

Solution to problem set 3

1. Based on Howard, J.V. (1992), “A Social Choice Rule and Its Implementation in Perfect Equilibrium”, *Journal of Economic Theory* **56**, 142-159.

The following solution is taken from Osborne and Rubinstein’s solution manual to *A Course in Game Theory*. Let x^* be the Nash solution of $\langle X, D, \succsim_1, \succsim_2 \rangle$. We will show that the unique subgame perfect equilibrium is x^* . First, note that if player 1 chooses x^* at the first stage, then player 2 can do no better than choose $(x^*, 1)$ at the second stage. This follows from observing that the outcome is either $p \cdot x$ or $p^2 \cdot x^*$ (where (x, p) is the choice of player 2 at the second stage), and if $p \cdot x \succ_2 x^*$, then from the definition of the Nash solution, we have $p \cdot x^* \succ_1 x$, so that the outcome is $p^2 \cdot x^*$. Thus, all subgame perfect equilibrium outcomes are at least as good for player 1 as x^* .

Now, let y be the choice of player 1 in the first stage. By choosing (x, p) for which $x \succ_1 p \cdot y$ in the second stage, player 2 can obtain the outcome $p \cdot x$. Let u_i for $i = 1, 2$ be a von Neumann-Morgenstein utility function that represents \succsim_i and satisfies $u_i(D) = 0$. This means that for any $p < u_1(x)/u_1(y)$ player 2 can achieve the payoff $pu_2(x)$. Thus, in an equilibrium player 2’s payoff is equal to $\max_{x,p} pu_2(x)$ subject to $p \leq \min\{u_1(x)/u_1(y), 1\}$. If $u_1(y) > u_1(x^*)$, then the solution of this problem is $(x, p) = (x^*, u_1(x^*)/u_1(y))$, in which case player 1’s payoff is less than $u_1(x^*)$. If $u_1(y) < u_1(x^*)$, then the solution of the problem is $(x, p) = (x, 1)$ where $x \sim_1 y$, hence player 1’s payoff is $u_1(y)$. Since player 1’s payoff in equilibrium is $u_1(x^*)$, neither case is thus an equilibrium. Finally, if $u_1(y) = u_1(x^*)$ but $y \neq x^*$, then player 2 chooses $(x^*, 1)$ and the outcome is x^* . Thus, in any subgame perfect equilibrium the outcome is x^* .

Note that in addition to the equilibrium in which player 1 chooses x^* and player 2 chooses $(x^*, 1)$, for any y with $y \prec_1 x^*$ there is an equilibrium in which player 1 chooses x^* and player 2 chooses $(y, 1)$.

2. Based on Rubinstein A. and A. Wolinsky (1992), “Renegotiation-Proof Implementation and Time Preferences”, *American Economic Review* **82**(3), 600-614.

(a) See the proof of Proposition 0, pp. 603-605.

(b) See the proof of Proposition 1, pp. 606-607.