V31.0018: Statistics (Lab)
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Solutions for Homework 2 (Chapter 4)

4.3 D: discrete
C: continuous

(a) D
(b) C
(c) C
(d) D
(e) C
(f) C

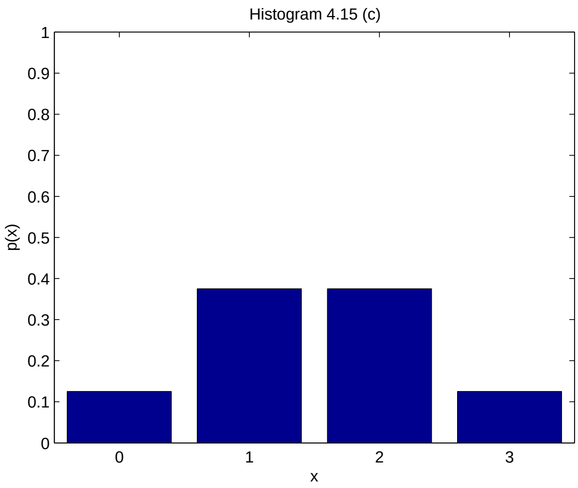
4.7 $x = 0, 1, 2, \ldots$
x is a discrete random variable.

4.13 (a) $P(x \leq 12) = P(x = 10) + P(x = 11) + P(x = 12) = .2 + .3 + .2 = .7$
(b) $P(x > 12) = .1 + .2 = .3$
(c) $P(x \leq 14 = 1$
(d) $P(x = 14) = .2$
(e) $P(x \leq 11 \text{ or } x > 12) = 1 - P(x = 12) = .8$

4.15 (a) HHH, HHT, HTH, HTT,
THH, THT, TTH, TTT
The numbers of heads, x, for each of these points is fairly obvious.

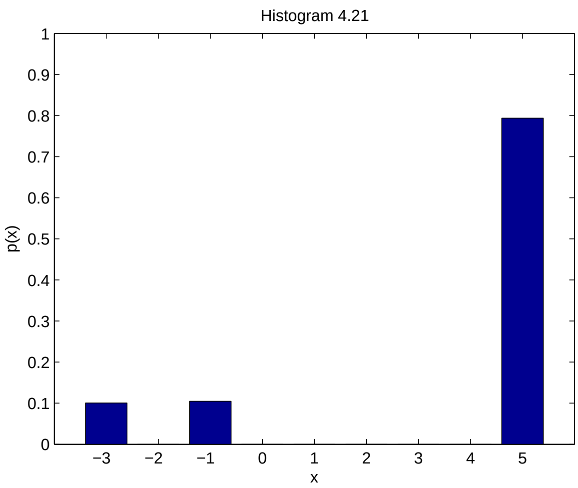
(b) $P(x = 0) = 1/8$
$P(x = 1) = 3/8$
$P(x = 2) = 3/8$
$P(x = 3) = 1/8$

(c)



(d) $P(x \geq 2) = 1/2$

4.21 $P(x = -3) = 68/678 = .100$
$P(x = -1) = 71/678 = .105$
$P(x = 5) = 539/678 = .794$



4.25 The mother passes on an X chromosome for sure, the father with probability one half. Hence the probability of a given child being a boy is one half (if the assumptions of the problem are correct...).

$$P(\text{at least one boy}) = 1 - P(3 \text{ girls}) = 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = \frac{7}{8}$$

4.35 (a) analogous to 4.21

(b)

$$E(x) = \sum_{i=1}^{9} p(x = i) \times i =$$

$$= p(x = 1) \times 1 + p(x = 2) \times 2 + \ldots + p(x = 9) \times 9 = 4.6485$$

(c) Lower numbers are more likely to be selected by people as the first significant digit in a number if they are told to select a number "at random" (see also exercise 3.23).

4.39 x: net payoff
Bet on red number succesful: $x = -5 + 10 = +5$, $P(x = +5) = 18/38$
Black or green outcome: $x = -5 + 0 = -5$, $P(x = -5) = 20/38$

$$E(x) = \frac{18}{38} \times 5 + \frac{20}{38} \times (-5) = -.263$$

This is why casinos are a profitable business!

4.40 x: net payoff from the lottery (see 4.39!)

$$E(x) = \frac{1}{23,000,000} \times 6,999,999 + \frac{22,999,999}{23,000,000} \times (-1) = -.70$$

The organizer of the lottery makes a profit of 70 cents for each participant in the lottery, if we abstract from the cost of running the lottery.

4.43 x: number of training units needed to master the program

(a)
- $E(x) = .1 \times 1 + .25 \times 2 + .4 \times 3 + .15 \times 4 + .1 \times 5 = 2.9$
  The mean is the average number of units that a student will need to master the program.
- $median(x) = 3$, since $P(x < 3) = .35$ and $P(x \leq 3) = .75$
  The faster half of the students masters the program before the third unit or exactly after having absolved the third unit.

4.53 (a) $n = 25,\ p = .7:$

$$P(x < 10) = P(x \leq 9) = \sum_{x=1}^{9} p(x) = p(1) + p(2) + \ldots + p(9) =$$

$$= .000(n = 25, p = .7)$$

(b) $n = 15,\ p = .9:$

$$P(x \geq 10) = 1 - P(x \leq 9) = 1 - \sum_{x=1}^{9} p(x) = 1 - .002 = .998$$

(c) $n = 5,\ p = .2:$

$$P(x = 2) = P(x \leq 2) - P(x \leq 1) = .206 - .069 = .137$$

4.57 (a) The event *enter* or *don't enter* a treatment program is obviously binomial. Also, in a random sample it is reasonable to assume that the probabilities of the different smokers to enter the program are independent among each other.

(b) $p = .05 = 1/20$. Interpretation: Out of 20 smokers, roughly one enters a treatment program.

(c)

$$E(x) = n \times p = 200 \times .05 = 10$$

Interpretation: On average, 10 out of a sample of 200 randomly selected smokers enters a treatment program.

4.59 analogous to 4.57

4.67 (a) Assumptions needed:

- Independence between the quality of the different golf balls that are selected.
- The probability of being deficient is equal for all selected golf balls of a given brand.

(b)

$$E(x) = \mu_x = .1 \times 24 = 2.4$$

$$\sigma_x^2 = np_x q_x = np_x(1 - p_x) = 24 \times .1 \times .9 = 2.16$$

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{2.16} = 1.47$$

(c)

$$p_y = 1 - p_x = 1 - .1 = .9$$
$$q_y = 1 - p_y = .1$$
$$n_y = n_x = n = 24$$
$$E(y) = \mu_y = .9 \times 24 = 21.6$$
$$\sigma_y^2 = np_yq_y = 24 \times .9 \times .1 = \sigma_x^2 = 2.16$$
$$\sigma_y = \sigma_x\sqrt{2.16} = 1.47$$