

Exchange Rate Determination under Interest Rate Rules*

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October 2006

Abstract

We propose a theory of exchange rate determination under interest rate rules in a two-country optimizing-agent model. We first show that simple interest-rate feedback rules can determine a unique and stable equilibrium without any explicit reaction to the nominal exchange rate.

We characterize how the behavior of the exchange rate and the terms of trade depends in a critical way on the monetary regime chosen, though not necessarily on monetary shocks. We give a simple account of exchange rate volatility in terms of monetary policy rules, we provide an explanation of the relation between nominal exchange rate volatility and macroeconomic variability in terms of the monetary regime adopted by monetary authorities.

*This is a revised version of our previous working paper “Monetary Policy Rules and the Exchange Rate”. We are grateful to Maurice Obstfeld and Michael Woodford for useful comments and discussions. We thank also Kosuke Aoki, Giancarlo Corsetti, Martin Uribe, Paolo Pesenti, Lars Svensson and Kenneth Rogoff. We also thank Katsuyuki Shibayama for excellent research assistantship. We thank seminar participants at the Bank of England, INSEAD, ESSIM 2001 and University of Pennsylvania. All the errors are ours. Pierpaolo Benigno gratefully acknowledge financial support from the Alfred P. Sloan Doctoral Dissertation Fellowship. © Gianluca Benigno and Pierpaolo Benigno.

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Keywords: interest rate rules, exchange rate regimes.
JEL Classification Number: E52, F41.

This paper develops a theory of exchange rate determination under interest-rate rules. The framework is a two-country optimizing-agent model with sticky prices, incorporating elements from both the recent closed-economy literature on the effects of monetary policy, in the spirit of the Neo-Wicksellian framework (Woodford, 2003), and the open-economy literature on exchange rate determination. We design monetary policy regimes by the interaction of interest-rate rules followed by the monetary policymakers of both countries.¹

As a first step, the design of monetary policy rules is crucial for the determinacy of the equilibrium. We show that simple interest-rate feedback rules can determine a unique and stable equilibrium. The important result is that there is no need to include an explicit reaction toward the nominal exchange rate. In a floating regime, the Taylor's principle that monetary policy rules should be aggressive toward domestic inflation holds provided that the principle is applied to both rules. We further show that appropriately designed fixed-exchange rate regimes are not destabilizing.

Once the equilibrium is determinate, we examine how the dynamic behavior of the terms of trade and of the nominal exchange rate depends on the exchange rate regime adopted.

Under a floating regime, the exchange rate is, in general, non stationary. An important feature of our findings is that the source of perturbations in the nominal exchange rate are real shocks drawn from a stationary distribution. On the other hand, these real shocks generate a stationary pattern for the real macroeconomic variables of the model. The way monetary rules are designed is crucial for determining the non-stationary property of the nominal exchange rate: rules that target the level of the nominal exchange can stabilize its long-run value.

¹See Taylor (1999) for a collection of works on monetary rules.

This paper is related to early contribution that have analyzed interest-rate rules in open-economy macro models as Ball (1999), Ghironi (1998), McCallum and Nelson (1999), Monacelli (2004), Svensson (1999,2000) and Weeparana (1998).² In the recent years, after these works and our contribution, there has been an increasing number of studies in related open-economy models. One important example among others is the model of Laxton and Pesenti (2003) which presents features rich enough to be the base of more sophisticated models of open economies currently used by Central Banks and other institutions. De Fiore and Liu (2005) have studied the issue of determinacy in a small-open economy model in which monetary frictions are non-negligible, while Gali and Monacelli (2005) have focused on a continuum of small open economies.

The structure of the paper is the following: in the first section we present the model and we briefly discuss its most novel aspects. We start our analysis from the log-linear approximation to the equilibrium conditions. We then specify the monetary policy rules that we are considering. The analysis of equilibrium determinacy is addressed in Section 2. Section 3 describes the allocation under flexible prices. In Section 4 we explore the positive consequences of different rules for the dynamic behavior of the terms of trade and the nominal exchange rate. Section 5 concludes.

1 The Model

We consider a world economy composed by two countries labelled Home (H) and Foreign (F). The economy is populated by a continuum of agents on the interval $[0, 1]$. The population on the segment $[0, n)$ belongs to country H , while that on the segment $[n, 1]$ belongs to country F . A generic agent is both producer and consumer: a producer of a single differentiated product and a consumer of all the goods produced in both countries.

Each agent derives utility from consuming an index of consumption goods,

²For an empirical perspective, see Clarida et al. (1998) among others.

a composite index of the Home and Foreign goods, and from the liquidity services of holding money³, while derives disutility from producing a single differentiated product. Households maximize the expected discounted value of the utility flow.

We assume that markets are complete within and across countries by allowing agents to trade in a set of nominal state-contingent bonds that span all the states of nature (as in Chari et al., 2002).

In order to give a role to monetary policy for the short-run fluctuations of the economy, we introduce nominal price rigidity rationalized in a context of a monopolistically-competitive goods market. Nominal rigidities are modelled using a price-setting mechanism *a la* Calvo (1983).⁴ In each period a firm can set a new price with a probability, $1 - \alpha$, which is the same for all firms and is independent of the amount of time elapsed since it last changed price. When a firm has an opportunity to set a new price, it does so in order to maximize the expected discounted value of its net profits. Under this framework, $1/(1 - \alpha)$ represents the average duration of contracts within a country. We allow the degrees of rigidity to vary across countries.

We consider the short-term nominal interest rate as the instrument of policy which is set to react to other macroeconomic variables. We analyze the positive consequences of alternative monetary regimes defined in terms of different combinations of monetary policy rules.

In the next section we present the log-linear approximation of the structural equilibrium conditions of the model. Details on the model are in Benigno (2004) and Benigno and Benigno (2001).

³We assume that utility is separable in these three components.

⁴Yun (1996), in a closed-economy model, and Kollmann (1996), in an open-economy model, introduce Calvo's type of price-setting behavior into dynamic general equilibrium monetary models.

1.1 AD Block

The aggregate demand block is derived from the log-linear approximation to the first-order conditions of the representative consumers in the Home and Foreign countries.⁵

Intertemporal optimizing decisions on how to allocate wealth among consumption and the nominal state contingent securities imply first order conditions that equalize the marginal rate of substitution between current and one-period ahead consumption to the asset prices. This set of conditions implies a stochastic Euler equation – one for each country– that describes the intertemporal link between current and one-period ahead expected consumption and relates it to the risk-free real return in units of the consumption index.

The assumption of complete international markets, combined with the law of one price and the fact that the consumption index is common across countries, implies that there is perfect consumption risk sharing across countries. In a log-linear approximation the weighted average of the stochastic Euler equations implies an aggregate stochastic Euler equation of the form

$$E_t \hat{C}_{t+1} = \hat{C}_t + \rho^{-1} n (\hat{i}_t^H - E_t \pi_{t+1}^H) + \rho^{-1} (1 - n) (\hat{i}_t^F - E_t \pi_{t+1}^F), \quad (1)$$

where \hat{C} is aggregate consumption which is perfectly risk-shared across countries, \hat{i}^H and \hat{i}^F are the nominal interest rates in the Home and Foreign countries, π^H and π^F are the respective producer inflation rates (where $\pi_t^H \equiv \ln P_{H,t} / P_{H,t-1}$ and $\pi_t^F \equiv \ln P_{F,t} / P_{F,t-1}$) and ρ is the inverse of the intertemporal elasticity of substitution in consumption.⁶ Expected consumption growth depends on the consumption-based real interest rate, which can

⁵In what follows, given a variable X^H for country H and the respective variable X^F for country F , we define $X^W \equiv nX^H + (1 - n)X^F$ and $X^R \equiv X^F - X^H$. Instead \tilde{X} denotes the log deviations of the variable X from the steady state when prices are flexible, while \hat{X} denotes the log deviations of the variable X from the steady state when prices are sticky.

⁶We have denoted with P_H the price of the home produced goods in the home currency and with P_F the price of the foreign produced goods in the foreign currency.

be written as a weighted average of the Home and Foreign consumption-based real interest rates. In fact, n and $(1 - n)$ represent respectively the shares of Home and Foreign goods in total consumption index. In this model, n and $1 - n$ coincide with the population size of the Home and Foreign country, respectively.

By taking the difference between the two stochastic Euler equations we obtain the log-linear approximation of the uncovered interest parity:

$$\mathbb{E}_t \Delta \hat{S}_{t+1} = \hat{i}_t^H - \hat{i}_t^F, \quad (2)$$

where S is the nominal exchange rate (the price of foreign currency in terms of home currency).

Output in each country is determined according to

$$\hat{Y}_t^H = (1 - n)\hat{T}_t + \hat{C}_t + g_t^H, \quad \hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t + g_t^F, \quad (3)$$

where \hat{Y}^H and \hat{Y}^F denote output produced in country H and F ; g^H and g^F are country-specific demand shocks; T , the terms of trade, is defined as $T \equiv SP_F/P_H$. Each country's output is affected by aggregate consumption C . However, the country-specific demand shocks and the terms of trade can create dispersion of output across countries. Using (3), world output can be written as the sum of world consumption and demand shocks

$$\hat{Y}_t^W = \hat{C}_t + g_t^W.$$

Finally, terms of trade are related to the producer inflation differential and to the exchange rate changes as follows

$$\hat{T}_t = \hat{T}_{t-1} + \Delta \hat{S}_t + \pi_t^F - \pi_t^H. \quad (4)$$

We write our variables of interest in terms of deviation from the equilibrium level that would arise when prices were flexible. The world output gap is then defined as difference between its sticky-price equilibrium and the natural rate that arises under flexible prices

$$y_t^W = \hat{Y}_t^W - \tilde{Y}_t^W,$$

while the relative output gap is the difference between the sticky-price equilibrium terms of trade and its flexible-price level, \tilde{T}_t :

$$y_t^R = \hat{T}_t - \tilde{T}_t. \quad (5)$$

Using the definition of world output gap, we can rewrite equation (1) as

$$E_t y_{t+1}^W = y_t^W + \rho^{-1} n (\hat{i}_t^H - E_t \pi_{t+1}^H - \tilde{R}_t^W) + \rho^{-1} (1-n) (\hat{i}_t^F - E_t \pi_{t+1}^F - \tilde{R}_t^W), \quad (6)$$

where \tilde{R}_t^W represents the world natural rate of interest (see Woodford, 2003). It is the rate that would arise in the case prices were perfectly flexible and each country's inflation rate were zero. As it is shown in section 3, it is a combination of world supply and demand shocks. Equation (6) can be interpreted as a microfounded 'open-economy' IS curve.⁷

1.2 AS Block

The supply block of the model contains the two aggregate supply equations, one for each country, given by

$$\begin{aligned} \pi_t^H &= \lambda^H \widehat{mc}_t^H + \beta E_t \pi_{t+1}^H, \\ \pi_t^F &= \lambda^F \widehat{mc}_t^F + \beta E_t \pi_{t+1}^F, \end{aligned}$$

where \widehat{mc} represents the deviation of the real marginal costs from the steady state; λ^H and λ^F are combinations of the structural parameters of the model while β is the intertemporal discount factor in consumer preferences.⁸

Under the Calvo-style price-setting model, producer inflation rates exhibit a forward-looking behavior. They depend on the current and expected deviations of the real marginal costs from the steady state. The short-run

⁷For a closed-economy counterpart, see Kerr and King (1996) and Woodford (1996, 2003).

⁸We have defined $\lambda^i \equiv [(1-\alpha^i\beta)(1-\alpha^i)/\alpha^i] \cdot [1/(1+\sigma\eta)]$ for $i = H$ or F . We have that σ is the elasticity of substitution in consumption between the differentiated goods produced within a country while η is the elasticity of the disutility of producing the differentiated goods.

response of inflation to real marginal costs is related to the probability that in each period sellers adjust their prices. When the two countries have the same degree of nominal rigidity, producer inflation rates react similarly to movements in the respective real marginal costs. Using the equilibrium relations in the labor market, it is possible to write the real marginal costs as a function of the marginal rate of substitution between consumption and the production of the domestic goods. We can then write the aggregate supply equations as

$$\pi_t^H = \lambda^H[(1-n)(1+\eta)(\hat{T}_t - \tilde{T}_t) + (\rho + \eta)y_t^W] + \beta E_t \pi_{t+1}^H, \quad (7)$$

$$\pi_t^F = \lambda^F[-n(1+\eta)(\hat{T}_t - \tilde{T}_t) + (\rho + \eta)y_t^W] + \beta E_t \pi_{t+1}^F. \quad (8)$$

There are some interesting novelties of these AS equations that arise in open economies.

First, the real marginal costs are not proportional only to the output gap, because of the role played by the terms of trade. The smaller and more open is a country, the more relative prices influence the real marginal costs and thus the producer inflation rates. Focusing on the AS equation in country H , an increase in the terms of trade shifts the AS equation and increases inflation of country H through two channels. The first is the expenditure-switching effect: an increase in the price of goods produced in country F relative to goods produced in H boosts the demand of goods produced in country H , pushing up inflation in this country. The second channel works through a the reduction in the marginal utility of nominal income: the optimal response is to increase prices in order to offset the fall in revenues.

Second, the relation between real marginal costs and the terms of trade may create an intrinsic inertia in the real marginal costs. Indeed, conditional on the monetary policy regime chosen, terms of trade might be sluggish; it follows that the adjustment of real marginal costs to disturbances might also be sluggish and so inflation rates.

1.3 Interest Rate Rules

The model is closed by specifying the monetary policy rules followed by the two monetary authorities. Here we assume that the monetary authority controls the short-term nominal interest rate. In the classical Taylor rule, the instrument is set to react to domestic inflation and output gap. However, in an open-economy model, the specification of the rules is more controversial, because the set of variables toward which monetary policy can react is larger.⁹

Our strategy is to explore the consequences for the equilibrium allocation of simple rules that lead to equilibria that can be solved analytically, in order to understand more clearly the transmission mechanism of disturbances in open economies. We analyze three regimes which we label: i) a fixed exchange rate; ii) a floating exchange rate and iii) a managed exchange rate, although the borders among these regimes are less marked than what appears from these definitions.

In the first regime, we design rules that determine a fixed nominal exchange rate. It will be shown that, in principle, many fixed exchange rate regimes exist depending on the specification of the underlying rules.

Then we consider a floating regime, which is defined as a regime in which the interest rates in both countries do not react explicitly to the exchange rate. A class of policies with this characteristic is

$$\begin{aligned}\hat{i}_t^H &= \gamma \hat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \gamma^* \hat{i}_{t-1}^F + \phi^* \pi_t^F + \psi^* y_t^F,\end{aligned}$$

with ϕ , ϕ^* , ψ , ψ^* , γ and γ^* non-negative; we label this combination of rules as floating regime (I). Rules of this kind have been extensively used in the closed-economy literature.¹⁰ Each policymaker reacts to past movements in the interest rate, the current domestic producer inflation rate and output gap. When the coefficients γ and γ^* are zeros, this class boils down to classical

⁹Early contributions include Ball (1999), Ghibroni (1998), McCallum and Nelson (1999), Monacelli (2004), Svensson (2000) and Weeparana (1998).

¹⁰An exhaustive analysis is presented in Woodford (2003).

Taylor rules. Within the floating-exchange regime, we consider also rules in which the reaction is toward the domestic price level

$$\begin{aligned}\hat{i}_t^H &= \phi_p p_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi_p^* p_t^F + \psi^* y_t^F,\end{aligned}$$

which we label floating regime (II) where $\phi_p, \phi_p^*, \psi, \psi^*$ are non-negative and p^H and p^F are the log of the respective price level.

As a third regime, we analyze a managed exchange rate, a ‘dirty’ floating. We consider the cases in which one country reacts either to changes in the nominal exchange rate or to the deviations of the level of the exchange rate from a defined target. In the latter case, a managed exchange rate (I) is defined as a couple of rules of the form

$$\begin{aligned}\hat{i}_t^H &= \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi^* \pi_t^F + \psi^* y_t^F - \delta \hat{S}_t^*,\end{aligned}$$

with $\phi, \phi^*, \psi, \psi^*, \delta$ non-negative where \hat{S}_t^* denotes the log-deviation of the exchange rate from a target S^* , i.e. $\hat{S}_t \equiv \ln(S_t/S^*)$

A managed exchange rate (II) is defined as

$$\begin{aligned}\hat{i}_t^H &= \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi^* \pi_t^F + \psi^* y_t^F - \mu \Delta \hat{S}_t,\end{aligned}$$

with $\phi, \phi^*, \psi, \psi^*, \mu$ non-negative and where S^* is the exchange rate target.¹¹

1.4 Summing Up

Equations (2), (4), (6), (7) and (8) combined with the log-linear version of the interest rate rules characterize completely our log-linear equilibrium dynamics.

In this model we can identify three roles for the exchange rate.

¹¹This class of rules can be further enlarged by incorporating a response to past values of the interest rate.

First, the exchange rate affects relative prices and by this channel the demand of the produced goods. This is the expenditure-switching effect: an exchange rate depreciation shifts demand from goods produced in country F to goods produced in H . As an indirect impact, it also increases producer price inflation in country H .

Second, the exchange rate is linked to the nominal-interest-rate differential through the uncovered interest parity.

Third, if the instrument of monetary policy reacts directly or implicitly to the exchange rate, there exists another channel of transmission through which the exchange rate has a direct impact on the real return, consumption, inflation and economic activity.

2 Equilibrium Determinacy

A critical issue in an open-economy framework is the determinacy of the nominal exchange rate. Indeed, purely ‘sunspot-driven’ movements in the nominal exchange rate can induce indeterminacy in the ‘real’ side of the economy through the relative price channel. The specification of the monetary policy rules can be relevant for eliminating any dependence of the equilibrium allocation of the economy on fluctuations unrelated to fundamental disturbances.

In this section, we discuss how monetary policy rules should be designed in order to determine a unique and stable rational expectations equilibrium for the log-linear approximation to the equilibrium dynamics of our model. We first focus on the cases in which both countries have the same degree of nominal rigidity and the same coefficients on the targets in the rules.

In a closed-economy framework, Woodford (2003) derives the conditions under which interest rate rules imply a determinate equilibrium. In particular, Taylor rules in which the interest rate reacts only to the current inflation rate lead to a determinate equilibrium when the reaction toward inflation is aggressive, i.e. if the weight on the inflation deviations from the target is

larger than one. This is known as the Taylor's principle.

In an open-economy framework the richness of the set of relevant variables complicates the analysis. The interdependence among the rules followed by different countries, even in a non coordinated way, is critical in determining the characteristics of the equilibrium allocation. Moreover, just specifying a policy rule for one of the two countries can be no longer sufficient for the determinacy of the equilibrium.

We first analyze a class of interest rate rules that can implement a fixed exchange rate regime. If the Foreign nominal interest rate is tied to the Home nominal interest rate, i.e. $\hat{i}_t^F = \hat{i}_t^H$ the uncovered interest parity, equation (2), implies that the expected exchange-rate variations are always zero. However, this is not sufficient to determine the nominal exchange rate. In fact, consider a simple bounded process $\{\zeta_t\}_{t=0}^{+\infty}$ such that $E_{t-1}\zeta_t = 0$. Then, consistently with the rule followed, there exists an equilibrium in which the exchange rate depreciation follows the path $\Delta\hat{S}_t = \zeta_t$ and in which the exchange rate can be moved by exogenous disturbances that are not related to the fundamentals. One solution to this problem is to allow the nominal interest rate in the Foreign country to follow the Home nominal interest rate and at the same time to react to deviations of the exchange rate from the desired target:

$$\hat{i}_t^F = \hat{i}_t^H - \tau\hat{S}_t^*, \quad (9)$$

with $\tau > 0$ where $\hat{S}_t^* \equiv \ln(S_t/S^*)$ and S^* is the exchange rate target. On a path in which the foreign exchange rate tends to appreciate with respect to the target, the foreign interest rate is lowered, viceversa if the foreign exchange rate tends to depreciate above the target.

Substituting (9) into equation (2) and noting that $\Delta\hat{S}_{t+1} = \hat{S}_{t+1} - \hat{S}_t$, we obtain

$$E_t\hat{S}_{t+1} = (1 + \tau)\hat{S}_t.$$

In order to have a unique and bounded rational-expectation equilibrium for the nominal exchange rate, it is sufficient to have τ greater than 0. In this equilibrium $S_t = S^*$ at all dates t .¹²

¹²It is worth stressing that the explicit feedback toward the exchange rate is not operative

To close the determination of all the variables of the model, we have to specify a reaction function for the “leader”.¹³ There are many possible specifications of the interest rate rule for the Home country all compatible with the definition of a fixed exchange rate regime but with different implications for macroeconomic variability. Here we restrict the instrument rule of the leader to

$$\hat{i}_t^H = \phi\pi_t^H + \psi y_t^H.$$

We have the following proposition.

Proposition 1 *Let us assume that $\alpha^H = \alpha^F$, so that $\lambda^F = \lambda^H = \lambda$. Under a fixed exchange rate regime defined by a couple of rules of the following form*

$$\begin{aligned}\hat{i}_t^F &= \hat{i}_t^H - \tau \hat{S}_t, \\ \hat{i}_t^H &= \phi\pi_t^H + \psi y_t^H,\end{aligned}$$

where ϕ, ψ, τ are non-negative, there is equilibrium determinacy if and only if the following conditions hold

$$(\phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) > 0, \quad (10)$$

$$\tau > 0. \quad (11)$$

Proof: In the Appendix.

This result presents some interesting features.

First, it shows that in general fixed exchange rate regimes are not destabilizing. The restrictions to be satisfied are not too stringent: we have assumed that the interest rate rule of the “follower” reacts to deviations of the exchange rate from the target, while the instrument rule of the “leader” follows a simple Taylor rule.

in the equilibrium, but private sector should believe that the monetary policymaker is committed to this reaction function in a credible way. See Benigno et al. (2006) for a more general non-linear analysis.

¹³In what follows we will use the terminology leader and follower only as a reference but without any implications for the timing of monetary policy decisions.

Second, once the exchange rate is determined, the restrictions required for the determinacy of the equilibrium are the same as the ones found in closed economies under a Taylor-rule regime. In the case $\psi = 0$, the Taylor's principle holds.

We now characterize the determinacy of the equilibrium under the floating exchange rate regime (I).

Proposition 2 (*Floating Exchange Rate Regime I*) *Let us assume that $\alpha^H = \alpha^F$, so that $\lambda^F = \lambda^H = \lambda$. Under a floating exchange rate regime defined by a couple of rules of the following form*

$$\begin{aligned}\hat{i}_t^H &= \gamma \hat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \gamma \hat{i}_{t-1}^F + \phi \pi_t^F + \psi y_t^F,\end{aligned}$$

with ϕ , ψ and γ non-negative, there is equilibrium determinacy if and only if the following conditions hold

$$(\gamma + \phi - 1)(1 + \eta)\lambda + \psi(1 - \beta) > 0, \quad (12)$$

$$(\gamma + \phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) > 0. \quad (13)$$

Proof: In the Appendix.

When $\rho < 1$ (13) implies (12), viceversa when $\rho > 1$. These conditions are similar to the closed-economy counterpart. The only, but important, qualification is that both interest-rate rules should be simultaneously “aggressive”. Indeed, if only one country were following a policy of interest rate pegging, we would observe indeterminacy of equilibrium. The higher the smoothing parameter, the less aggressive monetary policy need to be with respect to inflation and output gap. When the weight on the output gap is zero, then determinacy simply requires that $\gamma + \phi > 1$.

We now move to the second floating exchange rate regime.

Proposition 3 (*Floating Exchange Rate Regime II*) *Let us assume that $\alpha^H = \alpha^F$, so that $\lambda^F = \lambda^H = \lambda$. Under a floating exchange rate regime defined by*

a couple of rules of the following form

$$\begin{aligned}\hat{i}_t^H &= \phi_p p_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi_p p_t^F + \psi y_t^F,\end{aligned}$$

with ϕ_p, ψ non-negative, there is equilibrium determinacy if and only if $\phi_p > 0$.

Proof: In the Appendix.

As in a closed-economy model, see Giannoni (2000) and Woodford (2003), when the interest rate rules react to the price level –here the producer price level– it is just necessary and sufficient that the reaction to this price level is at least positive.

We analyze the determinacy under the third regime, the managed exchange rate regime (I) and (II).

Proposition 4 (*Managed Exchange Rate Regime I*) *Let us assume that $\alpha^H = \alpha^F$, so that $\lambda^F = \lambda^H = \lambda$. Under a managed exchange rate regime defined by a couple of rules of the following form*

$$\begin{aligned}\hat{i}_t^H &= \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi \pi_t^F + \psi y_t^F - \delta \hat{S}_t^*,\end{aligned}$$

with ϕ, ψ, δ non-negative, there is equilibrium determinacy if and only if either

$$\delta > 0$$

and

$$(\phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) > 0,$$

or

$$\delta = 0$$

together with

$$(\phi - 1)(1 + \eta)\lambda + \psi(1 - \beta) > 0,$$

and

$$(\phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) > 0.$$

Proof: In the Appendix

In the managed exchange rate regime (II), we have the following proposition:

Proposition 5 (*Managed Exchange Rate Regime II*) *Let us assume that $\alpha^H = \alpha^F$, so that $\lambda^F = \lambda^H = \lambda$. Under a managed exchange rate regime defined by a couple of rules of the following form*

$$\begin{aligned}\hat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi\pi_t^F + \psi y_t^F - \mu\Delta\hat{S}_t,\end{aligned}$$

with ϕ, ψ, μ non-negative, there is equilibrium determinacy if and only if the following conditions hold

$$\begin{aligned}(\phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) &> 0, \\ (\phi + \mu - 1)(1 + \eta)\lambda + \psi(1 - \beta) &> 0.\end{aligned}$$

Proof: In the Appendix.

2.1 General case: different degrees of rigidity

In the previous section, we have restricted the analysis to the case in which the two countries have the same degree of nominal rigidity. In this section we explore how our results change in the more general case. Analytical results are difficult to obtain because the characteristic polynomial increases to orders which are not tractable. We present one result for a simple Taylor-rule case.

Proposition 6 *Under Taylor rules of the form*

$$\begin{aligned}\hat{i}_t^H &= \phi\pi_t^H, \\ \hat{i}_t^F &= \phi^*\pi_t^F,\end{aligned}$$

with ϕ, ϕ^ non-negative, there is equilibrium determinacy if and only if the following conditions hold*

$$\phi > 1,$$

$$\phi^* > 1.$$

Proof: In the Appendix

To get determinacy both interest rate rules should react more than proportionally to the respective domestic producer inflation rate. We find that the Taylor principle can be extended to this two-country model with the qualifications that both coefficients should be greater than one and that the relevant inflation rate is the producer inflation rate.

In what follows we analyze numerically the more general cases in order to understand the relationship between the degrees of nominal price rigidities and the coefficients on the interest rate feedback rules. We calibrate the model according to the parametrization used in Rotemberg and Woodford (1998). The parameters β , η , ρ , σ assume the values 0.99, 0.47, 0.16, 7.88 respectively. We allow for the degree of nominal price rigidity to be different across countries and we consider general rules of the form

$$\begin{aligned}\hat{i}_t^H &= \gamma \hat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \gamma^* \hat{i}_{t-1}^F + \phi^* \pi_t^F + \psi^* y_t^F,\end{aligned}$$

In Figure 1, we look at the combinations of coefficients on inflation and the output gap that would guarantee the determinacy of equilibrium for given degrees of nominal rigidities. We vary them in order to examine how cross country heterogeneity in terms of the degree of nominal rigidities affect the determinacy region. We consider the following combinations of $\{(\alpha^H, \alpha^F)\} = \{(0.9, 0.9); (0.5, 0.5); (0.9, 0.5); (0.9, 0.1); (0.5, 0.1)\}$. The determinacy regions correspond to the area located on the right of each border line. If we start from the case in which $(\alpha^H, \alpha^F) = (0.9, 0.9)$ and we reduce α^F then our determinacy region shrinks. This means that when we lower the degree of nominal price rigidity for a country by keeping the other one fixed then monetary policy should react relatively more aggressively towards domestic producer inflation and/or output. Interestingly the gap between α^H and α^F does not seem to matter much: indeed in the figure the borders of the determinacy region for $(\alpha^H, \alpha^F) = (0.9, 0.5)$ and $(\alpha^H, \alpha^F) = (0.9, 0.1)$

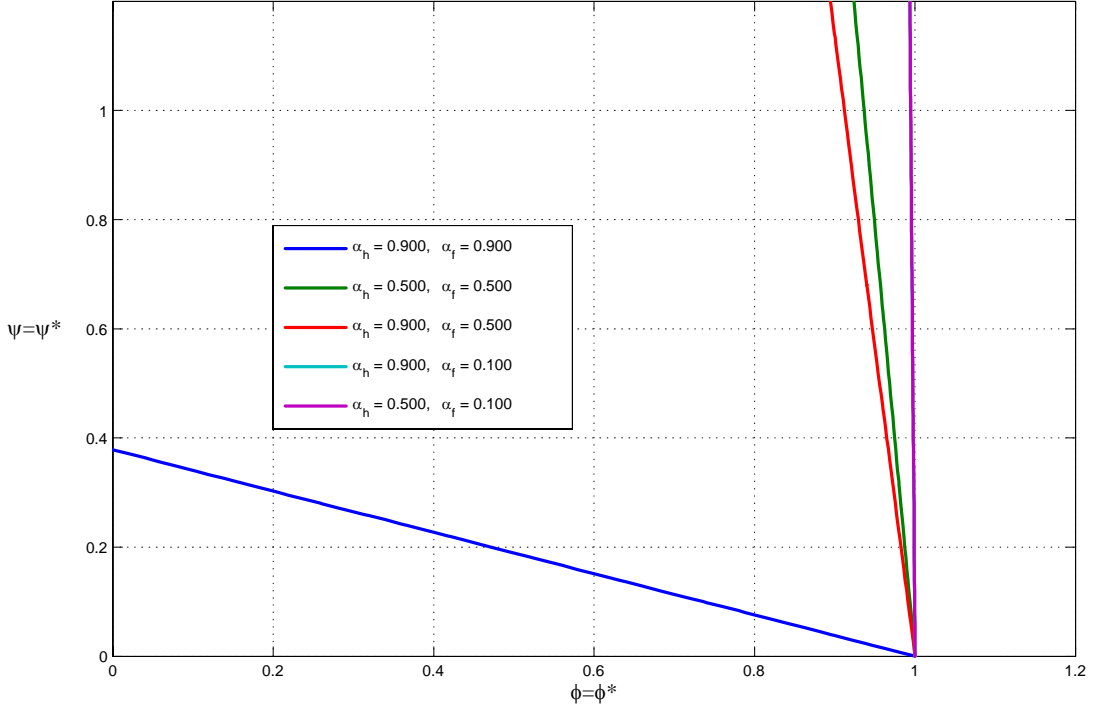
Figure 1: $\gamma = \gamma^* = 0$ 

Figure 1:

coincide. We also note from this figure that when $\psi = \psi^* = 0$ then the coefficient on inflation need to satisfy the Taylor principle (i.e. $\phi > 1$) as suggested in proposition 6.

In Figure 2 on the other hand we examine the combination of ϕ and ϕ^* that would guarantee determinacy of equilibrium for a given combination of the degree of price rigidities. We consider the following combination of $\{(\alpha^H, \alpha^F)\} = \{(0.7, 0.1); (0.7, 0.3); (0.7, 0.5); (0.7, 0.7); (0.7, 0.9)\}$. We now fix the reaction to output gap to $\psi = 0.2$ and let $\gamma = \gamma^* = 0$. We first note that the determinacy border is approximated by the $\min\{\phi, \phi^*\}$ function. Intuitively this means that monetary policy needs to be sufficiently aggressive in both countries in order to have a determinate equilibrium (as proposition 6 would suggest for the special case). When α^F decreases, for a

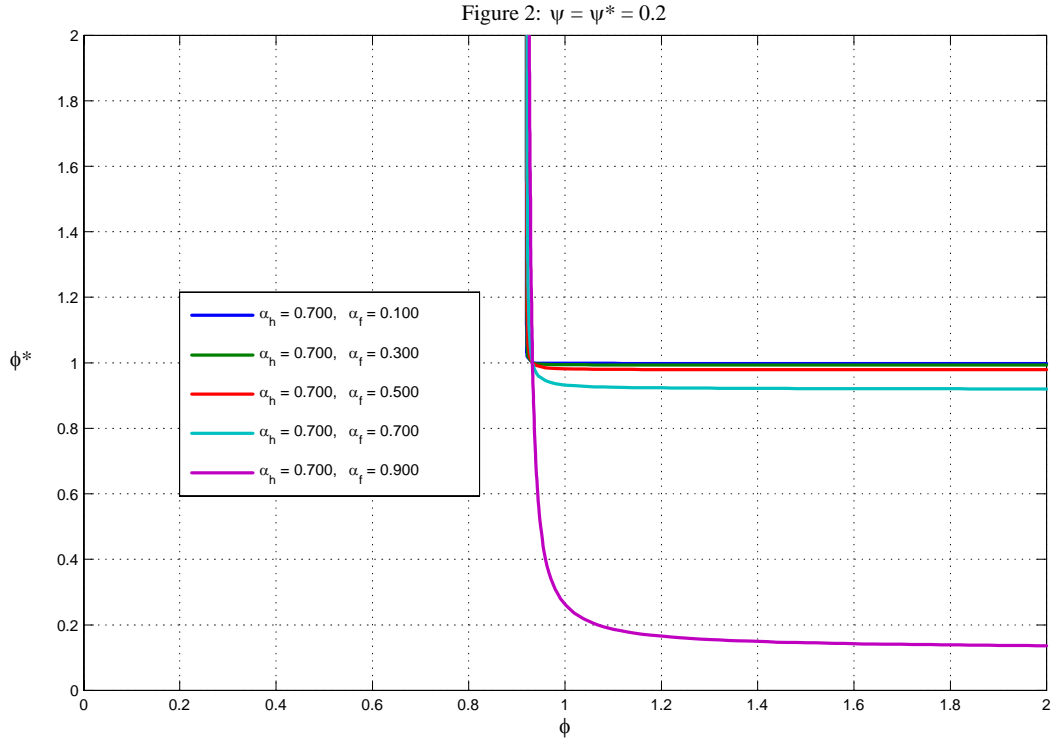


Figure 2:

given α^H , monetary policy in the F country need to become less aggressive towards inflation and the determinacy region expands along the vertical line corresponding to $\phi \approx 0.93$.

A similar message arises once we consider the coefficients on the output gap (see Figure 3). Here we fix $\phi = \phi^* = 0.95$ and as before $\gamma = \gamma^* = 0$. Indeed a higher degree of nominal price rigidity in the F country for a given α^H would imply a less aggressive reaction towards the output gap in the F country. As before the determinacy region will expand along the vertical line corresponding to $\psi \simeq 0.15$.

Figure 3: $\phi = \phi^* = 0.95$

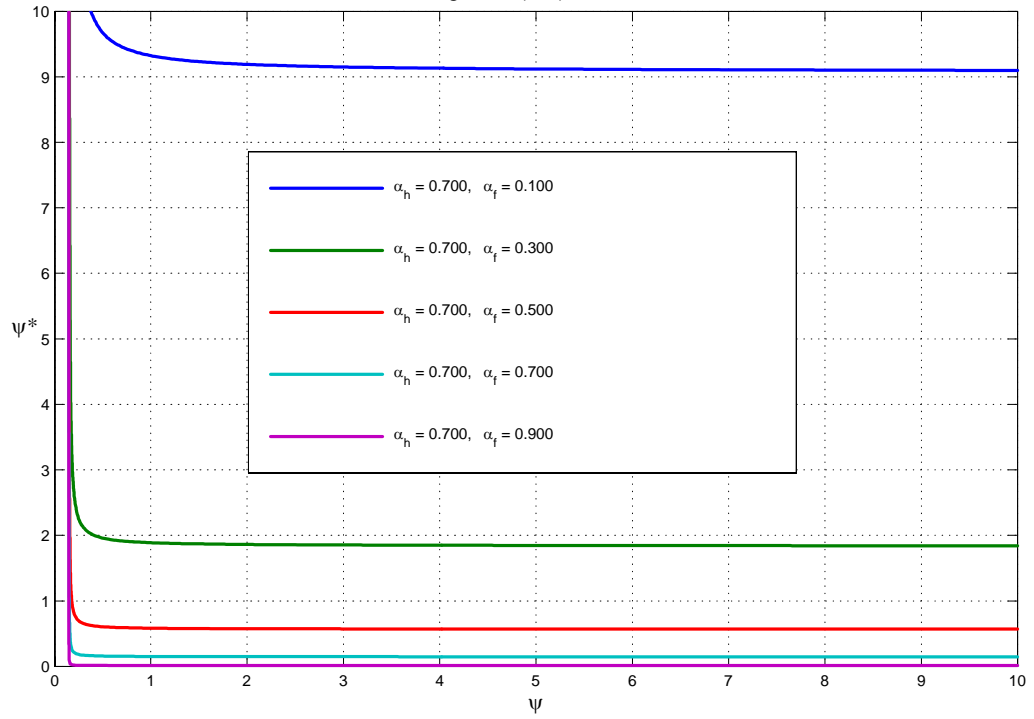


Figure 3:

3 Flexible price equilibrium

In any flexible price equilibrium, only real shocks affect real variables.¹⁴ In fact the equilibrium path of real variables is described by

$$\begin{aligned}\tilde{C}_t^W &= \frac{\eta}{\eta + \rho}(a_t^W - g_t^W), \\ \tilde{T}_t &= \frac{\eta}{1 + \eta}(g_t^R - a_t^R), \\ \tilde{Y}_t^H &= (1 - n)\tilde{T}_t + \tilde{C}_t^W + g_t^H, \\ \tilde{Y}_t^F &= -n\tilde{T}_t + \tilde{C}_t^W + g_t^F,\end{aligned}$$

where a and g are respectively supply (originated from productivity disturbances) and demand shocks. Only world shocks affect the natural rate of world consumption, while only relative shocks perturb the natural rate of the terms of trade.

Consider now the equilibrium condition

$$\tilde{T}_t = \tilde{T}_{t-1} + \Delta\tilde{S}_t + \tilde{\pi}_t^F - \tilde{\pi}_t^H.$$

There are infinitely many equilibria which differ because of the different decomposition of the changes in the terms of trade into exchange rate depreciation and inflation rate differential. The monetary policy regime is crucial in determining this split. Here we analyze a particular equilibrium in which the producer inflation rate is zero in each country. As explained in Benigno (2004), this particular flexible price equilibrium represents the efficient outcome that a central planner would optimally choose in the case prices are sticky and the monopolistic distortions are small in size.¹⁵ Under the assumption of zero producer inflation rates, the nominal exchange rate follows directly the path of the natural terms of trade. In fact, equation (4) implies that

$$\Delta\tilde{S}_t = \tilde{T}_t - \tilde{T}_{t-1},$$

¹⁴This is a consequence of the assumption of additive separability in the consumer's utility between consumption and real money balances.

¹⁵See Benigno and Benigno (2006) for results under general assumptions.

and, given the initial conditions $\tilde{S}_{-1} = \tilde{T}_{-1} = 0$, that

$$\tilde{S}_t = \tilde{T}_t.$$

An increase in the productivity of the Foreign country relative to that of the Home country reduces the disutility of labor in the Foreign economy. An appreciation of the Foreign terms of trade (a reduction in T) allows Foreign goods to be more competitive. All the adjustment is obtained by an appreciation of the nominal exchange rate (i.e. S decreases). Instead, an exogenous increase in the demand of Foreign goods relative to Home goods can be equilibrated by a depreciation in the Foreign terms of trade that shift part of the increased demand to the Home country.

By using the uncovered interest rate parity, it is possible to express the natural nominal (and real) interest-rate differential as

$$\tilde{i}_t^H - \tilde{i}_t^F = \text{E}_t\{\tilde{T}_{t+1} - \tilde{T}_t\} \equiv \tilde{R}_t^R, \quad (14)$$

which is driven only by the expected changes in the natural terms of trade; the world natural nominal interest rate instead is purely driven by world demand and supply shocks, in fact

$$n\tilde{i}_t^H + (1 - n)\tilde{i}_t^F = \rho\text{E}_t\{\tilde{C}_{t+1} - \tilde{C}_t\} \equiv \tilde{R}_t^W. \quad (15)$$

Each country's natural nominal interest rate is obtained by combining (14) and (15):

$$\begin{aligned} \tilde{i}_t^H &= \tilde{R}_t^W + (1 - n)\tilde{R}_t^R, \\ \tilde{i}_t^F &= \tilde{R}_t^W - n\tilde{R}_t^R. \end{aligned}$$

4 Terms of Trade and Exchange Rate Dynamics Under Alternative Regimes

In this section, we focus on the sticky-price equilibrium and continue to restrict the analysis to the case in which the two countries have the same

degree of nominal rigidities. We analyze how the path of the terms of trade and the exchange rate is affected by different monetary policy regimes.

A natural benchmark of comparison is the flexible-price allocation in which the producer inflation rates are zero in both countries (see the previous section).

In the sticky-price equilibrium, a non efficient path of the terms of trade determines a dispersion of output gap across countries as can be seen from equation (5).

The output-gap differential has also an immediate consequence on the path of the inflation rate differential. In fact, from the aggregate supply equation we obtain

$$\pi_t^F - \pi_t^H = \lambda(1 + \eta)E_t \left\{ \sum_{j=0}^{+\infty} \beta^j [y_{t+j}^F - y_{t+j}^H] \right\},$$

where the inflation differential depends on the expected path of the output gap differential. It is then implicit that a non-efficient path of the terms of trade creates a spread in the inflation differential across the economies. Instead, efficiency would require to absorb any asymmetric transitory shock with no variation in the output gaps and inflation differentials.

In the next sub-section, we investigate the path of the exchange rate and the terms of trade under a fixed exchange rate system.

4.1 Fixed Exchange Rate Regime

Combining equations (7) and (8) and imposing the equilibrium condition of a fixed exchange rate regime, we obtain that the terms of trade behaves according to a second-order stochastic difference equation which has always a unique and stable rational-expectation solution of the form

$$\hat{T}_t = \lambda_1 \hat{T}_{t-1} + \lambda_1 \lambda (1 + \eta) E_t \sum_{j=0}^{+\infty} (\beta \lambda_1)^j \tilde{T}_{t+j}, \quad (16)$$

where λ_1 is the stable eigenvalue, with $0 < \lambda_1 < 1$. Furthermore if the process followed by \tilde{T}_t is Markovian of the form

$$\tilde{T}_t = \rho_1 \tilde{T}_{t-1} + \varepsilon_t, \quad (17)$$

with $0 < \rho_1 < 1$, then (16) can be simplified to

$$\hat{T}_t = \lambda_1 \hat{T}_{t-1} + \nu \tilde{T}_t,$$

where we have $0 < \nu < 1$.¹⁶ It is important to note that the dynamic properties of the terms of trade – when the degrees of rigidity are equal across countries – do not depend on the underlying rules that characterize the fixed exchange rate and are only affected by real asymmetric shocks.

Moreover the terms of trade are inertial and their short-run response to natural terms of trade shocks is less than proportional. In particular λ_1 is a monotone increasing function of the overall degree of nominal rigidity. As the rigidity decreases, the inertia in the terms of trade decreases. On the other hand, ν is a monotone decreasing function of the degree of nominal rigidities, and at the limit, with perfect flexibility, ν approaches one. As the degree of rigidity decreases, the short run response approaches the flexible-price response. By dampening the variability of the exchange rate, the variability of the terms of trade is reduced:

$$\text{var}(\hat{T}_t) = \frac{1 + \rho_1 \lambda_1}{1 - \rho_1 \lambda_1} \frac{\nu^2}{1 - \lambda_1^2} \text{var}(\tilde{T}_t) < \text{var}(\tilde{T}_t)$$

where again the variability of the terms of trade increases monotonically as the degree of rigidity decreases and at the limit approaches the variability that efficiency would require.

4.2 Floating Exchange Rate Regime

First, we restrict the analysis to the classical Taylor rules

$$\begin{aligned} \hat{i}_t^H &= \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi \pi_t^F + \psi y_t^F. \end{aligned}$$

¹⁶ ν is defined as $\nu \equiv \lambda_1 \lambda (1 + \eta) / (1 - \beta \lambda_1 \rho_1)$.

As we show in the Appendix B, the equilibrium paths of the terms of trade, the interest rate differential and the exchange rate depreciation is given by

$$\begin{aligned}\hat{T}_t &= c_1 \tilde{T}_t, \\ \pi_t^F - \pi_t^H &= c_2 \tilde{T}_t, \\ \Delta \hat{S}_t &= -\hat{T}_{t-1} + c_3 \tilde{T}_t.\end{aligned}$$

An interesting implication for the dynamic properties of the terms of trade is that the inertia is completely eliminated. Instead changes in the exchange rate react negatively to past movements in the terms of trade (with a unitary coefficient). Moreover, $c_1 > c_2$ as well as $c_1 > c_3$. Instead c_3 becomes bigger than c_2 the more aggressive monetary policy is with respect to inflation and output (i.e. as ϕ and ψ increase). When the reaction toward inflation becomes infinite – i.e. as $\phi \rightarrow +\infty$ –, the parameters converge in the following way $c_1 \rightarrow 1$, $c_2 \rightarrow 0$ and $c_3 \rightarrow 1$.

Differently from the fixed exchange rate system, the weights that monetary policy put on inflation and output stabilization affect the dynamic properties of the terms of trade. In fact, under the floating regime, the volatility of the terms of trade is

$$\text{var}(\hat{T}_t) = c_1^2 \text{var}(\tilde{T}_t) < \text{var}(\tilde{T}_t),$$

and approaches the efficient volatility as the parameters ϕ or ψ increase.

At a first pass this analysis can address some empirical regularities.

For example, Obstfeld (1997) shows that in shifting from a fixed exchange rate system to a floating regime the volatility of the terms of trade increases. In our model, this result is ambiguous and depends on the coefficients of the interest-rate rules.¹⁷ When monetary policy is aggressive with respect

¹⁷Our result differs from Monacelli (2004). He addresses this issue in a dynamic small-open-economy model with an endogenous monetary policy by specifying a reaction function in which there is a feedback toward deviations of the nominal exchange rate from a target. He labels as a floating exchange rate regime a regime in which there is no feedback. He finds that the terms of trade variability is a decreasing function of the strengthen of the feedback.

to inflation (high values of ϕ) then the terms of trade volatility is higher under the floating regime. For higher values of ϕ , the volatility approaches the efficient volatility, while the volatility under fixed exchange rate system is independent of the parameters of the rule. However, for lower values of ϕ , the volatility of the terms of trade under the floating regime decreases and can be lower than the volatility under the fixed exchange rate regime.

Another empirical regularity documented by Obstfeld (1997) is that, in a floating regime, changes in the terms of trade display more variability than changes in the nominal exchange rate. Considering the terms of trade identity (4), we can write

$$\text{var}(\Delta\hat{T}) = \text{var}(\Delta\hat{S}) + \text{var}(\pi^R) + 2\text{cov}(\Delta\hat{S}, \pi^R).$$

The variability of the changes in the terms of trade is higher than that of changes in the nominal exchange rate if

$$\text{var}(\pi^R) > -2\text{cov}(\Delta\hat{S}, \pi^R),$$

which in our model is satisfied if and only if

$$c_2 < 2c_1(1 - \rho_1).$$

The latter condition holds for reasonable calibrations of the parameters, and in general is always satisfied provided ρ_1 is not high, e.g. $\rho_1 < 0.5$.

More interestingly in this simple example we can analyze the equilibrium path of the exchange rate. By exploiting the initial conditions $\hat{S}_{t-1} = \tilde{T}_{t-1} = \hat{T}_{t-1} = 0$, we note that at a generic time $\tau \geq t$ the exchange rate is equal to the sum of current and past perturbations to the natural terms of trade

$$\hat{S}_\tau = c_3\tilde{T}_\tau - \sum_{j=t}^{\tau-1} c_2\tilde{T}_j.$$

It follows that the exchange rate is a non-stationary variable. In the long run, temporary shocks do not die out and have a permanent effect. Here we denote with $\Delta E_t \hat{S}_\tau$ the innovation in the time t rational forecast of the time

τ nominal exchange rate following an innovation in the shock process at time t , i.e.¹⁸

$$\Delta E_t \hat{S}_\tau = E_t \hat{S}_\tau - E_{t-1} \hat{S}_\tau.$$

Assuming a process of the form (17) and a temporary perturbation ε_t at time t , we can write the innovation in the time t rational forecast of the time τ exchange rate as

$$\Delta E_t \hat{S}_\tau = c_3 \rho_1^{\tau-t} \varepsilon_t - c_2 \frac{1 - \rho_1^{\tau-t}}{1 - \rho_1} \varepsilon_t,$$

from which we obtain its long-run innovation

$$\Delta E_t \hat{S}_\infty = -c_2 \frac{1}{1 - \rho_1} \varepsilon_t. \quad (18)$$

The nominal exchange rate displays a non-stationary behavior.

In fact, $\Delta E_t S_\infty$ is the innovation in the rational forecast of the long-run nominal exchange rate, i.e. the innovation of the stochastic trend in the definition of Beveridge and Nelson (1981), and it measures the magnitude of the non-stationary component.

As well, we can compute the short-run unexpected response, which is given by $\Delta E_t \hat{S}_\infty = c_3 \varepsilon_t$. The long run response of the nominal exchange rate has the opposite sign with respect to the short run.

Following a positive perturbation to the natural terms of trade (i.e. an increase in \tilde{T}), the Home exchange rate suddenly depreciates – but less than proportionally. Instead, in the long run, it experiences an appreciation. The short-run reaction has the same sign as the efficient path, though the magnitude is smaller.

Consider for example a positive productivity shock that affects the Home country. The efficient equilibrium would require an appreciation of the Home terms of trade and a depreciation of the Home exchange rate (T and S should increase). In fact, there is a need to shift the demand to the Home produced goods, in order to equilibrate the disutility of working across countries.

However, under the floating regime, the Home terms of trade depreciate less while the exchange rate first depreciates and then appreciates. The time

¹⁸The difference operator is applied to the conditional expectations operator.

at which the appreciation is reached depends on the nature of the process \tilde{T} . If it follows a white-noise process, the appreciation occurs in the period immediately after the shock, while as ρ_1 increases the appreciation is delayed. The long-run appreciation is a function both of the parameters ρ_1 and c_2 . As the monetary policy rules become more aggressive, the long-run appreciation is reduced while the short-run response is amplified.

The magnitude of the non-stationary behavior is then a function of the values of the parameters of the policy rule chosen. Moreover, in the short run, the correlation between expected changes in the terms of trade and the exchange rate is positive, while becomes negative in the long-run.¹⁹

The intuition for the non-stationary behavior of the exchange rate is directly related to the state equation

$$\hat{T}_t = \hat{T}_{t-1} + \Delta \hat{S}_t + \pi_t^F - \pi_t^H.$$

Although the terms of trade is stationary and is expected to revert to the initial value, there is nothing, under the floating regime specified, that restrict the exchange rate to revert to the pre-shock value, unless the producer inflation rates are always stabilized to zero. The long-run value of the terms of trade is the same as the initial value but has a different decomposition between the exchange rate and price components. Taylor rules in which the reaction toward inflation is infinite can produce long-run stationarity of the exchange rate.

An important implication of this analysis is that, even if financial and monetary shocks can further exacerbate the excess volatility of the nominal exchange rate, real shocks do have an important role and can be source of the excess volatility of the nominal exchange rate. In particular, they can generate persistent effects. And we do not need to rely on a non-stationary distribution of such shocks. Nominal exchange rates are non stationary following stationary real shocks, while the same perturbations generate a stationary distribution for the real macroeconomic variables.

¹⁹This might also explain the empirical finding that the correlation between changes in the terms of trade and the exchange rate is not so strong, as shown in Obstfeld (1997).

Here we investigate if and how the non-stationary behavior changes when the floating regime is defined by smoothing rules of the form

$$\begin{aligned}\hat{i}_t^H &= \gamma \hat{i}_{t-1}^H + \phi \pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \gamma \hat{i}_{t-1}^F + \phi \pi_t^F + \psi y_t^F.\end{aligned}$$

As it is shown in Appendix B, the equilibrium path of the terms of trade and the exchange rate is of the form

$$\begin{aligned}A^s(L)\hat{T}_t &= R^s(L)\tilde{T}_t, \\ A^s(L)\Delta\hat{S}_t &= U^s(L)\tilde{T}_t,\end{aligned}$$

where $A^s(L)$, $R^s(L)$ are first-order polynomials while $U^s(L)$ is of second order. Note that when the inertia coefficient γ is zero, there is no autoregressive component in $A^s(L)$.

We investigate if the inertia originated from the response of the interest rates to past values can pin down a stationary behavior for the nominal exchange rate. We use the same calibration as in the previous section except that we set $\alpha^H = \alpha^F = 0.66$ and we also assume that the autocorrelation parameter in (17) is $\rho_1 = 0.4$. In figure 4 we plot the innovation in the rational forecast of the long-run exchange rate following a perturbation to the natural terms of trade, under different rules in this class. We set ϕ equal to 1.5, while ψ assumes values in the interval between 0 and 1. We allow the coefficient γ to vary between 0 and 10. For low values of the smoothing coefficient, in general for values below one, the exchange rate experiences a permanent appreciation. The opposite happens when the smoothing rules are super inertial. Again, only for particular parameters, the exchange rate becomes stationary. For example, this the case if γ is equal to 1, when ψ is equal to 0.

In particular this latter result is not surprising since under this particular parametrization the interest rate rules can be written as

$$\begin{aligned}\hat{i}_t^H &= \hat{i}_{t-1}^H + \phi(p_t^H - p_{t-1}^H), \\ \hat{i}_t^F &= \hat{i}_{t-1}^F + \phi(p_t^F - p_{t-1}^F).\end{aligned}$$

Figure 4: Innovation in the Rational Forecast of the Long-run Exchange Rate with Interest-Rate-Smoothing Regimes

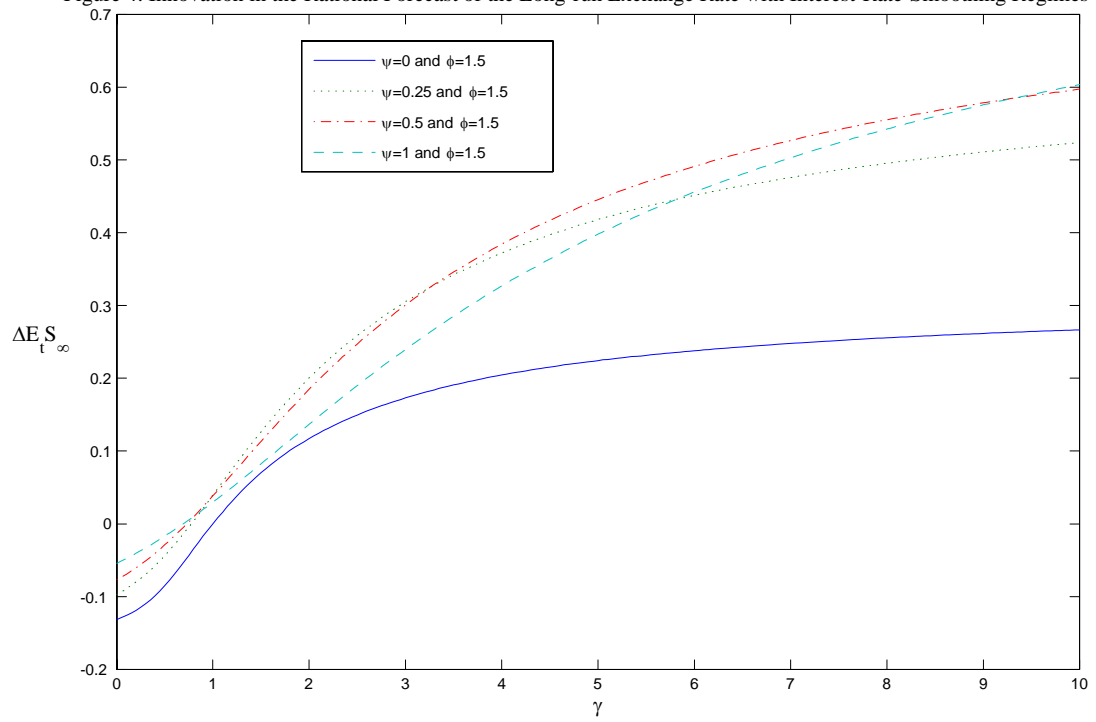


Figure 4:

as if the level of the interest rate is reacting to the log of the producer price level. Indeed, this result of absence of stationary of the nominal exchange rate disappears when we consider rules in the class of floating exchange-rate regime (II)

$$\begin{aligned}\hat{i}_t^H &= \phi_p p_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi_p p_t^F + \psi y_t^F.\end{aligned}$$

Indeed we show in Appendix B that in this case the equilibrium path of the terms of trade and exchange rate is given by

$$\begin{aligned}A^p(L)\hat{T}_t &= R^p(L)\tilde{T}_t, \\ A^p(L)\hat{S}_t &= U^p(L)\tilde{T}_t,\end{aligned}$$

where $A^p(L)$, $R^p(L)$ and $U^p(L)$ are first-order polynomials. It follows that the nominal exchange rate becomes stationary even under a floating exchange-rate regime in the case in which the rules react to the price level.

4.3 Managed Exchange Rate Regimes

We now analyze the case in which monetary policy rules react to the nominal exchange rate.

The main result of this section is that the existence of a small feedback on the level of the exchange rate induces a stationary path for the nominal exchange rate while a feedback on exchange rate changes is in general unsuccessful.

As shown in Appendix B, under a managed exchange rate (I), where the feedback is on the level, it is possible to obtain a solution for \hat{T}_t and \hat{S}_t of the form

$$\begin{aligned}A(L)\hat{T}_t &= R(L)\tilde{T}_t, \\ A(L)\hat{S}_t^* &= U(L)\tilde{T}_t,\end{aligned}$$

where $A(L)$ is a second-order polynomial, while $R(L)$ and $U(L)$ are first-order polynomials.

The exchange rate is then a stationary variable that converges to the target S^* and behaves according to an ARMA process in which the AR component is of second order. It is the case that such form of managed exchange rate succeeds in stabilizing the long-run fluctuations of the exchange rate.²⁰

Instead, under the managed exchange rate (II), where the interest rate reacts also to the exchange rate depreciation, we obtain a solution for \hat{T}_t and $\Delta\hat{S}_t$ of the form

$$\begin{aligned} A^m(L)\hat{T}_t &= R^m\tilde{T}_t, \\ A^m(L)\Delta\hat{S}_t &= U^m(L)\tilde{T}_t, \end{aligned}$$

where $A^m(L)$ and $U^m(L)$ are first-order polynomials, while R^m is a constant. The source of inertia in this case arises because of the explicit target on the exchange rate changes. However, from the solution of $\Delta\hat{S}_t$, changes in the nominal exchange rate are a stationary variable, but nothing assures that the level is stationary. In particular this class of managed exchange rates can pin down a determined equilibrium for the exchange rate, but it does not necessarily imply a stationary value, unless particular parametrization are assumed. Figure 5 plot the innovation in the rational forecast of the long-run nominal exchange rate under different rules belonging to this class for the same parametrization as in figure 4. We set ϕ equal to 1.5, while ψ assumes values in the interval between 0 and 1. We allow the coefficient μ to vary between 0 and 10. When ψ is equal to zero, the exchange rate experiences always a long-run appreciation following a positive perturbation to the natural terms of trade, unless the feedback on the exchange rate depreciation becomes infinite. However, when the rules react explicitly also to the output gap, a strong response to the exchange rate implies a long-run permanent depreciation. The sign of the long-run behavior of the exchange rate

²⁰We can further show that the terms of trade variability is monotone non-decreasing in the parameter δ . For low values of δ the variability of the terms of trade under this regime is lower than under the fixed exchange rate.

Figure 5: Innovation in the Rational Forecast of the Long-run Exchange Rate with Managed-Exchange-Rate Regimes (II)

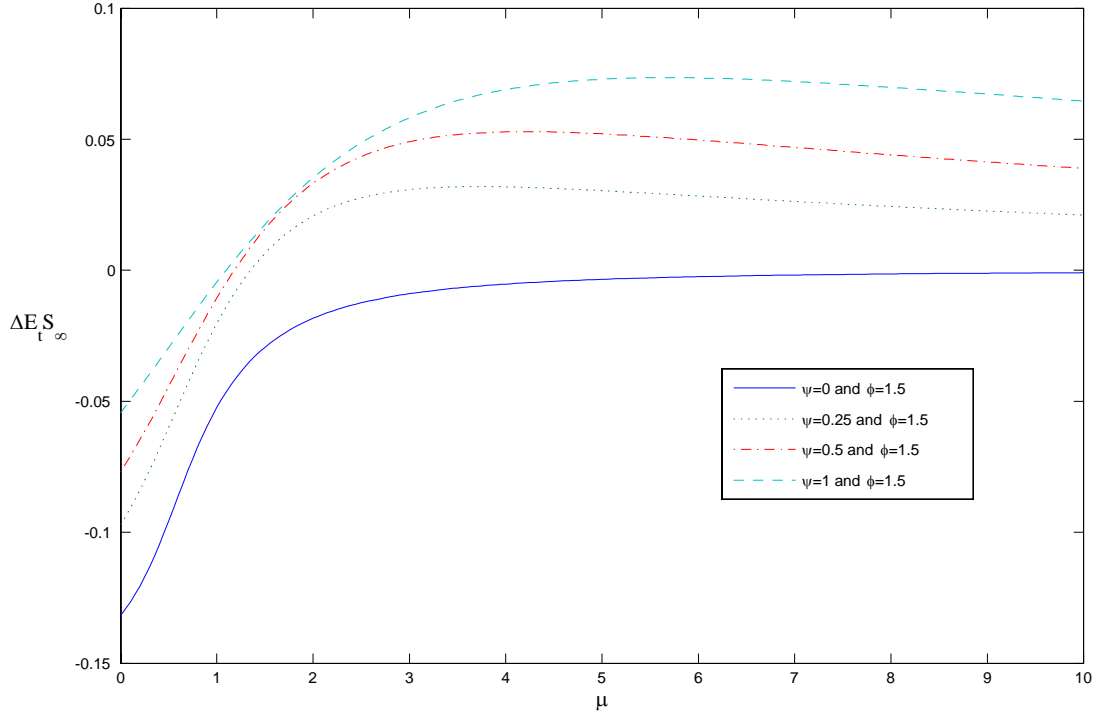


Figure 5:

depends on the parameters of the rule. Only under special parametrization the exchange rate reverts to its initial steady state.

4.4 Some Comparisons

In this section, we analyze in a calibrated economy the impulse response functions of the exchange rate and the terms of trade following a perturbation to the natural terms of trade. We consider the following regimes in comparisons to the efficient equilibrium:

1. A fixed exchange rate system;
2. A floating exchange-rate regime (I), a classical set of Taylor rules, with parameters $\phi = 1.5$, $\psi = 0.5$, $\gamma = 0$;

3. A floating exchange-rate regime (I) with parameters $\phi = 0.3$, $\psi = 0.1$ and $\gamma = 0.8$;
4. A floating exchange-rate regime (I) with parameters $\phi = 0.1$, $\psi = 0.025$ and $\gamma = 1.2$.
5. A managed exchange rate regime (I) with $\phi = 1.5$, $\psi = 0.5$ and $\delta = 0.5$;
6. A managed exchange rate regime (II) with parameters $\phi = 1.5$, $\psi = 0.5$ and $\mu = 0.5$;

Note that in the rule 3), the parameters that indicate the short-run reaction to inflation and the output gap have been adjusted to imply the same long-run response as under the classical Taylor rule 2). In rule 4) these parameters have been further lowered because of the super-inertial component implied by the smoothing coefficient. Rules 2), 3) and 4) belong to the class of floating regimes. Instead, rules 5) and 6) belong to the class of managed exchange rate regimes.

First, we analyze the case of no persistence in the natural terms of trade process, i.e. $\rho_1 = 0$.²¹ In Figure 6, we plot the impulse response of the terms of trade to a temporary positive shock to the natural terms of trade, for each of the rules considered. In the efficient equilibrium and in the Taylor-rule regime there is no endogenous persistence in the economy independently of the contract length; moreover, we note that the short-run response of the terms of trade under the Taylor rule is smaller than under efficiency. A rule with interest-rate smoothing implies a weaker short-run response than the Taylor-rule regime while determines an inertial behavior in the terms of trade. In the fixed exchange rate regime and in the managed exchange rate regimes (I) and (II) the response is dampened.

Figure 7 shows the impulse response of the nominal exchange rate.

²¹In what follows the parametrization is the same as in figure 4 except for the parameters mentioned in the text.

Figure 6: Impulse Response of the Terms of Trade to a Shock to the Natural Rate of the Terms of Trade, $\rho_1=0$

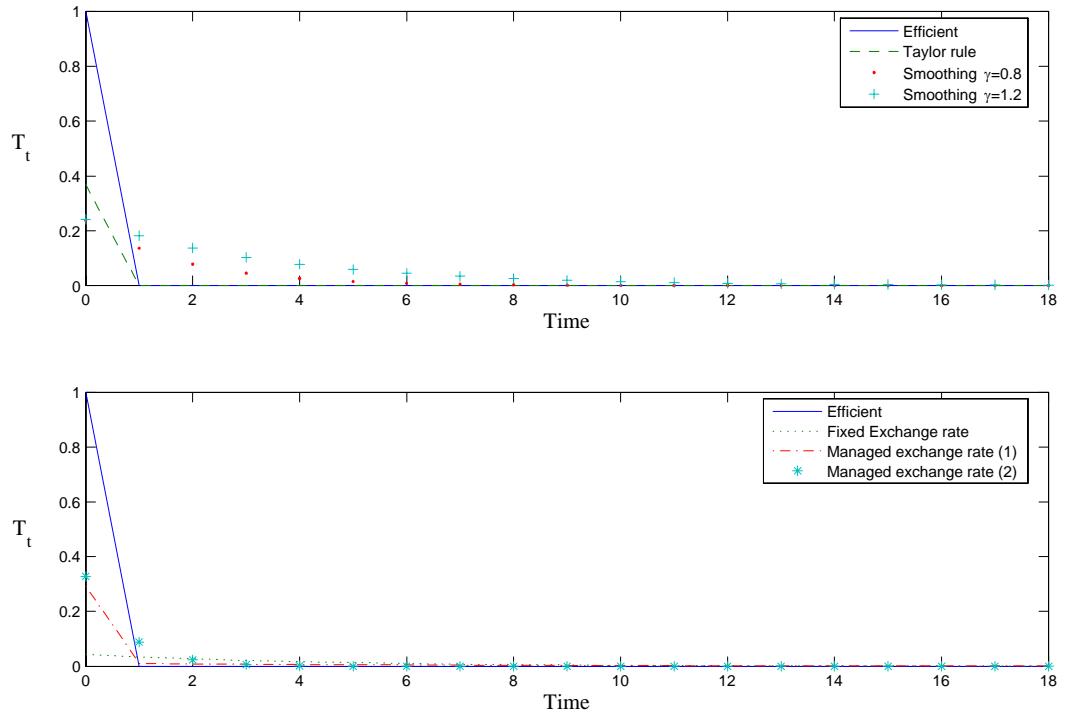


Figure 6:

Under the Taylor-rule regime, the exchange rate depreciates and suddenly appreciates without reverting to the initial equilibrium. By introducing interest rate smoothing, the short-run response is dampened, and the shock is propagated over time. The long-run value depends on the smoothing coefficient: when the coefficient γ is equal to 0.8, the nominal exchange rate is close, in the long run, to the initial value, while with a coefficient 1.2 it depreciates persistently. Under this regime, the exchange rate depreciates instantaneously and remains around the depreciated value for all periods. Once the shock occurred, the expected exchange rate depreciation or appreciation is approximately zero. The managed exchange rate regime (I) corrects for the non-stationary path and implies an inertial adjustment toward the equilibrium value.²² In the case of managed exchange rate (II), in the long run, the exchange rate persistently appreciates, though less than under the Taylor-rule regime.

In Figure 8 and 9 we repeat the same experiment, but assuming autocorrelation in the natural terms of trade, $\rho_1 = 0.4$. Consistently with the theoretical findings, the inertial behavior of the terms of trade, under the efficient path and the Taylor-rule regime, is a consequence of the exogenous inertia in the natural terms of trade. The response of the terms of trade under the fixed exchange rate system presents a hump-shaped pattern and in a short period of time overshoots the efficient path. The managed exchange rate systems (I) and (II) present a dampened short-run response but an higher persistence than the Taylor-rule regime. In a similar way, floating regimes with interest-rate smoothing display an amplified persistence. These regimes present also overshooting of the terms of trade above the natural terms of trade. A similar path, but less evident, arises under the managed exchange rate regime (I).

Looking at the long-run response of the exchange rate, a high smoothing coefficient can induce a persistent exchange rate depreciation, while when γ is equal to 0.8 the nominal exchange rate is stationary. Under the Taylor

²²We are assuming that the target coincides with the initial value.

Figure 7: Impulse Response of the Exchange Rate to a Shock to the Natural Rate Terms of Trade, $\rho_1=0$

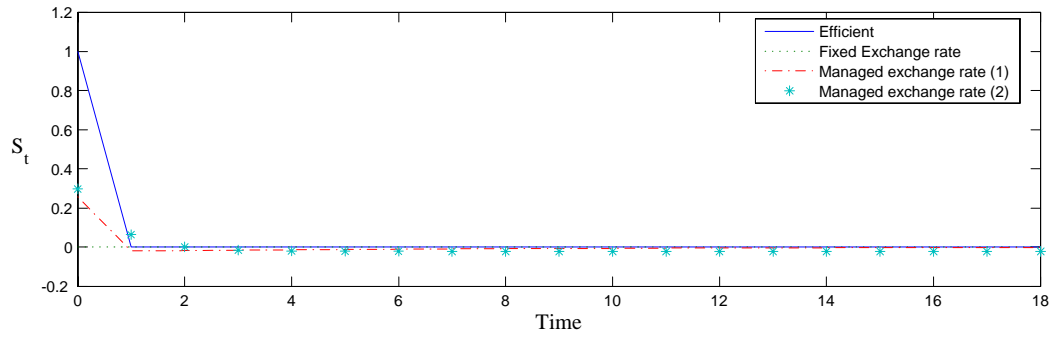
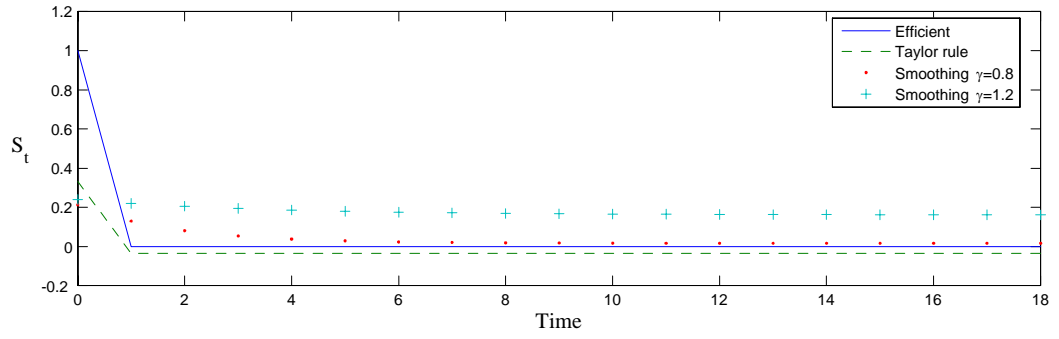


Figure 7:

Figure 8: Impulse Response of the Terms of Trade to a Shock to the Natural Rate of the Terms of Trade, $\rho_1=0.4$

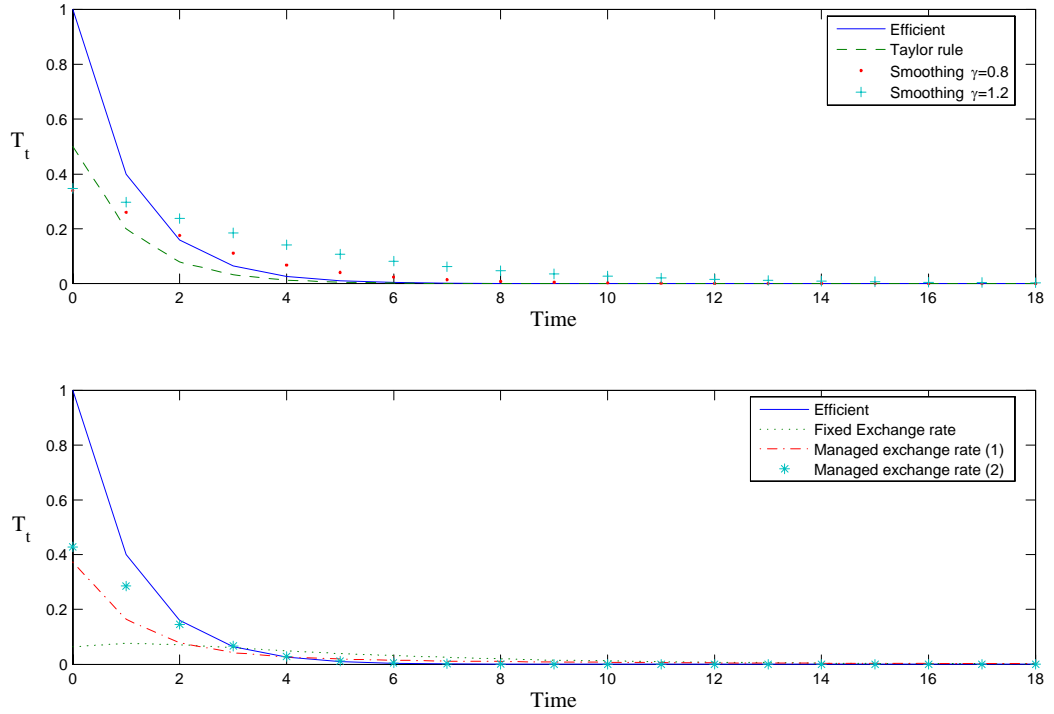


Figure 8:

Figure 9: Impulse Response of the Exchange Rate to a Shock to the Natural Rate Terms of Trade, $\rho_1=0.4$

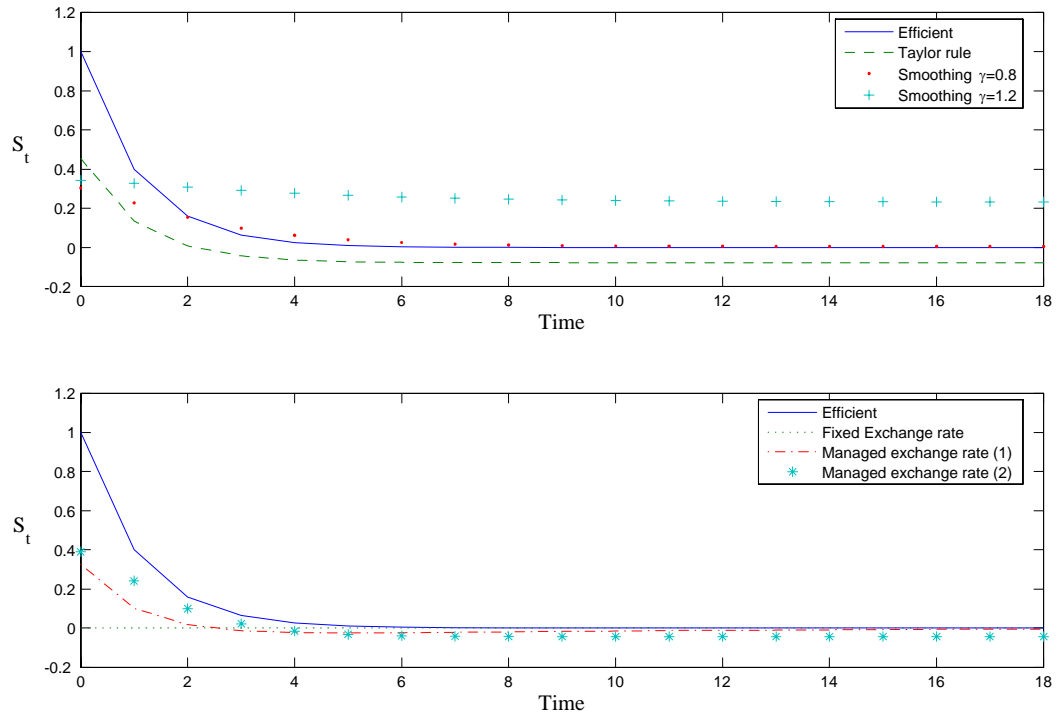


Figure 9:

rules and the managed exchange rate regime (I), the exchange rate first depreciates and then appreciates, but in the managed exchange rate regime (I) the exchange rate reverts to the target. Instead, under the managed exchange rate (II) there is a long-run appreciation less than under the Taylor-rule regime. Consistently with the previous analysis, note that the property of stationary exchange rate obtained when $\gamma = 0.8$ is not robust. With different smoothing parameters, the exchange rate would be non stationary, as it is the case with $\gamma = 1.2$.

5 Conclusions

This paper provides a simple framework for addressing empirical and policy issues in an open economy framework. We have emphasized the crucial role of policy rules in determining a different pattern for the nominal exchange rate and the terms of trade.

Here we want to underline how the simplicity and the flexibility of this model make it suitable for analyzing many other important issues that we have neglected at first pass. The extension to the analysis of monetary policy rules to an open economy framework provide new insights on the desirability of alternative rules and raises a number of issues of great interest, among which the choice of exchange rate regime, the potential benefit from monetary policy coordination, the optimal response to shock originating from abroad and the choice of consumer price indexes versus domestic inflation targeting.

References

- [1] Ball, Laurence, 1999, “Policy Rules for Open Economy”, in John Taylor (ed.) *Monetary Policy Rules*, University of Chicago Press, 1999, pp. 127-144
- [2] Benigno Pierpaolo, 2004, “Optimal Monetary Policy in a Currency Area”, *Journal of International Economics* 63(2): 293-320.
- [3] Benigno, Gianluca and Pierpaolo Benigno, 2001, “Monetary Policy Rules and the Exchange Rate”, CEPR working paper No. 2807.
- [4] Benigno, Gianluca and Pierpaolo Benigno, 2006, “Designing Targeting Rules for International Monetary Policy Cooperation,” *Journal of Monetary Economics*, 53(3): 473-506.
- [5] Benigno, Gianluca, Pierpaolo Benigno and Fabio Ghironi, 2006, “Interest Rate Rules for Fixed Exchange Rate Regimes”, forthcoming *Journal of Economic Dynamics and Control*.

- [6] Beveridge, Stephen and Charles R. Nelson, 1981, "A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle", *Journal of Monetary Economics* 7: 151-74.
- [7] Calvo, G. A., 1983, Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* 12, 383-398.
- [8] Chari, V.V., Kehoe, P. J., McGrattan, E. R., 2002. "Can sticky price models generate volatile and persistent real exchange rates?" *Review of Economic Studies* 69, 533-563.
- [9] Clarida, Richard, Jordi Galí and Mark Gertler, 1998, "Monetary Policy Rules in Practice: Some International Evidence", *European Economic Review*, 42: 1033-1068.
- [10] De Fiore, Fiorella and Liu, Zheng, 2005. "Does trade openness matter for aggregate instability?," *Journal of Economic Dynamics and Control*, vol. 29(7): 1165-1192.
- [11] Galí, Jordi and Tommaso Monacelli, 2005, "Monetary Policy and Exchange Rate Volatility in a Small Open Economy ", *Review of Economic Studies*, 72: 707-734.
- [12] Ghironi, Fabio, 1998, "Alternative Monetary Policy Rules: the Case of Canada", BC Econ. Dept. Working Paper 466.
- [13] Giannoni, M., 2000, "Optimal interest-rate rules in a forward-looking model and inflation stabilization versus price-level stabilization", unpublished manuscript, Federal Reserve Bank of New York.
- [14] Kerr, William and Robert G. King, 1996, "Limits on Interest Rate Rules in the IS Model," *Quarterly Review*, vol. 82, no. 2 (Spring 1996): 47-75.
- [15] Kollman, Robert, 1998, "International Financial Markets and Business Cycles: A General Equilibrium Approach with Money and Nominal Rigidities," unpublished, University of Paris XII.

- [16] Laxton, Douglas and Paolo Pesenti, 2003, “Monetary Rules for Small, Open, Emerging Economies,” *Journal of Monetary Economics* 50 (5) July 2003, pp. 1109-1146
- [17] McCallum, Bennet T. and Edward Nelson, 1999, “Nominal Income Targeting in an Open-Economy Optimizing Model”, *Journal of Monetary Economics*, 43: 553-578.
- [18] Monacelli, Tommaso, 2004, “Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy”, *Journal of International Economics*, 62, 191-217.
- [19] Obstfeld Maurice, 1997, “Open Economy Macroeconomics: Developments in Theory and Policy”, NBER Working Paper No. 6319.
- [20] Rotemberg, Julio J., and Michael Woodford, 1998, “Interest-Rate Rules in an Estimated Sticky-Price Model,” in J.B. Taylor, ed., *Monetary Policy Rules*, Chicago: University of Chicago Press.
- [21] Svensson, Lars E., 1999, “Inflation Targeting as a Monetary Policy Rule”, *Journal of Monetary Economics* 43: 607-655.
- [22] Svensson, Lars E., 2000, “Open Economy Inflation Targeting”, *Journal of International Economics*, 50: 155-183.
- [23] Taylor, John, 1999, “*Monetary Policy Rules*”, Chicago: University of Chicago Press.
- [24] Weeparana, Akila, 1998, “The Performance of Simple Monetary Policy Rules in a Model of Asymmetric Large Open Economies,” unpublished, Stanford University.
- [25] Woodford, Michael, 1996, “Control of the Public Debt: A Requirement for Price Stability?”, NBER Working Paper No. 5684.
- [26] Woodford, M., 2003, “Interest and prices: foundations of a theory of monetary policy”, Princeton University Press: Princeton.

- [27] Yun Tack, 1996, “Nominal price rigidity, money supply endogeneity, and business cycles”, *Journal of Monetary Economics* (37)2 pp. 345-370.

Appendix

Appendix A

Proof of the propositions of Section 2.

Proof of Proposition 1

In what follows, we disregard the stochastic disturbances in the structural equations since they will not matter for equilibrium determinacy. As discussed, in the text, the condition $\tau > 0$ is necessary and sufficient to have a unique and stable path for the nominal exchange rate. Under the assumption that $\alpha_H = \alpha_F$ we can take the difference between equation (7) and (8) to obtain

$$\pi_t^H - \pi_t^F = \lambda(1 + \eta)\hat{T}_t + \beta E_t(\pi_{t+1}^H - \pi_{t+1}^F) + \text{o.t.} \quad (\text{B.1})$$

where with o.t. we denote ‘other terms’ that are not relevant for the analysis of determinacy. Using (4) and assuming that $\Delta\hat{S}_t = 0$ —as it is implied by the fixed exchange rate—we can substitute out for $\pi_t^H - \pi_t^F$ in (B.1) and obtain

$$\hat{T}_t - \hat{T}_{t-1} = -\lambda(1 + \eta)\hat{T}_t + \beta E_t(\hat{T}_{t+1} - \hat{T}_t) + \text{o.t.}$$

which is a second-order stochastic difference equation in \hat{T}_t with a characteristic polynomial of the form

$$P(\zeta) = \beta\zeta^2 - [1 + \lambda(1 + \eta) + \beta]\zeta + 1.$$

All the roots are positive and since $P(0) > 0$ and $P(1) < 0$, there is always one root within the unit circle. It follows that the path of \hat{T}_t is determined. To determine the other endogenous variables of the model, we note that we can write (6) as

$$E_t y_{t+1}^H = y_t^H + \rho^{-1}(\hat{y}_t^H - E_t \pi_{t+1}^H) + \text{o.t.} \quad (\text{B.2})$$

and (7) becomes

$$\pi_t^H = \lambda(\rho + \eta)y_t^H + \beta E_t \pi_{t+1}^H + \text{o.t.} \quad (\text{B.3})$$

since \hat{T}_t is already determined. We can then combine (B.2) and (B.3) in a system of the form

$$AE_t z_{t+1} = Bz_t + \text{o.t.}$$

where the vector z_t is defined as $z_t \equiv [\pi_t^H \ y_t^H]$ and the matrices A and B are given by

$$A = \begin{bmatrix} \rho^{-1} & 1 \\ \beta & 0 \end{bmatrix} \quad B = \begin{bmatrix} \rho^{-1}\phi & 1 + \rho^{-1}\psi \\ 1 & -\lambda(\rho + \eta) \end{bmatrix}.$$

The characteristic polynomial can be obtained by

$$\det[A\zeta - B] = 0$$

which can be written as

$$P(\zeta) = \beta\zeta^2 - [1 + \beta + \beta\psi\rho^{-1} + \lambda(\rho + \eta)\rho^{-1}]\zeta + 1 + \lambda\phi(\rho + \eta)\rho^{-1} + \psi\rho^{-1}.$$

Determinacy requires that the two roots are outside the unit circle. For Descartes' sign rule, all roots are positive. Since $P(0) > 0$, $P'(0) < 0$ and $P''(0) > 0$ all roots have positive real part. Since $P(0) > 0$ it follows that it is necessary that $P(1) > 0$, which requires that

$$(\phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) > 0.$$

This is also a sufficient condition since there cannot be two real or complex roots within the unit circle for the products of the two roots is greater than $1/\beta$ and then greater than 1.

Proof of Proposition 2

Under the assumptions of the proposition, we can take the weighted average of equations (7) and (8) and obtain

$$\pi_t^W = \lambda(\rho + \eta)y_t^W + \beta E_t \pi_{t+1}^W + \text{o.t.} \quad (\text{B.4})$$

where $\pi_t^W = n\pi_t^H + (1 - n)\pi_t^F$. Equation (6) implies that

$$E_t y_{t+1}^W = y_t^W + \rho^{-1}(\hat{i}_t^W - E_t \pi_{t+1}^W) + \text{o.t.}, \quad (\text{B.5})$$

where $\hat{i}_t^W = n\hat{i}_t^H + (1-n)\hat{i}_t^F$, while the interest rate rules imply that

$$\hat{i}_t^W = \gamma\hat{i}_{t-1}^W + \phi\pi_t^W + \psi y_t^W. \quad (\text{B.6})$$

We can combine (B.4), (B.5) and (B.6) in a system of the form

$$AE_t z_{t+1} = Bz_t + \text{o.t.}$$

where the vector z_t is defined as $z_t \equiv [y_t^W \ \pi_t^W \ \hat{i}_t^W]$ and the matrices A and B are given by

$$A = \begin{bmatrix} 1 & \rho^{-1} & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 + \rho^{-1}\psi & \rho^{-1}\phi & \rho^{-1}\gamma \\ -\lambda(\rho + \eta) & 1 & 0 \\ \psi & \phi & \gamma \end{bmatrix}.$$

The characteristic polynomial is given by

$$\det[A\zeta - B] = 0$$

and is of the form

$$P(\zeta) = \beta\zeta^3 - [1 + \beta + \rho^{-1}\psi\beta + \beta\gamma + \lambda(\rho + \eta)\rho^{-1}]\zeta^2 + [1 + \lambda(\rho + \eta)\rho^{-1}(\phi + \gamma) + \beta\gamma + \rho^{-1}\psi + \gamma]\zeta - \gamma.$$

To have determinacy there should be one root within the unit circle. Since $P(0) \leq 0$, $P'(0) > 0$, $P''(0) < 0$, $P'''(0) > 0$ all the roots have non-negative real part. Moreover since $P(0) \leq 0$ when $\gamma \geq 0$ and $P'(0) > 0$ it is necessary that $P(1) > 0$ which requires that

$$(\gamma + \phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) > 0. \quad (\text{B.7})$$

In this case either there is one real root or three real roots or one real and two complex roots within the unit circle. To see that this condition is also sufficient. We write the polynomial in a generic form

$$P(\zeta) = a\zeta^3 + b\zeta^2 + c\zeta + d.$$

In particular, for this polynomial we know that

$$\zeta_1\zeta_2\zeta_3 = -\frac{d}{a}.$$

When $\gamma \geq \beta$ then $\zeta_1\zeta_2\zeta_3 \geq 1$ and at least one root is out of the unit circle. But then condition (B.7) assures that only one root (a real root) is indeed within the unit circle. When $\gamma < \beta$ we need to show that

$$\left(\frac{d}{a}\right)^2 - \frac{d}{a}b + c - 1 > 0 \quad (\text{B.8})$$

as discussed in Woodford (2003, p. 673). In our case (B.8) boils down to require that

$$\frac{(\eta + \rho)}{\beta\rho} \left(\phi + \gamma - \frac{\gamma}{\beta}\right) + \frac{(1 - \gamma)}{\beta\rho}\psi + (1 - \gamma)(1 - \beta) \frac{1}{\beta} \left(1 - \frac{\gamma}{\beta}\right) > 0.$$

But then (B.7) implies that

$$(\gamma + \phi)(\rho + \eta)\lambda > (\rho + \eta)\lambda - \psi(1 - \beta)$$

so that

$$\begin{aligned} \frac{(\eta + \rho)}{\beta\rho} \left(\phi + \gamma - \frac{\gamma}{\beta}\right) + \frac{(1 - \gamma)}{\beta\rho}\psi + (1 - \gamma)(1 - \beta) \frac{1}{\beta} \left(1 - \frac{\gamma}{\beta}\right) > \\ \frac{(\eta + \rho)}{\beta\rho} \left(1 - \frac{\gamma}{\beta}\right) + \frac{(\beta - \gamma)}{\beta\rho}\psi + (1 - \gamma)(1 - \beta) \frac{1}{\beta} \left(1 - \frac{\gamma}{\beta}\right). \end{aligned}$$

In particular the second line of the above expression is indeed positive when $\gamma < \beta < 1$. It follows that (B.7) is also sufficient.

We can now take the difference between (7) and (8) and using the fact that

$$y_t^H - y_t^F = (\hat{T}_t - \tilde{T}_t), \quad (\text{B.9})$$

we obtain

$$\pi_t^R = \lambda(1 + \eta)y_t^R + \beta E_t \pi_{t+1}^R + \text{o.t.} \quad (\text{B.10})$$

Moreover we note that (4) and (B.9) imply

$$\Delta S_t = \pi_t^R + \Delta y_t^R + \text{o.t.}$$

so that we can write (2) as

$$E_t \pi_{t+1}^R + E_t \Delta y_{t+1}^R = \hat{v}_t^R. \quad (\text{B.11})$$

Finally the difference between the interest rate rules imply

$$\hat{i}_t^R = \gamma \hat{i}_{t-1}^R + \phi \pi_t^R + \psi y_t^R. \quad (\text{B.12})$$

We can combine (B.10), (B.11) and (B.12) in a system of the form

$$AE_t z_{t+1} = Bz_t + \text{o.t.}$$

where the vector z_t is defined as $z_t \equiv [y_t^R \ \pi_t^R \ \hat{i}_t^R]$ and the matrices A and B are given by

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 + \psi & \phi & \gamma \\ -\lambda(1 + \eta) & 1 & 0 \\ \psi & \phi & \gamma \end{bmatrix}.$$

The characteristic polynomial is given by

$$\det[A\zeta - B] = 0$$

which is of the form

$$P(\zeta) = \beta\zeta^3 - [1 + \beta + \psi\beta + \beta\gamma + \lambda(1 + \eta)]\zeta^2 + [1 + \lambda(1 + \eta)(\phi + \gamma) + \beta\gamma + \psi + \gamma]\zeta - \gamma.$$

To have determinacy there should be one root within the unit circle. Since $P(0) \leq 0$, $P'(0) > 0$, $P''(0) < 0$, $P'''(0) > 0$ all the roots have non-negative real part. Moreover since $P(0) \leq 0$ when $\gamma \geq 0$ and $P'(0) > 0$ it is necessary that $P(1) > 0$ which requires that

$$(\gamma + \phi - 1)(1 + \eta)\lambda + \psi(1 - \beta) > 0.$$

But this is also sufficient for a similar reasoning to the previous part of the proof in this proposition.

Proof of Proposition 3

Under the assumptions of the proposition, we can take the weighed average of equations (7) and (8) and obtain

$$\pi_t^W = \lambda(\rho + \eta)y_t^W + \beta E_t \pi_{t+1}^W + \text{o.t.} \quad (\text{B.13})$$

Equation (6) implies that

$$E_t y_{t+1}^W = y_t^W + \rho^{-1}(\hat{i}_t^W - E_t \pi_{t+1}^W) + \text{o.t.}, \quad (\text{B.14})$$

The interest rate rules imply that

$$\hat{i}_t^W = \phi_p p_t^W + \psi y_t^W, \quad (\text{B.15})$$

where

$$p_t^W = \pi_t^W + p_{t-1}^W \quad (\text{B.16})$$

We can combine (B.13) to (B.16) in a system of the form

$$A E_t z_{t+1} = B z_t + \text{o.t.}$$

where the vector z_t is defined as $z_t \equiv [y_t^W \ \pi_t^W \ p_t^W]$ and the matrices A and B are given by

$$A = \begin{bmatrix} 1 & \rho^{-1} & -\rho^{-1}\phi_p \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 + \rho^{-1}\psi & 0 & 0 \\ -\lambda(\rho + \eta) & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

The characteristic polynomial is given by

$$\det[A\zeta - B] = 0$$

which is of the form

$$P(\zeta) = \beta\zeta^3 - [1 + 2\beta + \rho^{-1}\psi\beta + \lambda(\rho + \eta)\rho^{-1}]\zeta^2 + [2 + \beta + \rho^{-1}\psi(1 + \beta) + \lambda(\rho + \eta)(\phi_p + 1)\rho^{-1}]\zeta - (1 + \psi\rho^{-1}).$$

To have determinacy there should be one root within the unit circle. Since $P(0) < 0$, $P'(0) > 0$, $P''(0) < 0$, $P'''(0) > 0$ all the roots have positive real part. Moreover since $P(0) < 0$ it is necessary that $P(1) > 0$ which requires that

$$\phi_p > 0.$$

But this is also sufficient since there cannot be three roots within the unit circle for the products of the roots is $(1 + \psi\rho^{-1})\beta^{-1}$ which is greater than 1.

As in the previous proposition, we can write the relative block of the model as

$$\pi_t^R = \lambda(1 + \eta)y_t^R + \beta E_t \pi_{t+1}^R + \text{o.t.},$$

$$E_t \pi_{t+1}^R + E_t \Delta y_{t+1}^R = \hat{i}_t^R.$$

Finally the difference of the interest rate rules imply

$$\hat{i}_t^R = \phi_p p_t^R + \psi y_t^R$$

where

$$p_t^R = \pi_t^R + p_{t-1}^R.$$

We can combine the above equations in a system of the form

$$A E_t z_{t+1} = B z_t + \text{o.t.}$$

where the vector z_t is defined as $z_t \equiv [y_t^R \ \pi_t^R \ p_t^R]$ and the matrices A and B are given by

$$A = \begin{bmatrix} 1 & 1 & -\phi_p \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 + \psi & 0 & 0 \\ -\lambda(1 + \eta) & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

The characteristic polynomial is given by

$$\det[A\zeta - B] = 0$$

and is of the form

$$P(\zeta) = \beta \zeta^3 - [1 + 2\beta + \psi\beta + \lambda(1 + \eta)]\zeta^2 + [2 + \beta + \psi(1 + \beta) + \lambda(1 + \eta)(\phi_p + 1)]\zeta - (1 + \psi).$$

To have determinacy there should be one root within the unit circle. Since $P(0) < 0$, $P'(0) > 0$, $P''(0) < 0$, $P'''(0) > 0$ all the roots have positive real part. Moreover since $P(0) < 0$ it is necessary that $P(1) > 0$ which requires that

$$\phi_p > 0. \tag{B.17}$$

But this is also sufficient since there cannot be three roots within the unit circle because the product of the roots is equal to $(1 + \psi)\beta^{-1}$ which is greater than 1.

Proof of Proposition 4

We start from the relative block model and note that using the interest rate rules equation (2) can be written as

$$E_t \hat{S}_{t+1} = \phi \pi_t^R + \psi y_t^R + (1 + \delta) \hat{S}_t^* \quad (\text{B.18})$$

Moreover (4) and (B.9) imply

$$\Delta y_t^R = \hat{S}_t^* - \hat{S}_{t-1}^* - \pi_t^R + \text{o.t.} \quad (\text{B.19})$$

We can then put (B.10), (B.18) and (B.19) in a system of the form

$$A E_t z_{t+1} = B z_t + \text{o.t.}$$

where the vector z_t is defined as $z_t \equiv [\hat{S}_t^* \ \pi_t^R \ y_t^R \ \hat{S}_{t-1}^*]$ and the matrices A and B are given by

$$A = \begin{bmatrix} 1 & 0 & -\psi & 0 \\ 0 & \beta & \lambda(1 + \eta) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \delta + 1 & \phi & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

The characteristic polynomial is given by

$$\det[A\zeta - B] = 0$$

and is of the form

$$\begin{aligned} P(\zeta) &= \beta \zeta^4 - [2\beta + \lambda(1 + \eta) + \psi\beta + 1 + \delta\beta] \zeta^3 + \\ &\quad + [\psi + \lambda(1 + \eta)(1 + \phi + \delta) + \delta(1 + \beta) + \psi\beta + 2] \zeta^2 - [\delta + \psi + \phi\lambda(1 + \eta) + 1] \zeta \end{aligned}$$

To have determinacy there should be two roots within the unit circle. Since $P(0) = 0$, $P'(0) < 0$, $P''(0) > 0$, $P'''(0) < 0$ and $P''''(0) < 0$ all the roots have non-negative real part. From $P(\zeta)$ it follows that one root is zero. Moreover $P'(0) < 0$ so it is necessary that $P(1) > 0$ which requires that $\delta > 0$ or that $P(1) = 0$ and

$P'(1) > 0$. The latter condition is required to avoid that there are two coincident roots at 1. In this case it is then necessary that $\delta = 0$ and

$$(\phi - 1)(1 + \eta)\lambda + \psi(1 - \beta) > 0.$$

But these conditions are also sufficient because the sum of the four roots is in any case greater than $2 + 1/\beta$ and then of 3.

In the aggregate block we obtain that equations (B.4) and (B.5) still hold but now

$$\hat{i}_t^W = \phi\pi_t^W + \psi y_t^W + \delta\hat{S}_t^*.$$

However since the path of \hat{S}_t^* is determined by the relative block of the model then the condition for determinacy boils down to the one already found, i.e. that

$$(\phi - 1)\lambda(\rho + \eta) + \psi(1 - \beta) > 0.$$

Proof of Proposition 5

We start from the relative block of the model and note that using the interest rate rules, equation (2) can be written as

$$E_t\Delta\hat{S}_{t+1} = \phi\pi_t^R + \psi y_t^R + \mu\Delta\hat{S}_t \quad (\text{B.20})$$

Moreover (4) and (B.9) imply

$$\Delta\hat{S}_t = \Delta y_t^R + \pi_t^R + \text{o.t.} \quad (\text{B.21})$$

We can then combine (B.20) and (B.21) to obtain

$$E_t\Delta y_{t+1}^R + E_t\pi_{t+1}^R = (\phi + \mu)\pi_t^R + \psi y_t^R + \mu\Delta y_t^R \quad (\text{B.22})$$

which together with (B.10) can be put in a system of the form

$$AE_t z_{t+1} = Bz_t + \text{o.t.}$$

where the vector z_t is defined as $z_t \equiv [y_t^R \ \pi_t^R \ y_{t-1}^R]$ and the matrices A and B are given by

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \psi + 1 + \mu & \phi + \mu & -\mu \\ -\lambda(1 + \eta) & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

The characteristic polynomial is given by

$$\det[A\zeta - B] = 0$$

which is of the form

$$P(\zeta) = \beta\zeta^3 - [1 + \beta + \psi\beta + \beta\gamma + \lambda(1 + \eta)]\zeta^2 + [1 + \lambda(1 + \eta)(\phi + \mu) + \beta\gamma + \psi + \mu]\zeta - \mu.$$

To have determinacy there should be one root within the unit circle. Since $P(0) \leq 0$, $P'(0) > 0$, $P''(0) < 0$, $P'''(0) > 0$ all the roots are non-negative. Moreover since $P(0) \leq 0$ and $P'(0) > 0$ it is necessary that $P(1) > 0$ which requires that

$$(\mu + \phi - 1)(1 + \eta)\lambda + \psi(1 - \beta) > 0.$$

But this is also sufficient for a similar argument to the sufficient conditions in proposition 2.

In the aggregate block we obtain that equations (B.4) and (B.5) still hold but now

$$\hat{i}_t^W = \phi\pi_t^W + \psi y_t^W + \mu\Delta\hat{S}_t.$$

However since the path of \hat{S}_t is determined by the relative block of the model then the condition for determinacy boils down to the one already found in proposition 2 under the assumption $\gamma = 0$,

$$(\phi - 1)(\rho + \eta)\lambda + \psi(1 - \beta) > 0.$$

Proof of Proposition 6

Using (B.9) we can write (7) and (8) as

$$\pi_t^H = \lambda^H \{[(1 - n)(1 + \eta) + n(\rho + \eta)]y_{H,t} + (1 - n)(\rho - 1)y_{F,t}\} + \beta E_t \pi_{t+1}^F, \quad (\text{B.23})$$

$$\pi_t^F = \lambda^F \{[n(1 + \eta) + (1 - n)(\rho + \eta)]y_{F,t} + n(\rho - 1)y_{H,t}\} + \beta E_t \pi_{t+1}^H. \quad (\text{B.24})$$

The Euler equation can be written

$$\begin{aligned} E_t[ny_{H,t+1} + (1 - n)y_{F,t+1}] &= ny_{H,t} + (1 - n)y_{F,t} + \rho^{-1}n(\phi\pi_t^H - E_t\pi_{t+1}^H - \tilde{R}_t^W) \\ &\quad + \rho^{-1}(1 - n)(\phi^*\pi_t^F - E_t\pi_{t+1}^F - \tilde{R}_t^W), \end{aligned} \quad (\text{B.25})$$

where we have already substituted in the interest rate rule. Equation (2) together with (4) and (B.9) imply

$$E_t\{\Delta y_{H,t+1} - \Delta y_{F,t+1} + \pi_{t+1}^H - \pi_{t+1}^F\} = \phi\pi_t^H - \phi^*\pi_t^F + E_t\tilde{T}_{t+1} - \tilde{T}_t$$

We can then write the system in the form

$$AE_t z_{t+1} = Bz_t + \text{o.t.}$$

where $x_t = [\pi_t^H \ \pi_t^F \ y_{H,t} \ y_{F,t}]$ and

$$A = \begin{bmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ \rho^{-1}n & \rho^{-1}(1-n) & n & 1-n \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & -\lambda^H((1-n)(1+\eta) + n(\rho+\eta)) & -\lambda^F(1-n)(\rho-1) \\ 0 & 1 & -\lambda^F n(\rho-1) & -\lambda^H(n(1+\eta) + (1-n)(\rho+\eta)) \\ \rho^{-1}n\phi & \rho^{-1}(1-n)\phi^* & n & (1-n) \\ \phi & -\phi^* & 1 & -1 \end{bmatrix}$$

The characteristic polynomial is given by

$$\det[A\zeta - B] = 0$$

and is of the form

$$P(\zeta) = a\zeta^4 + b\zeta^3 + c\zeta^2 + d\zeta + e$$

where

$$a = -\beta^2$$

$$b = \rho^{-1}(k_1\lambda^F + k_2\lambda^H) + 2\beta(1+\beta)$$

$$d = \beta^{-1}b + \rho^{-1}\lambda^H(1+\beta)k_2\phi + \rho^{-1}\lambda^F(1+\beta)k_1\phi^* + \lambda^H\lambda^F\rho^{-1}k_3(\phi + \phi^*)$$

$$c = -(1+\beta^2+4\beta) - \lambda^F\rho^{-1}(1+\beta+\beta\phi^*)k_1 - \lambda^H\rho^{-1}(1+\beta+\beta\phi)k_2 - \lambda^H\lambda^F\rho^{-1}k_3$$

$$e = -1 - \lambda^H\lambda^F\rho^{-1}k_3\phi\phi^* - \lambda^F\rho^{-1}\phi^*k_1 - \lambda^H\rho^{-1}\phi k_2$$

and we have defined

$$k_1 \equiv \rho + (1 - n)\eta + n\rho\eta,$$

$$k_2 \equiv \rho + (1 - n)\eta\rho + n\eta,$$

$$k_3 \equiv (\rho + \eta)(1 + \eta).$$

To have determinacy we need all roots to be outside the unit circle. Since $P(0) < 0$, $P'(0) > 0$, $P''(0) < 0$, $P'''(0) > 0$, $P''''(0) < 0$ all roots have positive real part. Moreover since $P(0) < 0$ is necessary that $P(1) < 0$. Noting that

$$P(1) = -\text{sign}(\phi - 1)(\phi^* - 1)$$

we need that ϕ^* and ϕ are both above or below one. Of the two possibilities the requirement that they are both above one is also sufficient.

Indeed if $P(1) < 0$ there can be either four or two or zero roots within the unit circle. Since the products of the roots is $e/a > 1/\beta^2 > 1$ there cannot be four roots, either complex or real, within the unit circle. To exclude the possibility that there are two real roots within the unit circle we note that $P(0) < 0$, $P'(0) > 0$, $P''(0) < 0$, $P'''(0) > 0$, $P''''(0) < 0$ and $P(1) < 0$, $P'(1) > 0$ if $\phi > 1$ and $\phi^* > 1$, $P''(1) < 0$ if $\phi > 1$ and $\phi^* > 1$, $P'''(1) > 0$, $P''''(1) < 0$. When $\phi > 1$ and $\phi^* > 1$ the number of sign changes in the sequence $\{P(0), P'(0), P''(0), P'''(0), P''''(0)\}$ is the same as the number of sign changes in the sequence $\{P(1), P'(1), P''(1), P'''(1), P''''(1)\}$, this property implies that there are no real roots within the unit circle, following Budan-Fourier theorem. We need to exclude the remaining possibility that there are two complex roots within the unit circle. But this cannot be a possibility if on top of the above reasoning

$$(\zeta_1\zeta_2 - 1)(\zeta_1\zeta_3 - 1)(\zeta_1\zeta_4 - 1)(\zeta_2\zeta_3 - 1)(\zeta_2\zeta_4 - 1)(\zeta_3\zeta_4 - 1) > 0. \quad (\text{B.26})$$

Suppose that ζ_1 and ζ_2 are complex and within the unit circle. It follows that $\zeta_1\zeta_2 < 1$. Moreover since ζ_3 and ζ_4 are out of the unit circle then $\zeta_3\zeta_4 > 1$. No matter whether ζ_3 and ζ_4 are complex or real $(\zeta_1\zeta_3 - 1)(\zeta_1\zeta_4 - 1)(\zeta_2\zeta_3 - 1)(\zeta_2\zeta_4 - 1) > 0$ because in the case they are complex the couple $\zeta_1\zeta_3$ and $\zeta_2\zeta_4$

are complex conjugate as well as the couple $\zeta_1\zeta_4$ and $\zeta_2\zeta_3$ while in the case ζ_3 and ζ_4 are real then $\zeta_1\zeta_3$ and $\zeta_2\zeta_3$ are complex conjugate as well as $\zeta_2\zeta_4$ and $\zeta_3\zeta_4$. It follows that the above expression (B.26) cannot be positive. We can show that (B.26) requires that

$$1 + \frac{e^3}{a^3} - \frac{1}{a^2} (d - b) \left(d - \frac{e}{a} b \right) - \frac{c}{a} \left(\frac{e}{a} - 1 \right)^2 > 0$$

which we can show –after some tedious algebra– to be satisfied when $\phi > 1$ and $\phi^* > 1$.

Appendix B

In this appendix we derive the results of section 4 in the text.

Floating exchange-rate regime (I): Taylor rules.

In this appendix, we assume that both countries have the same degree of nominal rigidities. Under the Taylor-rules regime specified by the rules

$$\begin{aligned} \hat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi\pi_t^F + \psi y_t^F, \end{aligned}$$

there is complete separation between the determination of world variables and relative variables. It is the case that the equilibrium paths of the terms of trade, the exchange rate and the inflation rate differential can be obtained from the following equilibrium conditions

$$\pi_t^F - \pi_t^H = -\lambda(1 + \eta)(\hat{T}_t - \tilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H), \quad (\text{C.27})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta \hat{S}_t, \quad (\text{C.28})$$

$$E_t \Delta S_{t+1} = \phi(\pi_t^H - \pi_t^F) + \psi(\hat{T}_t - \tilde{T}_t), \quad (\text{C.29})$$

where condition (C.27) is obtained by subtracting (7) from (8); equation (C.28) is equation (4) in the text while the third equation is (2) in which the interest rate rules have been substituted. Given the Markovian nature of the process \tilde{T}_t , a

rational expectations equilibrium assumes the following form

$$\begin{aligned}\hat{T}_t &= b_1\hat{T}_{t-1} + c_1\tilde{T}_t, \\ \pi_t^F - \pi_t^H &= b_2\hat{T}_{t-1} + c_2\tilde{T}_t, \\ \Delta S_t &= b_3\hat{T}_{t-1} + c_3\tilde{T}_t.\end{aligned}$$

where the coefficients satisfy the following restrictions

$$\begin{aligned}b_1 &= 1 + b_2 + b_3 \\ c_1 &= c_2 + c_3, \\ b_1b_3 &= -\phi b_2 + \psi b_1, \\ c_1b_3 + \rho_1c_3 &= -\phi c_2 + \psi c_1 - \psi, \\ c_1(\beta b_2 - \lambda(1 + \eta)) + \beta c_2\rho_1 &= c_2 - k_T, \\ b_1(\beta b_2 - \lambda(1 + \eta)) &= b_2.\end{aligned}$$

In a unique and stable rational expectations equilibrium (stability requires that $|b_1| < 1$), it is always the case that $b_1 = b_2 = 0$ while $b_3 = -1$.

The coefficients c_1 , c_2 and c_3 are given by

$$\begin{aligned}c_1 &= \frac{(\phi - \rho_1)\lambda(1 + \eta) + \psi(1 - \beta\rho_1)}{(\phi - \rho_1)\lambda(1 + \eta) + (1 - \rho_1)(1 - \beta\rho_1) + \psi(1 - \beta\rho_1)}, \\ c_2 &= \frac{(1 - \rho_1)\lambda(1 + \eta)}{(\phi - \rho_1)\lambda(1 + \eta) + (1 - \rho_1)(1 - \beta\rho_1) + \psi(1 - \beta\rho_1)}, \\ c_3 &= \frac{(\phi - 1)\lambda(1 + \eta) + \psi(1 - \beta\rho_1)}{(\phi - \rho_1)\lambda(1 + \eta) + (1 - \rho_1)(1 - \beta\rho_1) + \psi(1 - \beta\rho_1)}.\end{aligned}$$

Floating exchange-rate regime (I)

Under the class of interest-rate smoothing rules

$$\begin{aligned}\hat{i}_t^H &= \gamma\hat{i}_{t-1}^H + \phi\pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \gamma\hat{i}_{t-1}^F + \phi\pi_t^F + \psi y_t^F,\end{aligned}$$

the relevant equilibrium conditions for the determination of the terms of trade and

the exchange rate are

$$\pi_t^F - \pi_t^H = -\lambda(1 + \eta)(\hat{T}_t - \tilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H), \quad (\text{C.30})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta \hat{S}_t, \quad (\text{C.31})$$

$$E_t \Delta \hat{S}_{t+1} = \phi(\pi_t^H - \pi_t^F) + \gamma(\hat{i}_{t-1}^H - \hat{i}_{t-1}^F) + \psi(\hat{T}_t - \tilde{T}_t), \quad (\text{C.32})$$

$$\hat{i}_t^H - \hat{i}_t^F = \gamma(\hat{i}_{t-1}^H - \hat{i}_{t-1}^F) + \phi(\pi_t^H - \pi_t^F) + \psi(\hat{T}_t - \tilde{T}_t), \quad (\text{C.33})$$

where condition (C.30) is obtained by subtracting (7) from (8); equation (C.31) is equation (4) in the text while (C.32) is (2) in which the interest rate rules have been substituted. Equation (C.33) is the difference between the two interest rate rules. The above set of equations can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1}^s \\ z_t^s \end{bmatrix} = \begin{bmatrix} M_1^s & M_2^s \\ M_3^s & M_4^s \end{bmatrix} \begin{bmatrix} y_t^s \\ z_{t-1}^s \end{bmatrix} + \begin{bmatrix} m_1^s \\ m_2^s \end{bmatrix} \tilde{T}_t,$$

where $y_t^{s'} = [\pi_t^R \quad \Delta \hat{S}_t]$, $z_{t-1}^{s'} = [\hat{i}_{t-1}^R \quad \hat{T}_{t-1}]$, M_j^s are 2×2 matrices, while m_j^s are 2×1 vectors. Under the conditions for determinacy, there are two eigenvalues outside the unit circle. Let denote these eigenvalues as ω_1 and ω_2 collected in the diagonal matrix Ω . Let V a 2×4 matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that $VM^s = \Omega V$. Furthermore we decompose V in two 2×2 matrices with $V = [V_1 \ V_2]$. Now, if \tilde{T}_t follows a Markovian process of the form (17), we have that the solution for y_t is of the form

$$y_t^s = \Psi_1^s z_{t-1} + \Psi_2^s \tilde{T}_t,$$

where $\Psi_1^s \equiv -V_1^{-1}V_2$ and $\Psi_2^s \equiv V_1^{-1}(I\rho - \Omega)^{-1}Vm$. We then obtain

$$\begin{aligned} z_t^s &= M_3^s y_t^s + M_4^s z_{t-1}^s + m_2^s \tilde{T}_t, \\ &= (M_3^s \Psi_1^s + M_4^s) z_{t-1}^s + (M_3^s \Psi_2^s + m_2^s) \tilde{T}_t, \\ &= Z_1^s z_{t-1}^s + Z_2^s \tilde{T}_t, \end{aligned}$$

where Z_1^s and Z_2^s have been appropriately defined. We have shown that in a pure floating regime the terms of trade do not introduce any intrinsic inertia. The inclusion of a smoothing argument into the Taylor rules does not alter this

property. In fact the second column of Z_1^s is of zeros. It follows that the only source of inertia in the system is coming only from the interest rate smoothing component. Reminding that $z_t^{s'} = [\hat{i}_t^R \ \hat{T}_t]$, it can be possible to obtain a solution for \hat{T}_t and \hat{i}_t^R of the form

$$\begin{aligned} A^s(L)\hat{T}_t &= R^s(L)\tilde{T}_t, \\ A^s(L)\hat{i}_t^R &= Q^s(L)\tilde{T}_t, \end{aligned}$$

where $A^s(L)$, $Q^s(L)$, $R^s(L)$ are first-order polynomials and in particular $A^s(L) = \det[I - Z_1^s L]$. While for $\Delta\hat{S}_t$ we obtain

$$A^s(L)\Delta\hat{S}_t = U^s(L)\tilde{T}_t,$$

where $U^s(L)$ is a second-order polynomials.

Floating exchange-rate regime (II)

Under the class of interest-rate rules

$$\begin{aligned} \hat{i}_t^H &= \phi p_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi p_t^F + \psi y_t^F, \end{aligned}$$

the relevant equilibrium conditions for the determination of the terms of trade and the exchange rate are

$$p_t^F - p_{t-1}^F - (p_t^H - p_{t-1}^H) = -\lambda(1 + \eta)(\hat{T}_t - \tilde{T}_t) + \beta E_t\{p_{t+1}^F - p_t^F - (p_{t+1}^H - p_t^H)\}, \quad (\text{C.34})$$

$$\hat{T}_t = p_t^F - p_t^H + \hat{S}_t, \quad (\text{C.35})$$

$$E_t\Delta\hat{S}_{t+1} = \phi(p_t^H - p_t^F) + \psi(\hat{T}_t - \tilde{T}_t), \quad (\text{C.36})$$

where condition (C.34) is obtained by subtracting (7) from (8) and using the relation between inflation rate and prices; equation (C.35) is equation (4) in levels while (C.36) is (2) in which the interest rate rules have been substituted.

The above set of equations can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1}^p \\ z_t^p \end{bmatrix} = \begin{bmatrix} M_1^p & M_2^p \\ M_3^p & M_4^p \end{bmatrix} \begin{bmatrix} y_t^p \\ z_{t-1}^p \end{bmatrix} + \begin{bmatrix} m_1^p \\ m_2^p \end{bmatrix} \tilde{T}_t,$$

where $y_t^{p'} = [p_t^R \ \hat{S}_t]$, $z_{t-1}^{p'} = [p_{t-1}^R]$, M_j^p are matrices of appropriate dimension, while m_j^p are vectors of appropriate dimensions. Under the conditions for determinacy, there are two eigenvalues outside the unit circle. Let denote these eigenvalues as ω_1 and ω_2 collected in the diagonal matrix Ω . Let V a 2×3 matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that $VM^p = \Omega V$. Furthermore we decompose V in $V = [V_1 \ V_2]$ where V_1 and V_2 are matrices of appropriate dimension. Now, if \tilde{T}_t follows a Markovian process of the form (17), we have that the solution for y_t is of the form

$$y_t^p = \Psi_1^p z_{t-1}^p + \Psi_2^p \tilde{T}_t,$$

where $\Psi_1^p \equiv -V_1^{-1}V_2$ and $\Psi_2^p \equiv V_1^{-1}(I\rho - \Omega)^{-1}Vm$. We then obtain

$$\begin{aligned} z_t^p &= M_3^p y_t^p + M_4^p z_{t-1}^p + m_2^p \tilde{T}_t, \\ &= (M_3^p \Psi_1^p + M_4^p) z_{t-1}^p + (M_3^p \Psi_2^p + m_2^p) \tilde{T}_t, \\ &= Z_1^p z_{t-1}^p + Z_2^p \tilde{T}_t, \end{aligned}$$

where Z_1^p and Z_2^p have been appropriately defined. Reminding that $z_t^{p'} = [p_t^R]$, it can be possible to obtain a solution for p_t^R of the form

$$A^p(L)p_t^R = Q^p \tilde{T}_t,$$

where $A^s(L) = 1 - Z_1^p L$ is a first-order scalar polynomial and Q^s is a scalar. While for \hat{S}_t and \hat{T}_t we obtain

$$A^p(L)\hat{S}_t = U^p(L)\tilde{T}_t,$$

$$A^p(L)\hat{T}_t = R^p(L)\tilde{T}_t,$$

where $U^p(L)$ and $R^p(L)$ are first-order polynomial.

Managed Exchange Rate (I)

Under the class of managed exchange rate (I)

$$\begin{aligned}\hat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi\pi_t^F + \psi y_t^F - \delta\hat{S}_t^*,\end{aligned}$$

the relevant equilibrium conditions for the determination of the terms of trade and the exchange rate are

$$\pi_t^F - \pi_t^H = -\lambda(1 + \eta)(\hat{T}_t - \tilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H), \quad (\text{C.37})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta\hat{S}_t^*, \quad (\text{C.38})$$

$$E_t\hat{S}_{t+1}^* = \phi(\pi_t^H - \pi_t^F) + \psi(\hat{T}_t - \tilde{T}_t) + (1 + \delta)\hat{S}_t^*, \quad (\text{C.39})$$

where condition (C.37) is obtained by subtracting (7) from (8); equation (C.38) is equation (4) in the text while the third equation is (2) in which the interest rate rules have been substituted. This set of equations can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1} \\ z_t \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \begin{bmatrix} y_t \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \tilde{T}_t,$$

where $y_t' = [\pi_t^R \ \hat{S}_t^*]$, $z_{t-1}' = [\hat{T}_{t-1} \ \hat{S}_{t-1}^*]$, M_j are 2×2 matrices, while m_j is a 2×1 vector. Under the conditions for determinacy, there are two eigenvalues outside the unit circle. Let denote these eigenvalues as ω_1 and ω_2 collected in the diagonal matrix Ω . Let V a 2×4 matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that $VM = \Omega V$. Furthermore we decompose V in two 2×2 matrices with $V = [V_1 \ V_2]$. Now, if \tilde{T}_t follows a Markovian process of the form (17), we have that the solution for y_t is of the form

$$y_t = \Psi_1 z_{t-1} + \Psi_2 \tilde{T}_t,$$

where $\Psi_1 \equiv -V_1^{-1}V_2$ and $\Psi_2 \equiv V_1^{-1}(I\rho - \Omega)^{-1}Vm$. Furthermore from the system (C.43), we have

$$\begin{aligned}z_t &= M_3 y_t + M_4 z_{t-1} + m_2 \tilde{T}_t, \\ &= (M_3 \Psi_1 + M_4) z_{t-1} + (M_3 \Psi_2 + m_2) \tilde{T}_t, \\ &= Z_1 z_{t-1} + Z_2 \tilde{T}_t,\end{aligned}$$

where Z_1 and Z_2 have been appropriately defined. Reminding that $z' = [\hat{T}_t \ \hat{S}_t^*]$, it can be possible to obtain a solution for \hat{T}_t and \hat{S}_t^* of the form

$$\begin{aligned} A(L)\hat{T}_t &= R(L)\tilde{T}_t, \\ A(L)\hat{S}_t^* &= U(L)\tilde{T}_t, \end{aligned}$$

where $A(L)$ is a second-order polynomial with $A(L) = \det[I - LZ_1]$ and $R(L)$ and $U(L)$ are first-order polynomials.

Managed exchange rate (II)

Under the class of managed exchange rate (I)

$$\begin{aligned} \hat{i}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{i}_t^F &= \phi\pi_t^F + \psi y_t^F - \mu\Delta\hat{S}_t, \end{aligned}$$

the relevant equilibrium conditions for the determination of the terms of trade and the exchange rate are

$$\pi_t^F - \pi_t^H = -\lambda(1 + \eta)(\hat{T}_t - \tilde{T}_t) + \beta E_t(\pi_{t+1}^F - \pi_{t+1}^H) \quad (\text{C.40})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta\hat{S}_t \quad (\text{C.41})$$

$$E_t\Delta\hat{S}_{t+1} = \phi(\pi_t^H - \pi_t^F) + \psi(\hat{T}_t - \tilde{T}_t) + \mu\Delta\hat{S}_t, \quad (\text{C.42})$$

where condition (C.40) is obtained by subtracting (7) from (8); equation (C.41) is equation (4) in the text while the third equation is (2) in which the interest rate rules have been substituted. The above set of equations can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1}^m \\ \hat{T}_t \end{bmatrix} = \begin{bmatrix} M_1^m & M_2^m \\ 1' & 1 \end{bmatrix} \begin{bmatrix} y_t^m \\ \hat{T}_{t-1} \end{bmatrix} + \begin{bmatrix} m_1^m \\ 0 \end{bmatrix} \tilde{T}_t, \quad (\text{C.43})$$

where $y_t' = [\pi_t^R \ \Delta\hat{S}_t]$, M_1^m is a 2×2 matrix, $1'$ and M_2^m are 2×1 vectors, while m_1^m is a 2×1 vector. Under the conditions for determinacy, there are two eigenvalues outside the unit circle. Because there is one predetermined endogenous variable (\hat{T}_{t-1}), the system has a unique bounded solution if and only if exactly two eigenvalues of the 3×3 matrix lie outside the unit circle. Let denote these

eigenvalues as ω_1 and ω_2 collected in the diagonal matrix Ω . Let V a 2×3 matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that $VM^m = \Omega V$. Following the same steps as in the previous case, we obtain

$$y_t^m = \Psi_1^m \hat{T}_{t-1} + \Psi_2^m \tilde{T}_t,$$

where $\Psi_1^m \equiv -V_1^{-1}V_2$ and $\Psi_2^m \equiv V_1^{-1}(I\rho - \Omega)^{-1}Vm$. Furthermore from the system (C.43), we have

$$\begin{aligned} \hat{T}_t &= 1'y_t + \hat{T}_{t-1} \\ &= (1'\Psi_1 + 1)\hat{T}_{t-1} + (1'\Psi_2)\tilde{T}_t, \\ &= Z_1^m \hat{T}_{t-1} + Z_2^m \tilde{T}_t, \end{aligned}$$

where Z_1^m and Z_2^m have been appropriately defined. So that we obtain that the solution for \hat{T}_t is of the form

$$A^m(L)\hat{T}_t = R^m \tilde{T}_t$$

where

$$A^m(L) = 1 - LZ_1^m$$

and $R^m = Z_2^m$. Note that $|Z_1^m| < 1$ and Z_1^m would be zero if there is no weight on exchange rate depreciation. We obtain then that

$$A^m(L)\Delta\hat{S}_t = U^m(L)\tilde{T}_t$$

where $U^m(L)$ is a second-order polynomial.