Solution to midterm exam

1. *Full extraction of the surplus* - this question is based on **Crémer and McLean (88)** and **Neeman (2004)**.

Consider running a simple second-price auction where ties are broken by tossing a fair coin. In this private values setting there exists a dominant strategy equilibrium in which agents bid their true valuations and the good is sold to the highest valuation agent (hence, the auction is strategy-proof and ex-post efficient). Note that high valuation buyers retain some informational rent. In particular, they expect a payoff of $\frac{2}{3}(h-l)$. Can the seller extract more surplus from the buyers, while still retaining the incentive to tell the truth?

Consider augmenting the above auction by asking each bidder to pay a transfer that depend *only* on the other agent's bid. Since an agent cannot affect these transfers with his own bid, it is still weakly dominant for him to bid his true value. Let t_b^i denote the transfer paid to agent i when agent j submits a bid $b \in \{l, h\}$. For each i, the pair of transfers (t_l^i, t_h^i) are chosen to be the solution to the following equation:

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} t_h^i \\ t_l^i \end{pmatrix} = \begin{pmatrix} \frac{2}{3}(h-l) \\ 0 \end{pmatrix}$$

Since Π_i has full row rank there exists a solution:

$$\begin{aligned} t_h^i &= -\frac{2}{3}(h-l) \\ t_l^i &= \frac{4}{3}(h-l) \end{aligned}$$

Hence, by adding these lotteries the seller is able to bring each type of buyer to his IR constraint. In other words, the seller is able to extract the entire interim expected surplus from each buyer.

2. *The multiple equilibrium problem in Bayesian Nash implementation* - this question is based on an example from **Palfrey (1992)**.

a. Note first that the optimal rule is a simple majority rule. Consider implementing this rule with a direct mechanism in which agents are asked to simultaneously report their favorite outcome. Fix some pair of strategies for agents j and k (note that a strategy here is a function that assigns an action to each possible preference relation over a and b). Whatever those strategies may be, agent i is indifferent between saying a or b when he is not pivotal, and he strictly prefers to say his favorite outcome.

b. Consider a mechanism with outcome function g for which the optimal rule is a Bayesian Nash equilibrium outcome. This means that the mechanism induces a Bayesian game in which there is a Nash equilibrium $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ whose outcome is the alternative preferred by the majority of players. Let $\sigma_i(x)$ denote player i's action whenever his top alternative is x (note this mechanism need not be a direct revelation game, so that a player's action may be a vector of items as in the canonical Nash mechanism). Let $\hat{\sigma}$ be a strategy profile in which $\hat{\sigma}_i(a) = \hat{\sigma}_i(b) = \sigma_i(a)$. Assume, by contradiction, that $\hat{\sigma}$ is *not* an equilibrium. Then this means that some player i has an action m such that $g(m, \sigma_{-i}(a)) = b$. But this implies that whenever player i prefers b to a, and the other players prefer a to b and use the equilibrium strategies σ_{-i}, player i can guarantee the outcome b by choosing m. However, if at least one of the other players preferred b to a and player i chose m, then the outcome might be a and not b.

Consider the worst case scenario in which $g(m, \sigma_{-i}(a)) = b$, but $g(m, \sigma_j(b), \sigma_k(a)) = a$ and $g(m, \sigma_{-i}(b)) = a$. If player i were to play $\sigma_i(b)$, he would get a lottery in which a is chosen with probability q^2 and b is chosen with probability $1 - q^2$. If i were to deviate to m, then in the worst case scenario, he would get a lottery in which b is chosen with probability q^2 and a is chosen with probability $1 - q^2$. If $q > \sqrt{0.5}$, then the second lottery first order stochastically dominates the first. This means that if $q > \sqrt{0.5}$, player i has a profitable deviation in contradiction to our assumption that σ is an equilibrium.

3. *Sequential screening* - this question is based on **Courty and Li (1992)** and **Eliaz and Spiegler (2007)**.

Suppose the restaurant manager could observe the customer's type, i.e. his prior belief θ on the satiated state. What is the optimal contract to offer this customer? In each state customer will never spend more than \$20. Hence, the highest surplus the restaurant can ever hope to extract from this customer is \$20 in each state. Because the customer will always eat a main course, this can be achieved with a contract in which the price of the main course is \$20 and the dessert is given for free (another way to describe this contract is an "all-you-can-eat buffet" for \$20). Since this is the first-best contract for a customer of type θ, and since it is independent of his type, it is the optimal contract to offer to all customer types. In other words, the optimal menu of contracts is $\{(20, 0)\}$, i.e., there is no price discrimination.

References

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[5] Palfrey, T. (1992): "Implementation in Bayesian Equilibrium: The Multiple Equilibrium Problem in Mechanism Design", in Advances in Economic Theory, Volume 1, edited by J. J. Laffont, Cambridge: Cambridge University Press, 291-292.