Problem Set: Common priors and speculative trade

1. Consider two firms whose profits can be any integer, positive or negative. Each firm is informed only of its profit. Let E be the event that firm 2's profit is higher. That is, E is the set of all the states (i, j) where $i < j$. In each state no firm can tell whether or not 2's profit is higher. Therefore, it is common knowledge in each state that both firms are ignorant of E.

Show that there is no nonempty finite event F at which it is common knowledge that the agents are ignorant of E, and moreover, this holds true at F also after F becomes common knowledge.

2. Consider two competing detectives who are both after a suspect in a criminal case. Let Ω be a finite set consisting of all the potential suspects. A state of the world $\omega \in \Omega$ is the individual who committed the crime. Let $P_i(\omega)$ represent the set of individuals that detective i suspects to be the potential criminals when individual ω committed the crime. Assume the detectives' information structures are partitional.

(a) Prove that it cannot be common knowledge that the height of the shortest suspect of detective 1 differs from the height of the shortest suspect of detective 2.

(b) Prove that it cannot be common knowlege that there is a bespectacled individual among the suspects of detective 1 but not among the suspects of detective 2.

3. Assume two players have information structures, which are not partitional but satisfy only the following conditions: for all $i = 1, 2$,

(i) For all $\omega \in \Omega$, $\omega \in P_i(\omega)$ (i.e., if i knows the set X, then X is true)

(ii) For all $\omega \in \Omega$ and for all $\omega' \in P_i(\omega)$, $P_i(\omega') \subset P_i(\omega)$ (i.e., if i knows X, he also knows that he knows X).

Let x be a random variable on Ω and α a number. Show that there can be a state at which it is common knowlege that, conditional on his information, 1 believes that the expectation of x is strictly above α and, conditional on his information, 2 believes that the expectation of x is strictly below α.

4. Consider two risk-neutral agents, 1 and 2. In the absence of private information, they agree that it is equally likely that the interest rate will rise or fall tomorrow. But suppose agent 1 has observed a signal correlated with the interest rate movement. In particular, if he observes signal x^u, he believes that the discount rate will rise with probability $u_1 > \frac{1}{2}$, and if he observes signal x^d, he believes that the discount rate will rise with probability $d_1 < \frac{1}{2}$. Agent 2 has not observed the signal, but if he had, his probabilities of an interest rise would be u_2 and d_2, respectively. The two agents are considering making a bet on whether agent 1's signal is correct or not. Show that if $u_2 > u_1$ and $d_2 < d_1$, then any agreeable bet is not incentive-compatible.

5. Assume there are nine states of the world, labeled $\omega_1, \ldots, \omega_9$. Consider two risk-neutral agents with the following partitional information structures:

$$\begin{aligned} I_1 &= \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5, \omega_6\}, \{\omega_7, \omega_8, \omega_9\}\} \\ I_2 &= \{\{\omega_1, \omega_4, \omega_7\}, \{\omega_2, \omega_5, \omega_8\}, \{\omega_3, \omega_6, \omega_9\}\} \end{aligned}$$

Both agents agree that each state is equally likely. Suppose agent 1 receives private information that the true state of the world, ω^*, is in $\{\omega_4, \omega_5, \omega_6\}$, while agent 2 receives private information that the true state of the world is in $\{\omega_3, \omega_6, \omega_9\}$. The agents wish to bet on whether or not the true state of the world is in the event $E = \{\omega_1, \ldots, \omega_6\}$. They contemplate whether or not to make the following bet: if $\omega^* \in E$, agent 2 pays one unit to agent 1, and if $\omega^* \notin E$, agent 1 pays one unit to agent 2.

The agent use the following sequential procedure in order to decide whether or not to sign this bet. First, agent 2 offers the bet to agent 1, who decides whether or not to accept this bet. If agent 1 refuses, the bet is not signed. If agent 2 agrees, then agent 2 decides whether he still wishes to sign the bet, given agent $1's$ willingnesss to sign the bet. The agents then take turn saying yes or no the bet. Each time an agent says yes, the other agent updates his posterior belief.

Will the agents continue saying yes to this bet? If not, in how many rounds will the bet be refused?