Problem Set 10

Foundations of Financial Markets

Answer key

1. (a) According to the B&S model:

$$P_0 = Xe^{-rT}[1 - N(d_2)] - S_0 e^{-deltaT}[1 - N(d_1)]$$

with d_1 and d_2 defined in the answer to question 3. First of all we need to compute the dividend yield: $\delta = \frac{D}{S_0} = \frac{1.92}{64} = 0.03$. The time to expiration is $T = 1/3 = .33$. We can compute all the pieces of the formula:

$$\begin{aligned} d_1 &= \frac{ln\,(64/62) + (.05 - .03 + .21^2/2) \times .33}{.21\sqrt{.33}} = .38 \\ d_2 &= .3775 - .21\sqrt{.33} = .26 \\ N(d_1) &= .6471 \\ N(d_2) &= .6011 \end{aligned}$$

Hence: $P_0 = \$1.96$.

(b) The hedge ratio of the option equals:

$$\Delta = N(d_1) - 1 = -.3529$$

(c) The hedge ratio tells us the sensitivity of the put price to changes in the stock price:

$$\frac{\Delta P}{\Delta S} = -.3529$$

Hence if the stock price changes by \$1, the put price will drop by -\$0.3529. The new put price will be $P' = 1.96 - .3529 = \$1.61$.

2. Your initial margin must be

$$0.15 \times \$0.994 \times 15,000 = \$2236.50$$

Marking to market will result in the following profits/losses on each day:

Day	Futures Price (cents per lb.)	Profit or Loss (cents per lb.)	Daily Proceeds
0	99.4	-	-
1	101.2	-1.8	-270
2	100.3	0.9	135
3	99.2	1.1	165
4	99.6	-0.4	-60
5	98.8	0.8	120
6	99.2	-0.4	-60
7	98.6	0.6	90
8	98.1	0.5	75
			195

This clear equals the profit that would be realized by reversing the trade at time 8 without accruing profits and losses as-you-go

$$\$(0.994 - 0.981) \times 15,000 = \$195$$

3. (a) You will get a margin call at

$$0.10 \times 0.994 \times 15,000 = \$1,491.00$$

   That is when the margin falls below 10% of the futures position.

   (b) If the price increases by 3 cents, the loss is $\$0.03 \times 15,000 = \$450$. The remaining margin equals the initial margin minus the loss: $\$2,236.50 - 450 = \$1,786.50$, which is above the maintenance margin. No margin call.

   (c) If the price goes up by 5 cents, the loss is $0.05 \times 15,000 = \$750$. The remaining margin is $\$2,236.50 - 750 = \$1,486.50$, which is below the maintenance margin. You will get a margin call.

4. (a) The price of the futures contract should be

$$F_0 = S_0 \times (1 + r_f/12)^2 = \$440 \times (1 + .03/12)^2 = \$442.20$$

   (b) An arbitrage opportunity exists because the current futures price is higher than the fair price. Here is how to construct the arbitrage.

   Today
   Buy 1 troy oz of palladium at $440
   Borrow at the risk free rate $440
   Enter a short futures contract on 1 troy oz. of palladium (today it's free!)
   Total cost is $0.

   At maturity
   1 troy oz of palladium is worth P_T
   Pay back the loan: -$\$440 \times (1 + 0.03/12)^2 = 442.20$
   Close the futures contract: $\$444 - P_T$
   Total profit is: $P_T - 442.20 + 444 - P_T = 1.8$

(c) The final price is irrelevant: the profit is always equal to $F_0 - P_0 \times (1 + r_f/12)^2$!

(d) If the futures price is \$440 there is still an arbitrage because the fair market value of the futures should be lower. Here is how to benefit from the arbitrage opportunity:

Today
Short sell 1 troy oz of palladium at \$440
Lend at the risk free rate \$440 (this is a negative payoff)
Enter a long futures contract on 1 troy oz. of palladium (today it's free!)
Total cost is \$0.

At maturity
1 troy oz of palladium is worth P_T: this is a negative payoff
Collect on the loan: $\$440 \times (1 + 0.03/12)^2 = 442.20$
Close the futures contract: $-\$440 + P_T$
Total profit is: $-P_T + 442.20 - 440 + P_T = 2.20$

5. (a) The cash flows from this strategy are as follows

Today
Buy the stock at S_0
Borrow at the risk free rate S_0
Enter a short futures contract on the stock (today it's free!)
Total cost is \$0.

At maturity
One share of the stock is worth $S_T + D$
Pay back the loan: -$S_0 \times (1 + r_f)$
Close the futures contract: $F_0 - S_T$
Total profit is: $S_T + D - S_0 \times (1 + r_f) + F_0 - S_T = -S_0 \times (1 + r_f) + F_0 + D$

(b) Because this position costs zero dollars, it must deliver zero dollars at maturity. That is

$$D - S_0 \times (1 + r_f) + F_0 = 0$$

or $F_0 = S_0 \times (1 + r_f) - D$.

(c) Let $d = D/S_0$. Then it follows from part b that:

$$\begin{aligned} F_0 &= S_0 \times (1 + r_f) - D \times \frac{S_0}{S_0} \\ &= S_0 \times (1 + r_f) - d \times S_0 \\ &= S_0 \times (1 + r_f - d) \end{aligned}$$