be in practice, we fit the model to data from the market for new cars in the United States. We take data on the number of new cars sold from the Bureau of Economic Analysis. $\hat{n}$ is the log of this number. The BEA also has data on the average purchase price of new cars. We normalize this number by the price index for new cars obtained from the Bureau of Labor Statistics, and take logs to get $\hat{S}$. $\hat{q}$ is the sum of $\hat{n}$ and $\hat{S}$. We construct the relative price of new cars, $\hat{p}$, by dividing the CPI index for new cars by the GDP deflator for non-durable goods and taking logs.

Although most of the data is available at a monthly frequency, we estimate the model at a quarterly frequency. The monthly data contain a lot of noise and are dominated at times by movements in inventories, which we have not modelled. Aggregating lessons these problems, and yields sensible results. We leave the inclusion of inventory movements for future work.

We restrict the analysis to the period 1967:1 to 1990:1. We begin in 1967 because there are some violent movements in the average price of used cars in the early 1960's, that appear to be more problems with the data than real economic phenomenon. We end in 1990 because there is a trend break in the series sometime in the late eighties when minivans and SUV's begin to replace the station wagon. Whereas station wagons were categorized as cars, Minivans and SUV's are categorized as light trucks.

We choose to work with seasonally adjusted data. In principle the model should work as well with seasonally adjusted data. Including seasonality, however, complicates the error structure without adding anything to the analysis.

4.1 The response of prices

We begin with the response of prices, since if the supply curve is elastic the models are identical. To estimate γ, we regress $\hat{p}$ on $\hat{q}$. We instrument for the demand for autos using the current and lagged change in non-durable consumption, the consumer price index for energy, the federal funds rate, as well as the lagged number of purchases. All instruments were expressed in logs. The consumption Euler equation implies that non-durable consumption should be proportional to the marginal utility of wealth, which acts as a demand shock in our model. The lagged number of purchases should be correlated with s_{t-1} and hence the current number of purchases. The federal funds rate and the price of energy were included under the hypothesis that they primarily shift durable demand.

Table 1 presents the second state results from the TSLS estimation.

Table 1

Instrumental Variables Estimation of

the Effect of Durable Demand on the Relative Price of Durables.

variable	coef.	s.e.	prob
$\hat{q}_t$	**.171**	.036	.000
constant	**4.548**	.207	.000
trend	**-.018**	.003	.000
100*trend^2	**.012**	.005	.001
10000*trend^3	-.002	.002	.293

The coefficient on $\hat{q}_t$ is significantly different from zero. The t-statistic is about 4.75. We conclude that supply is not perfectly elastic and there is a potential for the two models to differ. Given the positive value for γ, it is difficult to justify modelling the demand for durable goods under the assumption of exogenous prices as is the practice in much of the literature.[11]

This result is fairly stable. The coefficient on $\hat{q}$ is little changed if only lagged instruments are used, or if only the consumption of non-durables is used as an instrument. It is also similar to other results in the literature. Bils and Klenow [1998] find that the prices of a large number of durable goods are procyclical. Adda and Cooper [2002b] estimate a structural model of the market for new cars. Using data for France and the United States they find that they need a positive correlation between their demand shock and their price shock in order to fit the data.

We next estimate α and x, since these can be observed from the data.

4.2 The Number of Purchases

The number of purchases $\hat{n}$ is related to the change in the purchase trigger $\hat{s}$. If $\gamma > 0$, then $\hat{s}$ follows an AR(1). It is easy to see that $\hat{n}$ also follows an AR(1) and that the autoregressive coefficient is x_{cl}.

$$\hat{n}_t = x_{cl}\hat{n}_{t-1} + y_{cl}(\eta_{ct} + \eta_{\lambda t})$$

According to the model the error in this equation is independent of $\hat{n}_{t-1}$. We can therefore estimate x_{cl} from the data on $\hat{n}$ using OLS.

Table 2 fits an AR(1) to our data on $\hat{n}$.

Table 2

Estimation of $\hat{n}_t = x_{cl}\hat{n}_{t-1} + \varepsilon_t$

variable	coef.	s.e.	prob
$\hat{n}_{t-1}$	**.711**	.075	.000
constant	**1.631**	.749	.032
trend	.027	.036	.416
100*trend^2	-.037	.046	.433
10000*trend^3	.016	.021	.439

x_{cl} is estimated fairly precisely. It is significantly different from both zero and one. This relationship is also very stable. Further lags are insignificant. Dropping the trend variables has no effect on the autoregressive coefficient, neither does including the change in non-durable or the price of energy in the regression.[12]

4.3 The elasticity of demand

We can calibrate the elasticity of demand α from the reaction of the average size of purchases to price

$$\Delta \hat{S}_t = \frac{1}{\alpha - 1}(\Delta \hat{p}_t + \Delta \hat{\lambda}_t) \tag{16}$$

We estimate equation (16) using data on non-durable consumption to control for changes in the marginal utility of wealth. We also include the change in the federal funds rate and the change in the price of energy as controls. The results are reported in table 3.

Table 3

Estimation of Equation (16)

variable	coef.	s.e.	prob
$\Delta \hat{p}_t$	**-.403**	.179	.027
constant	.003	.042	.936
trend	-.001	.002	.787
100*trend^2	.001	.003	.609
10000*trend^3	-.001	.001	.490
Δcons	**.551**	.274	.047
Δp_{energy}	-.086	.072	.232
Δffunds	.000	.001	.824

The coefficient on $\Delta \hat{p}_t$ is fairly stable. It does not change if we estimate the equation in levels, omit the trend terms, or omit the other controls. The coefficient implies a value of α approximately equal to -1.5.

4.4 Fit

Whether or not the models differ depends on how one interprets the data. We do two experiments. First, we use the data to back out parameters of the Caplin-Leahy model and then use the mapping described above to derive the corresponding parameters of the representative agent model. Given our estimates of $\alpha = -1.5$, $x = .71$ and $\gamma = .17$ and we can calculate Δ_{cl} given values for c and β. We calibrate $\beta = .99$, which is consistent with a real interest rate of 4% per annum and select a number of values for $c \in [0, 1]$. It turns out that Δ_{cl} is relatively insensitive to the choice of c. For $c = 1$, we obtain $\Delta_{cl} = .094$. For $c = .2$, we obtain $\Delta_{cl} = .091$. The former implies an annual depreciation rate of 32.6%, whereas the latter implies 31.7%. Both of these are nearly identical to the calculations of 33.33% per annum estimated for autos by Jorgenson an Sullivan [1981]. The parameterization of the representative agent model that mimics the Caplin-Leahy model has a value for Δ_r of .074 if $c = .2$ and .076 if $c = 1$, and a value for β of .971 if $c = .2$ and .977 if $c = 1$. These depreciation rates are approximately 15% lower and are consistent with annual rates of depreciation of 26.5 and 27.2. These discount factors are consistent with real interest rates in the neighborhood of 12% per annum. Seen in this light the models appear very different.

The second look that we take is to use our derived parameters for the Caplin-Leahy model and insert these into the representative agent model, to derive x_{rep}. This exercise yields a value for x_{rep} of .68 if $c = .2$ and .67 if $c = 1$. These values are within one standard deviation of the estimate of x. Seen this way, similar parameterizations of the two models yield similar results.

5 Conclusions

The search for microfoundations for macroeconomics has brought to the fore the trade-off between realism and tractability in macroeconomic modelling. We need models that reflect the choices that agents actually make in order to make accurate measurements, to forecast and to predict and evaluate the effects of policy experiments. Models that are too realistic, however, quickly become as incomprehensible as the world that they are trying to explain.

In this paper we have developed an approximation of discrete choice that is simple enough that we can solve for the equilibrium dynamic of a market. We found that the discrete choice model and the representative agent shared similar dynamics, but that their parameterization potentially differed. While, in the case of the U.S. automobile market these difference did not appear to be too great, care should be taken in parameterizing the representative agent model. Parameters that appear reasonable on a microeconomic level may not be appropriate for a representative agent who proxies for a group of consumers facing adjustment costs. This distinction may be especially important when conducting policy experiments, since in this case the representative agent model that mimics the discrete choice model might change with the change in policy regime.

At this point it might be useful to comment on a number of potential effects of discrete adjustment that are ruled out in our approximation. Most obviously, our assumption that there was enough time between purchases that heterogeneity in depreciation smoothed out lumps in the cross sectional distribution of holdings, ruled out echoes of previous booms in sales. In our view, this is probably not an important difference between the discrete adjustment model and the representative agent model, since individual heterogeneity is pervasive.

More important, in our view, is the distinction between one-sided and two-sided adjustment. We implicitly assumed that adjustment was one-sided, that is that agents only adjust from small cars to large cars. With one sided, dynamics, heterogeneity tends to flatten the distribution of holdings between the (S,s) bands. With two-sided adjustment this is no longer the case. The distribution of holdings tends to be tent-shaped: it peaks near the purchase target and slopes downward toward the triggers. Changes in the purchase triggers therefore lead to changes in the density near the trigger. This adds an additional source of dynamics. How these dynamics relate to the representative agent model remains an open question.

Whether dynamics are one-sided or two-sided depends on the context. Situations with strong drift, such as inflation in prices or depreciation in investment or durable goods, tend to be well modeled as one-sided. Our results apply mainly to these cases.

6 References

References

[1] Adda, Jérôme, and Russel Cooper (2000a), "Balladurette and Jeppette: A Discrete Approach," *Journal of Political Economy.*

[2] Adda, Jérôme, and Russel Cooper (2000b), "The Dynamics of Car Sales: A Discrete Choice Approach," mimeo., Boston University.

[3] Bar-Ilan, Avner, and Alan Blinder (1996), "Consumer Durables: Evidence on the Optimality of Usually Doing Nothing," *Journal of Money, Credit and Banking* **24**, 258-272.

[4] Bernanke, Ben, (1985), "Adjustment Costs, Durables, and Aggregate Consumption," *Journal of Monetary Economics*, 41-68.

[5] Bertola, Giuseppe, and Ricardo Caballero (1990), "Kinked Adjustment Costs and Aggregate Dynamics," *NBER Macroeconomics Annual 1990.*

[6] Bils, Mark, and Peter Klenow (1998), "Using Consumer Theory to Test Competing Business Cycle Models," *Journal of Political Economy* **106**, 233-261.

[7] Caballero, Ricardo (1993), "Durable Goods: An Explanation for their Slow Adjustment," *Journal of Political Economy* **101**, 351-384.

[8] Caballero, Ricardo, Eduardo Engel, and John Haltiwanger (1997), "Agggregate Employment Dynamics: Building from Microeconomic Evidence," *American Econmic Review*, **87**, 115-137

[9] Caplin, Andrew, and Daniel Spulber (1986), "Menu Costs and the Neutrality of Money," *Quarterly Journal of Economics* **102**, 703-725.

[10] Caplin Andrew, and John Leahy (1991), "State-Dependent Pricing and the Dynamics of Money and Output," *Quarterly Journal of Economics* **106**, 683-708.

[11] Caplin, Andrew, and John Leahy (1997), "Aggregation and Optimization with State-Dependent Pricing," *Econometrica*, **65**, 601-627.

[12] Caplin, Andrew, and John Leahy (2002a), "Durable Goods Cycles," New York University Working Paper.

[13] Caplin, Andrew, and John Leahy (2002b), "On the Relationship between sS Models and Representative Agent Models," New York University Working Paper.

[14] Chah, Eun Young, Valerie Ramey, and Ross Starr (1995), "Liquidity Constraints and Intertemporal Consumer Optimization: Theory and Evidence from Durable Goods," *Journal of Money, Credit and Banking,* **27**, 272-287.

[15] Cooper, Russell, John Haltiwanger (2000), "On the Nature of Capital Adjustment Costs," NBER Working Paper No. 7925.

[16] Cooper, Russell, John Haltiwanger, and Laura Power (1995), "Machine Replacement and the Business Cycle: Lumps and Bumps," NBER Working Paper No. 5260.

[17] Dotsey, Michael, Robert King, and Alexander Wolman (1999), "State Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics* **104**, 655-690.

[18] Fisher, Jonas, and Andreas Hornstein (2000), "(S,s) Inventory Policies in General Equilibrium," *Review of Economic Studies* **67**, 117-145.

[19] Jorgenson, Dale, and Martin Sullivan (1981), "Inflation and Corporate Capital Recovery," in C. Hulten, ed., *Depreciation , Inflation and the taxation of Income from Capital*, Washington: Urban Institute Press.

[20] Leahy, John, and Joseph Zeira (2000), "The Timing of Purchases and Aggregate Fluctuations," NBER Working Paper No. 7672.

[21] Mankiw, N. Gregory (1982), "Hall's Consumption Hypothesis and the Real Interest Rate," *Journal of Monetary Economics* **10**, 417-425.

[22] Rotemberg, Julio, and Michael Woodford (1997) "The Cyclical Behavior of Prices and Costs," Princeton University, mimio..

[23] Thomas, Julia (2001), "Is Lumpy Investment Relevant for the Business Cycle," University of Minnesota Working Paper.

7 Appendix A: Linearization

7.1 Representative Agent Model

We linearize the model about the non-stochastic steady state. We begin with the first order condition for the durable stock.

$$(\alpha - 1)K^{\alpha-1}\hat{k}_t = p\lambda(\hat{p}_t + \hat{\lambda}_t) - \beta(1-\Delta)p\lambda E_t(\hat{p}_{t+1} + \hat{\lambda}_{t+1})$$

Hats represent log-deviations from steady state values. Variables without time subscripts denote steady state values. Since $K^{\alpha-1} = [1-(1-\Delta)\beta]p\lambda$, in steady state, the first order condition becomes:

$$(\alpha - 1)[1-(1-\Delta)\beta]\hat{k}_t = (\hat{p}_t + \hat{\lambda}_t) - \beta(1-\Delta)E_t(\hat{p}_{t+1} + \hat{\lambda}_{t+1}) \tag{17}$$

Linearizing the definition of sales yields:

$$Q\hat{q}_t = K\hat{k}_t - (1-\Delta)K\hat{k}_{t-1}$$

or since $Q = \Delta K$ in steady state

$$\Delta\hat{q}_t = \hat{k}_t - (1-\Delta)\hat{k}_{t-1} \tag{18}$$

Log linearizing marginal cost

$$p\hat{p}_t = Q^{\gamma} c \left(\gamma \hat{q}_t + \hat{c}_t\right)$$

or, since $p = Q^{\gamma} c$,

$$\hat{p}_t = \gamma \hat{q}_t + \hat{c}_t \tag{19}$$

Together with the evolution of the shocks, equations (17)-(19) define the model.

Finally, substituting for price in the first order condition, yields the following second order difference equation:

$$\begin{aligned} p\lambda(\hat{c}_t + \hat{\lambda}_t) - \beta(1-\Delta)p\lambda E_t(\hat{c}_{t+1} + \hat{\lambda}_{t+1}) \quad &= \\ &\frac{\gamma(1-\Delta)}{\Delta} p\lambda \hat{k}_{t-1} + \left[(\alpha-1)K^{(\alpha-1)} - \frac{\gamma p\lambda}{\Delta} - \beta(1-\Delta)\frac{\gamma(1-\Delta)}{\Delta} p\lambda\right] \hat{k}_t \\ &+\beta(1-\Delta)\frac{\gamma p\lambda}{\Delta}\hat{k}_t \end{aligned}$$

which has the following soluiton

$$\hat{k}_t = x_{rep}\hat{k}_{t-1} + y_{rep}\hat{e}_t$$

where

$$x = \frac{-\left[(\alpha-1)K^{(\alpha-1)} - \frac{\gamma}{\Delta}p\lambda(1+\beta(1-\Delta)^2)\right]}{2\beta(1-\Delta)\frac{\gamma}{\Delta}p\lambda} - \sqrt{\left[\frac{(\alpha-1)K^{(\alpha-1)} - \frac{\gamma}{\Delta}p\lambda(1+\beta(1-\Delta)^2)}{2\beta(1-\Delta)\frac{\gamma}{\Delta}p\lambda}\right]^2 - \frac{1}{\beta}}$$

$$y = \frac{p\lambda(1-\beta(1-\Delta)\theta)}{-\left[(\alpha-1)K^{(\alpha-1)} + \frac{\gamma}{\Delta}p\lambda(1-\beta(1-\Delta)^2)\right] - \beta(1-\Delta)\frac{\gamma}{\Delta}p\lambda(x+\theta)}$$

7.2 Caplin-Leahy Model

We first linearize the first order condition for the optimal purchase size:

$$-\varepsilon_s(\hat{S}_t - \hat{\kappa}_t) + (\alpha-1)\hat{\kappa}_t = \hat{p}_t + \hat{\lambda}_t$$

Here $\varepsilon_s = -\frac{v''(S/K)S/K}{v'(S/K)}$. If the time between purchases is sufficiently long ε_s will be approximately equal to $1-\alpha$.[13] For simplicity, we adopt this approximation, so that the first order condition becomes:

$$(\alpha-1)\hat{S} = \hat{p}_t + \hat{\lambda}_t \qquad (20)$$

Linearizing sales

$$Q\hat{q}_t = \mu S\delta\hat{S}_t + \mu S(\hat{s}_t - \hat{s}_{t-1})$$

which since $Q = \mu S\delta$ becomes:

$$\hat{q}_t = \hat{S}_t + \frac{1}{\delta}(\hat{s}_t - \hat{s}_{t-1}) \qquad (21)$$

The marginal cost equation is the same as in the representative agent model:

$$\hat{p}_t = \gamma \hat{q}_t + \hat{c}_t \tag{22}$$

Finally, we linearize the first order condition for the purchase trigger:

$$\begin{aligned}
& v' S\kappa^{\alpha}(\hat{S}_t - \hat{\kappa}_t) + \alpha v \kappa^{\alpha} \hat{\kappa}_t - p\lambda S \hat{S}_t + (1-c)sp\lambda \hat{s}_t - (S-(1-c)s)\, p\lambda \left(\hat{p}_t + \hat{\lambda}_t\right) \\
= \ & s^{\alpha}\hat{s}_t + E_t\beta \Big[v' S\kappa^{\alpha}\left(\hat{S}_{t+1} - \hat{\kappa}_t\right) + \alpha v \kappa^{\alpha} \hat{\kappa}_{t+1} - p\lambda S \hat{S}_{t+1} \\
& \quad + (1-c)(1-\Delta)sp\lambda \hat{s}_t - (S-(1-c)(1-\Delta)s)\, p\lambda \left(\hat{p}_{t+1} + \hat{\lambda}_{t+1}\right)\Big]
\end{aligned}$$

Using the first order condition for S, the $\hat{S}_t$ terms cancel, leaving:

$$\begin{aligned}
& [\alpha v \kappa^{\alpha} - p\lambda S]\,(\hat{\kappa}_t - E_t \beta \hat{\kappa}_{t+1}) \\
& -p\lambda\,(S-(1-c)s)\left(\hat{p}_t + \hat{\lambda}_t\right) + p\lambda\,(S-(1-c)(1-\Delta)s)\,\beta E_t\left(\hat{p}_{t+1} - \hat{\lambda}_{t+1}\right) \\
= \ & [s^{\alpha} - (1-\beta(1-\Delta))\,(1-c)sp\lambda]\,\hat{s}_t
\end{aligned} \tag{23}$$

Finally, linearizing the frictionless capital stock yields

$$(\alpha - 1 - \gamma)\hat{\kappa}_t = \hat{\lambda}_t + \hat{c}_t \tag{24}$$

Equations (20)-(24) define the model.

To derive the second order difference equation in $\hat{s}_t$,we begin with equation (23). We replace $v(S/\kappa)\kappa^\alpha$ using the steady state relationship:

$$(1-\beta)\left[v(S/\kappa)\kappa^\alpha - Sp\lambda\right] + (1-\beta(1-\Delta))(1-c)sp\lambda = \frac{s_t^\alpha}{\alpha}$$

to get

$$\begin{aligned}
& \left[s^\alpha - (1-\beta(1-\Delta))(1-c)sp\lambda\right]\hat{s}_t \\
= & \left[\frac{s_t^\alpha}{(1-\beta)} + (\alpha-1)\,Sp\lambda - \frac{(1-\beta(1-\Delta))(1-c)}{(1-\beta)}sp\lambda\right](\hat{\kappa}_t - E_t\beta\hat{\kappa}_{t+1}) \\
& -p\lambda\left(S-(1-c)s\right)\left(\hat{p}_t + \hat{\lambda}_t\right) + p\lambda\left(S-(1-c)(1-\Delta)s\right)\beta E_t\left(\hat{p}_{t+1} + \hat{\lambda}_{t+1}\right)
\end{aligned}$$

Next combining the expressions (20)-(22) yields:

$$\hat{p}_t = \frac{\varepsilon_S}{\varepsilon_S+\gamma}\left(\frac{\gamma}{\delta}(\hat{s}_t - \hat{s}_{t-1}) - \frac{\gamma}{\varepsilon_S}\hat{\lambda}_t + \hat{c}_t\right)$$

which allows us to replace $\hat{p}_t$

$$\begin{aligned}
& -p\lambda\left(S-(1-c)s\right)\frac{\varepsilon_S}{\varepsilon_S+\gamma}\frac{\gamma}{\delta}\hat{s}_{t-1} \\
& +\left[s^\alpha - (1-\beta(1-\Delta))(1-c)sp\lambda + p\lambda\frac{\varepsilon_S}{\varepsilon_S+\gamma}\frac{\gamma}{\delta}\left((1-\beta)S - (1-\beta(1-\Delta))(1-c)s\right)\right]\hat{s}_t \\
& -p\lambda\left(S-(1-c)(1-\Delta)s\right)\beta E_t\frac{\varepsilon_S}{\varepsilon_S+\gamma}\frac{\gamma}{\delta}\hat{s}_{t+1} \\
= & \left[\left[\frac{s_t^\alpha}{(1-\beta)} + \alpha\left(S - \frac{(1-\beta(1-\Delta))(1-c)s}{(1-\beta)}\right)p\lambda\right] - p\lambda S\right](\hat{\kappa}_t - E_t\beta\hat{\kappa}_{t+1}) \\
& -p\lambda\left(S-(1-c)s\right)\left(\frac{\varepsilon_S}{\varepsilon_S+\gamma}\right)\left(\hat{\lambda}_t + \hat{c}_t\right) \\
& +p\lambda\left(S-(1-c)(1-\Delta)s\right)\beta\left(\frac{\varepsilon_S}{\varepsilon_S+\gamma}\right)E_t\left(\hat{\lambda}_{t+1} + \hat{c}_{t+1}\right)
\end{aligned}$$

Finally, we use (24) and the assumption that the shocks are permanent:

$$E_t\left(\hat{\lambda}_{t+1}+\hat{c}_{t+1}\right)=\hat{\lambda}_t+\hat{c}_t$$

to get

$$\begin{aligned}
&-p\lambda\left(S-(1-c)s\right)\frac{\varepsilon_S}{\varepsilon_S+\gamma}\frac{\gamma}{\delta}\hat{s}_{t-1}\\
&+\left[s^{\alpha}+p\lambda\frac{\varepsilon_S}{\varepsilon_S+\gamma}\frac{\gamma}{\delta}(1-\beta)S-\left(1+\frac{\varepsilon_S}{\varepsilon_S+\gamma}\frac{\gamma}{\delta}\right)((1-\beta(1-\Delta))(1-c)sp\lambda)\right]\hat{s}_t\\
&-p\lambda\left(S-(1-c)(1-\Delta)s\right)\beta E_t\frac{\varepsilon_S}{\varepsilon_S+\gamma}\frac{\gamma}{\delta}\hat{s}_{t+1}\\
=\;&\frac{1}{\alpha-1-\gamma}\left(s_t^{\alpha}-\left(1-\beta(1-\Delta)\right)(1-c)sp\lambda\right)\left(\hat{\lambda}_t+\hat{c}_t\right)
\end{aligned}$$

This is a second order difference equation has a solution is of the form

$$\hat{s}_t=x\hat{s}_{t-1}+y\hat{e}_t$$

where

$$\begin{aligned}
x\;=\;&\frac{s^{\alpha}+\chi(1-\beta)S-\left(1+\frac{\chi}{p\lambda}\right)((1-\beta(1-\Delta))(1-c)sp\lambda)}{2\left(S-(1-c)(1-\Delta)s\right)\beta\chi}-\\
&\sqrt{\left(\frac{s^{\alpha}+\chi(1-\beta)S-\left(1+\frac{\chi}{p\lambda}\right)((1-\beta(1-\Delta))(1-c)sp\lambda)}{2\left(S-(1-c)(1-\Delta)s\right)\beta\chi}\right)^2-\frac{(S-(1-c)s)}{\beta\left(S-(1-c)(1-\Delta)s\right)}}
\end{aligned}$$

and

$$y = \frac{1}{\alpha - 1 - \gamma} \frac{s_t^{\alpha} - (1 - \beta(1 - \Delta))\,(1 - c)sp\lambda}{s^{\alpha} + \chi(1 - \beta)S - (p\lambda + \chi)\,(1 - \beta(1 - \Delta))(1 - c)s + (S - (1 - c)(1 - \Delta)s)\,\beta\chi(1 + x)}$$

and $p\lambda \frac{\varepsilon_S}{\varepsilon_S + \gamma} \frac{\gamma}{\delta}$.

*Respectively, New York University and NBER, and Boston University and NBER. We are grateful to Orazio Attanasio, Russell Cooper, Chris Foote, Chris House, Alejandro Micco, David Laibson, Greg Mankiw, Torsten Persson, Thomas Sargent, Christopher Sims and two annonymous referees. for helpful comments and discussions. We gratefully acknowledge support from the NSF. Leahy thanks the Sloan Foundation for financial support.

Notes

[1]Discrete adjustment, particularly in the form of sS policies, have been studied in the context of pricing (Caplin and Spulber [1986], Caplin and Leahy [1991, 1997], and Dotsey, King and Wolman [1999]), labor hiring and firing (Caballero, Engel and Haltiwanger [1997]), investment (Cooper, Haltiwanger and Power [1995], Thomas [2001]), and the demand for durable goods (Bertola and Caballero [1990], Bar-Ilan and Blinder [1996], Adda and Cooper [2000a, 2000b], and Caplin and Leahy [2002a]).

[2]This paper draws heavily on work presented in Caplin and Leahy [2002a, 2002b]. The approximation is worked out in Caplin and Leahy [2002a]. The mapping between the representative agent model and the discrete choice model is worked out under more general assumptions in Caplin and Leahy [2002b].

[3]See, in particular, Bertola and Caballero [1990] and Cooper and Haltiwanger [2000]. Many of the papers cited above are also relevant to this issue.

[4]Adda and Cooper [2000b] argue that this extensive margin is more important to distributional dynamics than the intensive margin.

[5]See Cooper, Haltiwanger and Power (1995).

[6]It is ironic that in this case our model is a more fleshed out version of the model employed by Bar-Ilan and Blinder from which the above quote was taken.

[7]This is a fairly standard assumption in the literature on durable goods. It receives some empirical support from Bernanke [1985].

[8]For the marginal utility of wealth to follow a random walk it must be the case that the discount factor is equal to the interest rate.

[9]With perfect capital markets the marginal utlity of wealth will be equal across agents.

[10]Depreciation creates a natural tendency towards one-sided adjustment. If the adjustment cost is too small, however, increases in price or in the marginal utility of wealth may create sufficient incentive for agents to reduce durable holdings.

[11]Mankiw [1982], Bernanke [1985], Caballero [1993], Chah, Ramey and Starr [1995] all make this assumption. All of these papers (implicitly or explicitly) assume that the price of durable goods is independent of demand and find that the estimated parameters do not make sense within the context of the model. It is possible, however, that price is not perfectly elastic and that the estimated parameters need to be reinterpreted in light of the mapping between the representative agent model and the sS model.

[12]It is also interesting that lagged disposible income is insignificant. This equation therefore passes a Hall orthogonality test of the rational expectations permanent income hypothesis.

[13]If the time between purchases were fixed then ε_s would be exactly $1-\alpha$. The difference arises since an increase in S postpones the next purchase.