Education, Segregation and Marital Sorting: Theory and an Application to the UK*

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Abstract

This paper presents a model of the intergenerational transmission of education and marital sorting. Parents matter both because of their household income and because their human capital determines the distribution of a child's disutility from making an effort to become skilled. We show that an increase in segregation has potentially ambiguous effects on the proportion of individuals that become skilled in the steady state, and hence on marital sorting, the personal and household income distribution, and welfare. We calibrate the steady-state of our model to UK statistics We find that an increase in the correlation of spouses in their years of education will bring about a small increase in the proportion of skilled individuals when the relative supply of skilled individuals is variable at the family level and a decrease when this supply is fixed. Ex ante utility (of an unborn individual) increases in the first case and decreases in the second. The welfare effect of increased sorting is negative for unskilled individuals and positive for skilled individuals. Increased segregation always leads to an increase in welfare inequality between skilled and unskilled individuals.

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1. Introduction

The ability of individuals to sort or segregate themselves across various dimensions (e.g., race, income, and ability) and in different spheres (e.g. residence, workplace, schools, and households) is a topic of concern in many countries. Wilson (1987) and Reich (1991), for example, have worried that increased segregation at the community level or that the creation of new "gated communities" will lead to increased inequality in the US. In Europe, observers worry that the geographic and social marginalization of recent immigrants may lead to increased social tensions. In the UK, the stratification of the social system and the existence of a self-perpetuating social and educational elite is a perennial topic of debate.

There is some evidence, at least in the US, of increased sorting in several spheres. Jargowsky (1996, 1997), for example, shows that sorting by income at the neighborhood level appears to have increased across all groups in US metropolitan areas even after controlling for racial and ethnic group characteristics.[1] This has led to the fear, as expressed in Wilson (1987), that lower-income individuals in these groups will be left without positive peer effects provided by a more integrated middle class. Kremer and Maskin (1996) present some evidence that sorting by skill level in the workplace has increased and Mare (1991) shows that the probability that a high school graduate will marry a college graduate has decreased.

This paper is concerned with the effect of increased segregation on marital sorting, inequality and welfare. We present a model of marital sorting in which both environment (the degree of segregation) and preferences (the tradeoff individuals make between the quality of a match and household consumption) determine the degree of assortative matching. In the model, individuals are either skilled or unskilled (according to education decisions made when young) and have a given number of opportunities in which to form a household with another agent. The probability of an individual meeting a skilled or an unskilled individual depends in part on the degree of segregation (in schools, residence and work) in society. Whether a household is formed or not depends on an individual's future matching opportunities, on the quality of the match, and on the household income an individual would enjoy. Couples have children and these in turn decide whether to

[1] Jargowsky finds that the proportion of the population that lives in ghettos and other high-poverty neighborhoods has risen from 14.4 in 1970 to 17.4 percent in 1990. Among the black poor, the figure has risen from 26.1 percent to 33.5 percent and for the white poor this increase has been from 2.9 percent to 6.3 percent

become skilled or unskilled workers. A decision to become skilled (synonymous here with acquiring a given level of education) is costly, both in terms of resources and in terms of effort. To finance education, young individuals borrow in an imperfect capital market in which parental income plays the role of collateral. Thus parental income and the child's effort cost (the distribution of which is allowed to differ by family type), determine the proportion of children from each family type that in aggregate becomes skilled. These individuals then also meet and form households, have children, and so on.

We show that the steady state to which this economy converges will in general depend upon initial conditions. In particular, it is possible to have steady states with a high degree of sorting (skilled agents form households predominantly with others who are skilled; unskilled form households predominantly with unskilled) and high inequality Alternatively, there can be steady states with a low degree of sorting and low inequality.

We then examine the theoretical effects of an increase in segregation on the steady-state of the economy. We show that this depends not only on the profile of fertility and the propensity for children to become skilled (as in Fernández and Rogerson (2001)), but also on the degree to which this increase in segregation affects the expected utility differential between skilled and unskilled individuals and how this translates into a change in the relative supply of skilled individuals.

Next we examine the effects of an increase in segregation by parameterizing the steady state of the model to several key UK statistics. We use primarily the British Cohort Study to create a sample of parents and children that permits us to examine the degree of correlation of parents in education and the probability that a child from different parental types will become skilled. This, along with data on fertility by parental type, the skill premium, and the elasticity of substitution in production, allows us to find the steady-state proportion of skilled individuals in the economy. We next proceed to examine the effects of an increase in segregation first in a model in which all behavior (i.e., matching and the skill supply by each family type) is exogenously specified. We then add endogenous matching to the parameterized model and lastly we allow for variable skill supply at the family level. We find that the effects of increased segregation on the proportion of the population that is skilled depends on the specification of the model. Skilled workers, however, are always made better off and unskilled workers are always made worse off. From an ex ante utility point of view (i.e., when expected utility is calculated for a representative unborn individual using the weights that correspond to the frequency in the population), increased segregation increases welfare in the calibrated variable skill supply model.

The literature most closely related to this paper is Kremer (1997), Fernández and Rogerson (2001), and Fernández, Guner and Knowles (2001). The first two papers examine the consequences of greater marital sorting on long-run inequality. Kremer (1997) uses a linear model of intergenerational transmission of education to argue that even a large increase in marital sorting is unlikely to have important quantitative consequences for the distribution of income in the US in the long run. Fernández and Rogerson argue the opposite by showing that the intergenerational transmission process is likely to be non-linear. They show that once fertility differentials and the (non-linear) propensity of children from different family types to become more educated are taken into account, an increase in marital sorting would be likely to have a significant quantitative impact on the US income distribution. The contributions of this paper over the last two, in addition to examining UK rather than US data, are that it endogenizes sorting and allows for the skill supply at the family level to respond to changes in the differential utility of becoming skilled relative to unskilled.

Fernández et al. (2001) provide a model of marital sorting, fertility and inequality that generates multiple steady states, with greater inequality leading to more assortative matching and greater fertility differentials among high and low income families. They do not calibrate their model, however, but use it to motivate their empirical investigation. They examine a cross-section of 34 countries, and show that couples are more correlated in their years of education the greater the skill premium. As predicted by their model, they also find that countries with more sorting have lower per capita income. Lastly, there is also a small literature on endogenous matching but that basically abstracts from the endogeneity of the income distribution in the economy (the seminar paper is Becker (1973) and more recent papers include Cole, Mailath, and Postlewaite (1992), and Burdett and Coles (1997, 1999)). Aiyagari, Greenwood, and Guner (2000) and Greenwood, Guner and Knowles (2000) also examine marriage decisions, but they are primarily concerned with fertility and divorce decisions (which our model does not address) and use a computational approach to solve complicated individual decision problems.

In addition to the literature on marital sorting, there is also a rapidly growing literature that is concerned with various aspects of sorting in other spheres (see Fernández (2001a)) for a recent review of this literature). Alesina and La Ferrara (2000) examine the consequences of heterogeneity of communities for the formation of social capital. Using survey data on group membership in US localities, they find that participation in social activities is significantly lower in more diverse (racially or ethnically) and more unequal communities. Bisin and Verdier (2000) provide a model of intergenerational transmission of ethnic and religious traits. They find that an increase in segregation increases the fraction of assortative marriages. Fernández and Rogerson (1996) use a multi-community model to show that a local system of financing education is likely to lead to suboptimal sorting of income groups, with too few higher-income individuals living in poorer communities. There is also a large literature that examines the effects of borrowing constraints on human capital accumulation. The most relevant papers here are by Benabou (1996), Cooper (1997), Durlauf (1995), and Fernández and Rogerson (1997,1998) as they are concerned with the effects of sorting or stratification (into neighborhoods and schools) for the transmission of education and growth in the presence of borrowing constraints.

This paper is organized as follows. The next section sets up the model. Section 3 is devoted to a theoretical analysis of the effects of increased segregation. Section 4 parameterizes the steady state of the model to UK statistics. Section 5 examines the reaction of the parameterized model to an increase in segregation. Section 6 concludes.

2. The Model

We consider a two-period OLG model with the following timing of decisions. In the first period of life, young individuals decide whether to become one of two types–skilled (s) or unskilled (u)–with each type synonymous with the individual acquiring a given amount of education. Having made the education decision, the individual enters a "household matching market" and acquires a mate. In the second period the individual works, pays off debt associated with the education decision of the first period (if any), consumes and has children.

To simplify our model we take a couple's fertility as exogenous and, as will be seen shortly, we also abstract from bargaining problems within a family by assuming that the couple shares a joint household utility function over consumption. Thus, in the second period of life families simply consume their household income I minus whatever debt repayment they need to make. All the interesting economic action in the model concerns how individuals decide with whom to match given their skill type and how they decide upon a skill type in the first place. We first turn to a description of the labor market and the determinants of household income.

2.1. The Labor Market and Household Income

We assume that workers supply their labor inelastically in a perfectly competitive labor market. Let L_i, $i \in \{s, u\}$, denote the amount of labor of type i, hence the total labor force L is given by $L = L_s + L_u$. Given a strictly quasi-concave constant returns to scale production function

$$F(L_s, L_u) = LF(\lambda, 1-\lambda) \tag{2.1}$$

where $\lambda = \frac{L_s}{L}$, the assumption of a perfectively competitive labor market implies that wages depend only on the proportion of skilled individuals in the population, i.e.,

$$w_s(\lambda) = F_1(\lambda, 1-\lambda) \quad \text{and} \quad w_u(\lambda) = F_2(\lambda, 1-\lambda) \tag{2.2}$$

with $w_s' < 0$ and $w_u' > 0$.

Most often we will be concerned not with the skilled wage but with its value net of the monetary cost p of becoming a skilled worker, which we denote by $\widetilde{w}_s$. Assuming that individuals work only in the second period (whereupon they repay the cost of acquiring skills if they become skilled), and letting I_{ij} denote the household income for a couple composed by skill types $i, j \in \{s, u\}$, we obtain:

$$I_{ij}(\lambda) = \begin{cases} 2\widetilde{w}_s(\lambda), & \text{if } ij = ss \\ \widetilde{w}_s(\lambda) + w_u(\lambda), & \text{if } ij = su \\ 2w_u(\lambda), & \text{if } ij = uu \end{cases} \tag{2.3}$$

We next turn to the analysis of household matching.

2.2. Segregation and Household Matching

Our model of household sorting (or matching) is simple and yet allows both environment (how segregated schools and neighborhoods are) and preferences (how individuals trade off the income and the quality from a relationship) to matter. Individuals are assumed to have two rounds in which to find a match. The opportunities available in an individual's first round depend on the individual's environment. The opportunities available in the second round depend on the individual's skill type.

In the first round, individuals are either segregated or not according to the individual's exogenously specified environment. To make this differentiation stark, we assume that a fraction θ of individuals are perfectly segregated. Within this segment of society, segregation ensures that types do not mix, even when relatively young. Thus, skilled individuals (or individuals who will become skilled) meet only other skilled individuals in the first round; likewise, unskilled meet only other unskilled in the first round. The remaining $1-\theta$ fraction of the population is not segregated. For the unsegregated portion of the population, meeting is at random. Hence, if λ is the fraction of the population that is skilled, then the probability of a skilled and an unskilled individual "meeting" in the first round is given by $2\lambda(1-\lambda)$. For both the segregated and unsegregated portions of society, individuals must decide whether to match (form a household) with the person they met in the first round. If they decide not to match with that person they then enter the second round of matching. In this round, a skilled individual will meet another skilled individual with probability one; similarly, unskilled individuals meet only other unskilled individuals. This two-step process (as in Fernández, Guner and Knowles (2001)) is meant to reflect that, by and large, when one is younger one has access to a more varied group of individuals (in terms of their final educational attainment) than when one is older.[2]

The role of the environment is reflected in the degree to which individuals are segregated in the first stage. The degree of segregation of the population reflects, among other things, both the residential and schooling environment: e.g., how much society is divided into gated communities, exclusive suburbs, depressed ghettos; the degree of segregation of schools and within a school; the variance of school quality; the prevalence of exclusive private schools and low quality urban public schools; tracking by ability within a school, magnet schools, etc..We will be interested in examining the consequences of an increase in the degree of segregation, θ. Undoubtedly, θ is itself an endogenous variable, affected by the amount of inequality in society. It is, however, also amenable to public policy. That is, if admission to public schools is a function of residence (as in the UK) and/or are locally financed (as in the US), there is, for the same amount of inequality, a greater incentive to segregate.[3] Or, if highways are built allowing easy access

[2]Other matching models are also possible such as random matching over time with discounting. The formulation we have chosen is very tractable and allows us to get rid of the multiple equilibria that can result otherwise.

[3]See Fernández and Rogerson (1996, 1998) for an analysis of the consequences of a local system of school finance.

from suburbs to jobs in the city, it is easier to segregate into different neighborhoods than in their absence. Alternatively, the placing of some public low-income housing in richer communities or education policies that lead to a lower variance in the quality of schooling may allow a lower degree of segregation for the same degree of inequality.[4]

In order to determine who will match with whom, we need to specify utilities from forming a household. For now, we do not impose any structure other than: (i) Individuals share the same utility from a match. This ensures, for example, that there is no bargaining problem between spouses over the allocation of household income and thus simplifies the analysis of equilibrium; and (ii) Individuals obtain utility both from consumption and from the quality of the match. These two components are assumed to be additively separable. The assumptions above permit us to write the indirect utility from a partner in a match of quality q and household income I as:

$$V(I, q) = v(I) + q \tag{2.4}$$

where v is assumed to be concave.

We assume that quality is perfectly observable to both participants and that it is completely match specific. The draws of quality are independently distributed across rounds and participants. Thus, quality (like beauty, supposedly) is completely in the eye of the beholder, and furthermore is mutual (i.e., the two participants to a match do not disagree about the match quality). Lastly, solely to simplify algebraic presentation, we assume that quality is distributed identically across both types of individuals (skilled and unskilled) so that q is an iid draw from a cumulative distribution Q in each round of matching.

Note that the assumptions above imply that if two individuals of the same type meet in the first round, either because they belong to the segregated part of society or because they met someone of their own type at random, they will accept any match quality above the mean $\mu = E(q)$ of the quality distribution and

[4] The way we have modelled segregation is symmetric for skilled and unskilled inviduals. However, one could well model it as an unsymmetric phenomenon with a fraction θ_s of the skilled population and a potentially different fraction θ_u of the unskilled seceeding or being excluded. This would then imply that the unsegregated part of the population would now consist of a proportion $\frac{(1-\theta_s)\lambda}{(1-\theta_s)\lambda+(1-\theta_u)(1-\lambda)}$ of skilled individuals and a proportion $\frac{(1-\theta_u)(1-\lambda)}{(1-\theta_s)\lambda+(1-\theta_u)(1-\lambda)}$ of unskilled individuals. As we do not have data on the magnitudes of these parameters, we model it as symmetric.

reject any that is below. This follows from the fact that their household income is invariant to proceeding to the second round (whereupon they are guaranteed to meet someone of their own type). If, on the other hand, a skilled and unskilled individual meet in the first round (thus, they must belong to the non-segregated part of society), then a skilled individual earning $\widetilde{w}_s \geqslant w_u$ (a necessary condition for anyone to become skilled in equilibrium) will only accept the match if q is at least as large as q^*, given by:

$$q^* = v(I_{ss}) - v(I_{su}) + \mu \tag{2.5}$$

i.e., a quality of match such that a skilled individual is indifferent between obtaining the household income of two skilled individuals, I_{ss}, with an expected match quality of μ and settling for a lower household income of I_{su}.

The observations above immediately permit us to solve for the proportion φ of each type of household that will form in the population as a function of λ and of the population θ that is segregated. Note that, from (2.3), we can express q^* in (2.5) as a function solely of λ, i.e., $q^*(\lambda)$, and that the latter is not a function of θ.

$$\varphi_{ij}(\lambda_t;\theta) = \begin{cases} \theta\lambda_t + (1-\theta)[\lambda_t^2 + \lambda_t(1-\lambda_t)Q(q^*(\lambda_t))] & \text{if } ij = ss \\ (1-\theta)2\lambda_t(1-\lambda_t)(1-Q(q^*(\lambda_t)) & \text{if } ij = su \\ \theta(1-\lambda_t) + (1-\theta)[(1-\lambda_t)^2 + \lambda_t(1-\lambda_t)Q(q^*(\lambda_t))] & \text{if } ij = uu \end{cases} \tag{2.6}$$

Letting $\rho(\lambda;\theta) = \theta + (1-\theta)Q(q^*(\lambda))$, we can rewrite (2.6) above as:

$$\varphi_{ij}(\lambda_t,\theta) = \begin{cases} \lambda_t^2 + \lambda_t(1-\lambda_t)\rho(\lambda_t;\theta), & \text{if } ij = ss \\ 2\lambda_t(1-\lambda_t)(1-\rho(\lambda_t;\theta)), & \text{if } ij = su \\ (1-\lambda_t)^2 + \lambda_t(1-\lambda_t)\rho(\lambda_t;\theta), & \text{if } ij = uu \end{cases} \tag{2.7}$$

Remark 1. $\rho(\lambda;\theta) = \theta + (1-\theta)Q(q^*(\lambda))$ *is the degree of correlation of spouses in education (or skill type).*

It is useful to note the following features of this correlation: (i) it is independent of λ except via the endogenous dependence of q^* on λ; (ii) if there is complete segregation ($\theta = 1$), then $\rho = 1$; if there is no segregation ($\theta = 0$), then $\rho = Q(q^*)$; (iii) if individuals only cared about household income and not quality then $Q(q^*) = 1$ and $\rho = 1$; if individuals simply matched with whomever they met in the first round (i.e., $Q(q^*) = 0$), then the degree of correlation would be governed solely by the degree of segregation of the environment, and hence $\rho = \theta$.

2.3. The Education Choice

Next we consider a young individual's decision to become skilled. The expected utility from becoming skilled (gross of any disutility from effort, as will be discussed shortly), given that a fraction λ_{t+1} also becomes skilled, is given by:

$$\begin{aligned} V^s(\lambda_{t+1}) &= [\theta + (1-\theta)\lambda_{t+1}] \int_0^{\overline{q}} \max\left[V_{ss}(x, \lambda_{t+1}), V_{ss}(\mu, \lambda_{t+1})\right] dQ(x) \\ &\quad + (1-\theta)(1-\lambda_{t+1}) \int_0^{\overline{q}} \max\left[V_{su}(x, \lambda_{t+1}), V_{ss}(\mu, \lambda_{t+1})\right] dQ(x) \end{aligned} \quad (2.8)$$

whereas the expected utility of becoming an unskilled worker is:

$$\begin{aligned} V^u(\lambda_{t+1}) &= (1-\theta)\lambda_{t+1}[\int_0^{q*} V_{uu}(\mu, \lambda_{t+1}) dQ(x) + \int_{q*}^{\overline{q}} V_{su}(x, \lambda_{t+1}) dQ(x)] \quad (2.9) \\ &\quad + [\theta + (1-\theta)(1-\lambda_{t+1})] \int_0^{\overline{q}} \max\left[V_{uu}(x, \lambda_{t+1}), V_{uu}(\mu, \lambda_{t+1})\right] dQ(x) \end{aligned}$$

Note that the expected payoff from becoming skilled includes the monetary cost of acquiring an education as skilled wages in both (2.8) and (2.9) are expressed as net of p.

Next we turn to the dependence of a child's decisions on her family type. We introduce two different ways in which a family may matter (i) family background; (ii) borrowing constraints. We consider each in turn.

In addition to bearing a monetary cost if they choose to become skilled, individuals also face an idiosyncratic (non-monetary) effort cost γ_i. The distribution of this cost can be thought of as being influenced by parental background as reflected in the education of these, which can facilitate or make more difficult the child's acquisition of human capital. We allow therefore the distribution of the effort cost to depend on family type and indicate by $G_{ij}(\gamma)$ its cumulative distribution. To simplify exposition, we assume that the effort cost draws are perfectly correlated within a family and that γ enters linearly in a skilled individual's utility function, i.e., the payoff to individual i of becoming a skilled worker is given by $V_i^s = V^s - \gamma_i$.

Define by $\gamma^*(\lambda)$ the skilled-unskilled payoff differential generated when a fraction λ of the population becomes skilled, i.e.,

$$\gamma^*(\lambda_{t+1}) \equiv V^s(\lambda_{t+1}) - V^u(\lambda_{t+1}) \quad (2.10)$$

Note that an individual with idiosyncratic cost γ_i of becoming skilled who expects a fraction λ of her cohort to become skilled would also prefer to be skilled as long as $\gamma_i \leq \gamma^*(\lambda)$.

Next we turn to the effect of parental income on a child's decision to acquire an education. We assume that in order to finance the acquisition of skilled education, children need to access capital markets.[5] Rather than endogenize the interest rate, we assume that individuals have access to an international market for funds (in which the country is small) with an interest rate normalized to equal one. Capital markets are unable to monitor how these funds are spent so that parental income must act as collateral for children. We assume that children within the same family with $n-1$ siblings can borrow up to some (increasing) function $Z(I,n)$ of parental income, so that the (integer part) of the solution to:

$$Z(I,n) = \overline{m}p \tag{2.11}$$

yields the maximum number $m_{nij} = \lfloor \overline{m} \rfloor$ of skilled individuals a family with n children and income I_{ij} can produce. Note that m_{nij} is in general a function of λ as this determines family income.

For simplicity, we assume that fertility is exogenous, but we allow the number of children n to depend on family type by modeling it as a random draw from a cumulative distribution H_{ij}. We denote by η_{nij} the probability that a family of type ij has $n = \{0, 1, 2, ..., \bar{n}\}$ and use f_{ij} to denote their average fertility, i.e., $f_{ij} = \sum\limits_{n=0}^{\bar{n}} n\eta_{nij}$.

Putting together both parental income and background, yields

$$\Gamma_{ij}(\gamma^*(\lambda_{t+1});\lambda_t) = \frac{G_{ij}(\gamma^*(\lambda_{t+1}))}{f_{ij}} \sum_{n=0}^{n=\overline{n}} \min[m_{nij}(\lambda_t), n]\eta_{nij} \tag{2.12}$$

as the average fraction of children from parents of type ij that will become skilled given that on aggregate a fraction λ_{t+1} of the population also plans to become skilled (producing a differential in utility between skilled and unskilled of γ^*).

Note that in the absence of borrowing constraints, $\min[m_{nij}, n] = n$ and hence $\Gamma_{ij}(\gamma^*(\lambda_{t+1});\lambda_t) = G_{ij}(\gamma^*)$. Furthermore, note that if under this scenario the distribution of non-monetary costs were independent of family background, i.e.,

[5] A bequest model would have similar implications.

$G_{ij}(\gamma^*) = G(\gamma^*)\ \forall ij$, then all families would produce the same fraction of skilled children independent of type. In general, however, Γ_{ij} is a function both of λ_t (since parental income determines the maximum number of children who can afford to become skilled) and of the expected value of λ_{t+1} (which given rational expectations must also be the realized value), since this determines the incentive for individuals with different γ_i to desire to become skilled.

2.4. Equilibrium

An equilibrium at time t and $t+1$, for an initial division of the population into skilled and unskilled (λ_t), is given by marriage decisions as specified by (2.5), wages as specified by (2.2), and decisions by children to become skilled as specified by (2.12). It is easy to ensure the existence of an interior equilibrium by placing conditions on the production function or on the Z function such that even if $\lambda = 0$, some portion of children can afford to become skilled. This ensures that the economy does not get stuck in a poverty trap in which no one can afford to become skilled. To bound λ away from one it is sufficient to assume that technology is such that $F_1 < F_2$ for some $\lambda < 1$.[6] As shown in Fernández et al. (2001), $\gamma^*(\lambda_{t+1}) \equiv V^s(\lambda_{t+1}) - V^u(\lambda_{t+1})$ may not be decreasing in λ, raising questions as to whether equilibrium at a point in time is unique. For the parameter values we examined, however, equilibrium was always unique.

Equilibrium can be depicted as the intersection of two curves as shown in Figure 1. In this figure, the downward sloping curve gives the difference, γ^*, between V^s and V^u as a function of λ_{t+1}. The upward sloping line shows what γ^* would have to be such that a fraction λ_{t+1} would both be willing to and able to afford to become skilled, i.e., it solves for the value of γ such that $\lambda_{t+1} = \frac{\sum_{ij} \varphi_{ij}(\lambda_t)\Gamma_{ij}(\gamma;\lambda_t) f_{ij}}{\sum_{ij} \varphi_{ij}(\lambda_t) f_{ij}}$.[7] We denote the solution to the implicit equation as $\gamma = \Psi(\lambda_{t+1}; \lambda_t)$. This curve is upward sloping since an increase in γ will increase the supply of skilled labor from all family types.

[6] Thus, Inada conditions on the production function and the assumption that $Z(0) > 0$ are sufficient to ensure the existence of an interior equilibrium.

[7] Note that because of borrowing constraints, λ_{t+1} may reach an upper bound that is below one for some values of λ_t even if γ^* becomes infinite.

2.5. Dynamics

We now turn to specifying the dynamic evolution of this economy. The working population in the economy in period $t+1$ as a function of λ_t is given by:

$$L_{t+1}(\lambda_t) = \sum_{ij} \varphi_{ij}(\lambda_t) f_{ij} L_t \tag{2.13}$$

and the number of skilled individuals is:

$$L_{s,t+1}(\gamma^*(\lambda_{t+1}); \lambda_t) = \sum_{ij} \varphi_{ij}(\lambda_t) \Gamma_{ij}(\gamma^*(\lambda_{t+1}); \lambda_t) f_{ij} L_t \tag{2.14}$$

A steady state in this economy will be a skilled fraction of the population λ^* such that

$$\lambda_{t+1}(\gamma^*(\lambda_{t+1}); \lambda^*) = \frac{L_{s,t+1}(\gamma^*(\lambda_{t+1}); \lambda^*)}{L_{t+1}(\lambda^*)} = \lambda^* \tag{2.15}$$

Alternatively, we can express the steady state as:

$$\lambda_{t+1}(\gamma^*(\lambda_{t+1}); \lambda^*) = \frac{\sum_{ij} \varphi_{ij}(\lambda^*) \Gamma_{ij}(\gamma^*(\lambda_{t+1}), \lambda^*) f_{ij}}{\sum_{ij} \varphi_{ij}(\lambda^*) f_{ij}} = \lambda^* \tag{2.16}$$

In general this model may have multiple steady states as the initial condition of the economy may determine which steady state it converges to. This is due to a non-convexity in the form of a discrete amount needed to become skilled and to the presence of borrowing constraints. For example, for a given degree of segregation, an economy that starts out with a low fraction of skilled individuals and thus high inequality between skilled and unskilled, may end up in a steady state that reproduces those features due to the fact that *uu* type families will have low income and are more likely to be constrained. On the other hand, an economy with the same degree of segregation but with a large fraction of skilled individuals will have low inequality and low sorting and can end up in a steady state with a high fraction of skilled individuals as *uu* family income will be higher and thus they are less likely to be constrained.

In any case, the potential multiplicity of steady states is not the feature of the analysis we wish to emphasize. Our calibration procedure will select a steady state and we will be interested in the effect of a change in segregation we will be given that initial steady state. We now turn to an analysis of this.

3. The Effect of an Increase in Segregation

The effect of an increase in segregation in this model depends on: (i) whether the function that maps parental type into the number of children that become skilled next period has a positive cross-partial in each spouse's education level; (ii) whether fertility has a positive cross-partial in each spouse's education level; (iii) how the expected utility differential of skilled relative to unskilled individuals responds.

This can be seen most clearly by totally differentiating (2.16) with respect to θ and evaluating it at $\lambda_t = \lambda_{t+1} = \lambda^*$. Doing this yields:

$$\frac{d\lambda^*}{d\theta} = \frac{\lambda^*(1-\lambda^*)(1-Q(q^*))[(f_{ss}\Gamma_{ss} - 2f_{su}\Gamma_{su} + f_{uu}\Gamma_{uu}) - \lambda^*(f_{ss} - 2f_{su} + f_{uu})]}{D} + \frac{\sum_{ij} \varphi_{ij}(\lambda^*) f_{ij} \frac{\Gamma_{ij}(\lambda^*;\theta)}{G_{ij}(\lambda^*;\theta)} \frac{\partial G_{ij}(\gamma^*(\lambda^*;\theta))}{\partial \gamma^*} \frac{\partial \gamma^*}{\partial \theta}}{D} \tag{3.1}$$

where

$$D = \sum_{ij} f_{ij}\varphi_{ij}(\lambda^*;\theta) + \sum_{ij} f_{ij} \frac{d\phi_{ij}(\lambda^*;\theta)}{d\lambda^*}(\lambda^* - \Gamma_{ij}) - \frac{d\gamma^*}{d\lambda^*}\left(\sum_{ij} \varphi_{ij}(\lambda^*) f_{ij} \frac{\Gamma_{ij}(\lambda^*;\theta)}{G_{ij}(\lambda^*;\theta)} \frac{\partial G_{ij}(\gamma^*(\lambda^*;\theta))}{\partial \gamma^*}\right)$$

is required to be positive in order for the original steady-state to be locally stable.[8] Note that throughout this analysis we are assuming that any change in wages brought about by the change in λ does not alter how binding borrowing constraints are.

The first line of (3.1) shows the effect on the steady state value of λ keeping γ^* fixed. That is, it isolates the compositional effect of a θ change on λ^*. The second line keeps the composition fixed (i.e., the proportion of families that belong to each type) and examines the effect of a θ change on the incentives to become skilled, i.e., on γ^*. We consider the two lines in (3.1) separately.[9]

[8] Note that $\frac{d\phi_{ij}(\lambda^*;\theta)}{d\lambda^*}$ has two components: one is what happens to the proportion of couples of different types due to the change in λ^*; the other is what happens to these proportions as a result of the change in q^* brought about by the change in wages that result from the change in λ^*.

[9] The first line is the same as in Fernández and Rogerson (2001) but the second has no counterpart there.

Consider the first term in the square brackets: $(f_{ss}\Gamma_{ss}-2f_{su}\Gamma_{su}+f_{uu}\Gamma_{uu})$. This term shows the effect of replacing 2 su couples by one ss and one uu couple–which is what an increase in segregation does–on the number of children who become skilled, ceteris paribus. In the absence of fertility differentials (i.e., $f_{ij}=f\ \forall ij$), if the production of skilled children has a negative cross-partial in each parent's years of education (i.e., if the marginal increase in the number of skilled children brought about by an increase in one parent's education is decreasing in the years of education of the other parent–and, as we shall show for the UK data, it is) then the number of skilled children will be maximized by mixing s and u individuals in couples rather than by creating homogenous couples. Hence, ceteris paribus, an increase in sorting will tend to decrease the number of skilled children. With fertility differentials, what matters is the fertility weighted equivalent of the same expression, as shown above. For the UK, the expression is indeed negative.

The second term in parenthesis in the square brackets, $(f_{ss}-2f_{su}+f_{uu})$, reminds us that what we are interested in is not the number of skilled children per se, but rather their proportion in the population. This term gives the change in the aggregate economy's fertility rate as a result of an increase in segregation (i.e., the replacement of 2 su couples by one ss and one uu couple). If a couple's fertility has a positive cross-partial in each parent's years of education (i.e., if a couple's marginal fertility is increasing in the years of education of the other parent) then the number of children will be maximized by creating homogeneous couples rather than by mixing s and u individuals. In that case, this term is positive (recall that we are subtracting it) and thus reinforces the negative first term as it shows that average fertility rate would increase thus ensuring that the smaller *number* of skilled children also translates into a smaller *fraction* of skilled children. Again, for our UK statistics this term is indeed positive.

The sign of the second line is given by the sign of $\frac{\partial\gamma^*}{\partial\theta}$ which indicates what happens to the differential in the expected utility of skilled and unskilled individuals if, keeping λ constant, segregation is increased. At first blush, one might think that this term must be unambiguously positive. Skilled individuals should be made better off as increased segregation implies a greater probability of meeting other skilled individuals (and thus enjoying higher household income). This intuition is correct. Where one's intuition may be misleading is in concluding that unskilled individuals will necessarily be made worse off. The effect on V^u is actually ambiguous. Although unskilled individuals now have a lower chance of meeting skilled individuals (and thus of enjoying higher income if a match is formed), if this probability was low in the first place because most matches were being rejected, then there is a positive counterpart to increased segregation. This counterpart is the fact that unskilled individuals need not waste their time (their matching opportunities) on a match that will happen only with a low probability and if rejected leaves them with a match quality of expected value μ. Instead it allows a greater proportion of them to make better use of the two rounds by having their matches be only with other unskilled individuals and thus permitting them an expected match quality of $\int_{\mu}^{\overline{q}} qdQ + Q(\mu)\mu$. Hence, the expected quality of the match is increased with segregation, at the expense of a lower probability of obtaining higher household income.

More formally,

$$\frac{\partial \gamma^*}{\partial \theta} \equiv \frac{\partial (V^s - V^u)}{\partial \theta} = (1 - Q(q^*))[(q^* - \mu) - \lambda(v_{ss} - 2v_{su} + v_{uu})] + (1 - 2\lambda)[(Q(\mu) - Q(q^*))\mu + \int_{\mu}^{q^*} qdQ] \quad (3.2)$$

Note that concavity of v ensures that $v_{ss} - 2v_{su} + v_{uu}$ is negative and thus that the expression in the first square brackets is positive. The ambiguity comes from the second term where the expression in the square brackets is positive, reflecting the marginal gain in expected match quality for both skill types from an increase in segregation. On the margin, an increase in segregation changes an individual's expected match quality from $Q(q^*)\mu + \int_{\mu}^{q^*} qdQ$ to $\int_{\mu}^{\overline{q}} qdQ + Q(\mu)\mu$. This increase is the same for both types. For skilled individuals, however, the probability of meeting an unskilled individual was $(1-\theta)(1-\lambda)$, hence the increase in segregation decreases this probability by $(1-\lambda)$. For unskilled individuals, on the other hand, the probability of meeting a skilled individual was $(1-\theta)\lambda$, leading to a decreased probability of λ. Hence, which type gains most from the increase in expected match quality depends on whether $1 - \lambda$ is greater or smaller than λ, i.e., on the sign of $1 - 2\lambda$. Thus, in order for the gain to unskilled workers to be greater than that to skilled workers requires $\lambda > .5$. Note that this requirement makes it relatively unlikely that $\frac{\partial \gamma^*}{\partial \theta} < 0$ since $Q(q^*)$ is increasing in the wage differential and thus reaches its maximum at $\lambda = 0$. Thus, in general, one would expect an increase in segregation to increase the relative attractiveness of becoming skilled.

We now turn to the parameterization of our model.

4. Parameterizing the Model

In this section we parameterize our model by choosing parameter values so that the cross-sectional data generated by the steady-state of our model are consistent with those observed in actual UK data. We also discuss how we evaluate welfare changes.

Both in order to evaluate welfare and to be able to examine the endogenous response in sorting resulting from an increase in the degree of segregation in the economy, we will need to specify the household's utility function v. We choose to work with log utility, so:

$$v(I) = \log I \tag{4.1}$$

which implies

$$q^{*} = \log \frac{2\widetilde{w}_s}{\widetilde{w}_s + w_u} + \mu \tag{4.2}$$

Note that an increase in $\widetilde{w}_s$ will increase q^* whereas an increase in w_u will decrease q^*.

4.1. Parameter Values

There are several key parameters in our model. To begin with, we need to know how correlated spouses are in their type (here taken to mean their years of education), and the relationship between parental types and children's education. To obtain these statistics, we proceed by constructing a sample of parents and children. Our primary data source is the British Cohort Study (BCS). This survey began with all children born in one week in April 1970 in the UK. It reports parental education data in a retrospective questionnaire of parents conducted five years later in the BCS survey of 1975. Children's education is obtained when these are 26 years old from the BCS 1996 survey, where they report their highest qualification level attained. We map this variable into years of education by considering how long it usually takes to obtain the stated level. In order to include a child in the sample we require both the data on parental and children educational attainment to be present. After eliminating the categories of "not known, not stated, or non-applicable", we were left with a sample of 6361 children with their respective parents.[10] In general, if an individual indicated no qualifications, 10 years of educations were assigned. Individuals with any secondary qualifications were assigned 11 years; those with advanced vocational qualifications and some A level were assigned 13 years; those with a sub-degree level of further education (such as nursing and higher education diplomas) were assigned 15 years; whereas those with a degree and higher were assigned 16 years.[11]

From our BCS sample, we can calculate how correlated spouses are in their years of education (i.e., ρ). We find $\rho = .5$. Recall that the correlation of spouses in our model is given by $\rho = \theta + (1-\theta)Q(q^*)$. When all marital sorting is exogenous ($Q' = 0$) how we decompose this correlation into its two components, θ and $Q(q^*)$, is irrelevant. When marital sorting is endogenous though, it would be ideal if we had a measure of segregation which would allow us to distinguish between the "exogenous" component of marital sorting and the endogenous one. In the absence of such a measure (or of a statistic to match that would allow us to impute a value to this variable), we set the value of θ arbitrarily to equal .25. This implies $Q(q^*) = 1/3$.[12]

Next we turn to calibrating (locally) the slope of the Q distribution. Fernández, Guner and Knowles (2001) use household surveys to examine the relationship across countries between the degree of correlation of spouses in their years of education and various measures of the skill premium. For the non-Latin American countries in their sample, they find that $\frac{d\rho}{d(\frac{w_s}{w_u})} \approx .27$. From the definition of ρ and equation (4.2) we have:

$$\frac{d\rho}{d(\frac{\widetilde{w}_s}{w_u})} = (1-\theta)Q'\frac{dq^*}{d\frac{\widetilde{w}_s}{w_u}} = (1-\theta)Q'\frac{1}{\frac{\widetilde{w}_s}{w_u}(1+\frac{\widetilde{w}_s}{w_u})} \tag{4.3}$$

which, assuming $\frac{d\rho}{d(\frac{w_s}{w_u})} \approx \frac{d\rho}{d(\frac{\widetilde{w}_s}{w_u})}$ (we have no other way of obtaining an estimate otherwise), allows us to solve for $Q'(q^*)$ as a function of θ and $\frac{\widetilde{w}_s}{w_u}$.

The requirements of $\rho = .5$ and $q^* > \mu$ imply that for any positive choice of θ, Q must be a distribution such that the mean lies below the median. A simple distribution that can satisfy this requirement is a step function. Thus, we assume that Q is a step function with two steps, where the base of each step is given respectively by $[0, a_q]$ and $(a_q, v_q]$ and the areas of each step are given

[10] Each sibling in a pair of twins or higher multiple births was included as an independent observation.

[11] The details of the coding into years of education are available by request from the author.

[12] For robustness, we have examined other initial values of θ as well.

by h_q and $1 - h_q$ respectively. We arbitrarily set the area of first step, h, to equal $Q(q^*)/2$, which implies that the area of second step must equal $1 - Q(q^*)/2$. Furthermore, since q^* lies in the second interval, the height of the second step is given by Q' which from equation (4.3) equals $\frac{d\rho}{d(\frac{\tilde{w}_s}{w_u})}\frac{\frac{\tilde{w}_s}{w_u}(1+\frac{\tilde{w}_s}{w_u})}{(1-\theta)}$.This allows us to solve for the remaining unknown parameters of the distribution (i.e., a_q, v_q).[13]

Both for establishing the fertility profiles and the distribution of effort cost by parental types, it will be useful to establish a cutoff value at which we declare an individual to be skilled and below which unskilled. We choose to call all individuals with at least two A levels (or their equivalent) skilled and all those below are considered unskilled.[14]

As the BCS data does not have data on the fertility of the parents, we use the British Household Panel Survey (BHPS) of 1992 to examine this question. We construct a sample of women (with their partners) who would have been between the ages of 18 to 35 in 1970. The couple's fertility is then identified from the retrospective fertility data taken in 1992 and the education of the two partners in the couple is obtained.[15] We find that the average fertility of ss couples is 2.07; that of su couples is 2.11; and that of uu couples is 2.18.

We calibrate our fertility distributions by assuming that the fertility of a family type is a random draw from a binomial distribution over the two integers that bracket the average fertility rate for that type (i.e., either two or three for all types), such that the expected value of that distribution equals that family type's average fertility. Thus, $\eta_{2ss} = .93$, $\eta_{2su} = .89$, and $\eta_{2uu} = .82$.

Next we consider the choice of the G_{ij}'s, i.e., the distribution (by parental type) of a child's effort cost required to become skilled. Of course, we have no direct measure of these distributions. Using our sample of parents and children, however, we can find the average fraction of children from each family type that became skilled, i.e., the Γ_{ij}'s. We find that this fraction is .76 for type ss; .52 for type su; and lastly .23 for type uu. Thus, we require that at the steady state $\gamma^*(\lambda^*)$ we have $\Gamma_{ss}(\gamma^*(\lambda^*); \lambda^*) = .76$, $\Gamma_{su}(\gamma^*(\lambda^*); \lambda^*) = .52$, and $\Gamma_{uu}(\gamma^*(\lambda^*); \lambda^*) = .23$. Note that these numbers, however, do not tell us what each $G_{ij}(\gamma^*)$ is; knowledge

[13]Although the choice of distribution and some of its parameters are arbitrary, note that what we are really interested in is the (local) slope of the Q function. This is not arbitrary and is pinned down by eqn. (4.3).

[14]In our mapping from achievement to years of education, this is equivalent to considering skilled those individuals with more than 13 years of education.

[15]A bias of using this method is that we only obtain fertility data from couples that are still together in 1992.

of the latter also requires information as to the number of children of each family type that are constrained, i.e., of the m_{nij} in the equation below.

$$\Gamma_{ij}\left(\gamma^*(\lambda^*);\lambda^*\right) = \frac{G_{ij}(\gamma^*(\lambda^*))}{f_{ij}} \sum_{n=0}^{n=\overline{n}} \min[m_{nij}(\lambda^*), n]\eta_{nij}$$

Of course, when there is no change at the family level in their skilled labor supply in response to changes in relative wages, then the only pieces of information needed are the Γ_{ij}'s (as in Fernández and Rogerson (2001)) and how the latter are decomposed is irrelevant. When there is a family level supply response, then the most important information we require is how large this response is to a change in the skill premium. McVicar and Rice (2001) examine the time series evidence relating to participating in further education in England and Wales using cointegration analysis. For men, they find that a 1% increase in the gross weekly earnings of professional, managerial, and related occupations relative to the average gross weekly earnings of manual workers in full-time employment is associated with a .56% increase in the ratio of the participation rate relative to the non-participation rate in full-time higher education. We translate this finding into the assumption that $\frac{\partial \frac{\lambda}{1-\lambda}}{\partial \frac{w_s}{w_u}} \frac{\frac{w_s}{w_u}}{\frac{\lambda}{1-\lambda}} \approx .56$. Thus, we assume that when w_s increases from 54,000 to 54,540 (i.e., a 1% increase in $\frac{w_s}{w_u}$ assuming w_u stays constant) then $\frac{\lambda'}{1-\lambda'} = 1.56\frac{\lambda^*}{1-\lambda^*}$ where λ' is next period's proportion of skilled workers.

As before, we use a two-step function as our distributional assumption for the G_{ij}'s and assume that all three of these functions share the same base for each step $[0, a_\gamma]$ and $(a_\gamma, v_\gamma]$ but with potentially different relative areas h_{ij}, $1-h_{ij}$ on these 2 steps. Letting $a = \gamma^*/2$ (hence assuming that γ^* lies in the second step), we can use the statistics provided above, an assumption about m_{nij} and the relationship

$$\lambda' = \frac{\sum_{ij}\left(\phi_{ij} G_{ij}(\gamma^{*\prime}) \sum_{n=0}^{n=\overline{n}} \min[m_{nij}, n]\eta_{nij}\right)}{\sum_{ij} \phi_{ij} f_{ij}}$$

to calibrate the parameters of the distributions.

We specify borrowing constraints as $I - \overline{c}(n+2) \geq m_n p$ where $\overline{c}$ is some minimum consumption level required by each child in the family and $m_n \leq n$ solves for the maximum number of children that will be able to borrow p. As this is an inequality (and we are not using data to match either $\overline{c}$ or p, nor are we examining the effect of changes in constraints), then in general, any particular Γ_{ij} value does not have a unique decomposition into a parameter of a given distribution and a assumption about how binding is the borrowing constraint.[16] For any distribution G_{ij}, the requirement that $\Gamma_{ij} = \frac{G_{ij}(\gamma^*)}{f_{ij}} \sum_{n=0}^{n=\overline{n}} \min[m, n]\eta_{nij}$ can be decomposed several ways into a family background and a borrowing constraint component.[17] Table 1 below shows several such decompositions.

Table 1
Decomposition of Γ_{ij}'s into Distributions and Borrowing Constraints

	$m_{nij} = n$	$m_{3uu} = 2$	$m_{3uu} = 2, m_{3su} = 2$
μ_{ss}	0.5803	0.5554	0.5455
σ_{ss}	0.9571	0.9036	0.8822
μ_{su}	1.0540	1.0041	0.9320
σ_{su}	1.1746	1.1074	1.0685
μ_{uu}	1.6264	1.5076	1.4766
σ_{uu}	1.1580	1.0986	1.0707

In the table above, the columns report the means and standard deviations of the γ distributions (the G_{ij}'s) under different assumptions. The first column assumes that the average fraction of skilled children from different parental types seen in the data are the result of only biology/parental background and that borrowing constraints play no role. Of course, this requires that the mean effort cost is greatest for those whose parents are both unskilled and lowest for those with two skilled parents. The next column assumes that uu type parents with three children can at most afford to afford to educate two of them (i.e. $m_{uu3} = 2$), whereas the last column assumes that this situation also holds for su parents with three children. Note that as borrowing constraints play a larger role, the means of the distributions fall since the Γ_{ij}'s observed in the data can now be explained by borrowing constraints and not solely by the influence of parental educational background on the distribution of effort cost.

[16]We do not attempt to match p since what matters is not really the price one must pay to, say, go to college, but rather the price parents must pay to live in a neighborhood that allows their children to obtain a good quality primary and secondary education.

[17]For all our calculations we arbitrarily set $p = 1000$.

As our benchmark we will take the second column, i.e., we assume the existence of mild borrowing constraints in that only uu type parents with three children are affected by borrowing constraints. These are mild constraints since in the calibrated steady-state equilibrium it implies that only 1.6% of families in the population are affected (since in order to be constrained, uu families with 3 children must also have $\gamma < \gamma^*$).[18]

Lastly, we model the production function as a constant elasticity of substitution production function:

$$F(L_s, L_u) = A[bL_s^\delta + (1-b)L_u^\delta]^{\frac{1}{\delta}}$$

which allows us to write the relative wages of skilled to unskilled workers (the skill premium) as:

$$\frac{w_s}{w_u} = \frac{b}{1-b}\left(\frac{\lambda}{1-\lambda}\right)^{\delta-1}$$

Based on data in Machin (1999), we match a skill premium of 1.8 as our benchmark. We also make use of the survey by Katz and Autor (1999) which suggests that a reasonable elasticity of substitution to match lies between 1 and 2.5. We choose to match an elasticity of substitution of 1.5 as our benchmark case, implying $\delta = 1/3$. We also set the initial value of w_u equal to $30,000$ as a normalization.

We leave the discussion of the results of the parameterization of the model to the next section in which they are compared with the new steady state resulting from an increase in θ.

4.2. Welfare Analysis

Changes in the degree of segregation will bring about changes in welfare. There are several measures of welfare that are potentially of interest. First, we would like to know how the welfare of skilled and unskilled workers are affected, i.e., V^s and V^u (of course, for a particular individual i who decides to become skilled, her entire welfare is given by $V^s - \gamma_i$). It is useful to convert any change in utility to the percentage change in household incomes the individual would have to gain in the original steady state so as to be indifferent between it and the new steady

[18]For robustness, we have also performed all the exercises in this paper for the case in which there are assumed to be no binding constraints. The basic conclusions of our experiments are unchanged.

state associated with the increase in segregation. Thus, we can find Δ_i $i \in \{s,u\}$ such that

$$V^{i0}\left(\Delta_i\right) \equiv V^i\left(\widetilde{w}_s^0(1+\Delta_i), w_u^0(1+\Delta_i)\right) = V^i\left(\widetilde{w}_s^1, w_u^1\right) \equiv V^{i1}$$

where 0 indicates the original steady state and 1 indicates the new steady state. Thus, V^{i0} indicates the utility of individual of type $i \in \{s,u\}$ in the original steady state and $V^{i0}\left(\Delta_i\right)$ indicates the same individual's utility in that steady state when the household incomes she receives have been increased by a fraction Δ and everything else is held constant. So, for example, we would solve for Δ_s by solving for its value that set $V^{s0}(\Delta_s) = [\theta + (1-\theta)\lambda^0][\log 2\widetilde{w}_s^0(1+\Delta_s) + \int_{\mu}^{\overline{q}} qdQ + Q\left(\mu\right)\mu] + (1-\theta)(1-\lambda_{t+1})[(1-Q(q^{*0}))\log(\widetilde{w}_s^0 + w_u^0)(1+\Delta_s) + \int_0^{\overline{q}} qdQ + Q(q^{*0})(\log 2\widetilde{w}_s^0(1+\Delta_s)+\mu)]$ equal to V^{s1}. It is easy to show that

$$\log(1+\Delta_i) = V^{i1} - V^{i0}$$

Another interesting measure of welfare is an individual's ex-ante welfare. That is, we can think of the individual as having a probability of being born to a family type ij as given by $\frac{\varphi_{ij} f_{ij}}{\sum_{ij}\varphi_{ij} f_{ij}}$ and then becoming skilled with probability $\Gamma_{ij}(\gamma^*, \lambda)$ and obtaining expected utility $V^s - \frac{\int_0^{\gamma^*}\gamma dG_{ij}}{G_{ij}(\gamma^*)}$ in that case, and becoming unskilled with probability $1-\Gamma_{ij}(\gamma^*, \lambda)$ and obtaining expected utility V^u. Summing over all the family types, we obtain that an (unborn) individual's ex-ante utility, U, is given by:

$$U \equiv \lambda V^s + (1-\lambda)V^u - \frac{\sum_{ij}\varphi_{ij} f_{ij}\frac{\Gamma_{ij}(\gamma^*,\lambda)}{G_{ij}(\gamma^*)}\int_0^{\gamma^*}\gamma dG_{ij}(\gamma)}{\sum_{ij}\varphi_{ij} f_{ij}} \tag{4.4}$$

We can again convert the change in ex-ante expected utility in the pre and post increased segregation steady states by solving for the fraction Δ that an individual would have to obtain from all household incomes in the original steady state so as to be indifferent between the two steady states (U^0 and U^1). As before, it is easy to show that Δ solves $\log(1+\Delta) = U^1 - U^0$. Note that the last term in expression (4.4) is the expected disutility from the effort cost to become skilled. As we will need to refer to it in our welfare calculations, we shall henceforth denote it by $W(\gamma^*, \lambda)$.

4.3. Calibration Results

It is important to note that for the calibration of the model to the initial steady state λ^*, the decomposition of the correlation into an exogenous and an endogenous component and the specification of the supply response is irrelevant; that is, the initial calibration does not depend on how we parameterize the Q or G_{ij} distributions. Thus, the parameters A, b, δ, ρ and the steady-state value of λ are determined solely by the wage premium, the normalization of the initial value of the unskilled wage, the elasticity of substitution, the correlation of spouses in their years of education, and the Γ_{ij} values.

Table 2 below reports the results of calibrating the steady state of our model to the statistics discussed above. Table A1 in the Appendix reports the parameter values of the distributions and production function as well as all statistics used to calibrate them.

Table 2
Calibration of the Initial Steady State

ρ	λ^*	w_s	w_u	$std \log y$	std logf$y$	V_s	V_u	U
.5	0.49161	54000	30000	0.29385	0.25536	11.509	11.082	11.292

The results of the baseline calibration yields around 49% of individuals who are skilled. This is a bit lower than the 53.7% reported by the Department for Education and Employment (2001) as the fraction of 21 year olds that have attained either 2 A levels, an NVQ level 3, an advanced GNVQ (or equivalent) in England in the year 2000 but is above the 47.2% level of economically active adults (18-64 years old males, 18-59 years old females) with the equivalent qualifications in that same year. The standard deviation of log income is small relative to its actual value of .5 in the national labor force survey, but that is to be expected since all the variation in personal income in the model is produced by two wages only.

5. Response to an Increase in Segregation

We examine the response to an increase in segregation under three different possible scenarios. The first is one in which almost all behavior is exogenous in that both the family supply response is fixed and the degree of sorting is not affected by any changes in the environment. In the second scenario, we allow for sorting to respond to incentives, but keep the family supply response fixed. In the last scenario, both the supply of skilled labor at the family level and the degree of sorting respond to their environment.

5.1. Exogenous Sorting and No Family Supply Response

We first examine the consequences in changes in the degree of segregation in an economy in which almost all behavior is exogenously specified. That is, we explore the effect of an increase in environmental segregation (i.e., θ) assuming that there is no endogenous component to sorting, i.e., $Q'(q^*) = 0$. In addition we assume that there is no endogenous response of the Γ_{ij}'s to the change in sorting. This assumption can be made consistent with microeconomic foundations, but for the moment we can simply interpret this constancy as the outcome of some socioeconomic/genetic process such that the Γ_{ij}'s are constant.[19] Thus, changes in the relative supplies of skilled and unskilled labor are governed by changes in the relative frequency of different family types. This is a very useful benchmark as it is the most "mechanical" approach that still respects the fact that there are fertility differentials as well as propensities for children to become skilled that differ across family types.

Table 3 below reproduces the results of the steady-state calibration in the first row and the response (in the new steady state) to an increase in segregation (equivalent here to the correlation of spouses in their years of education) from .5 to .6 in the second row.

Table 3
Increase in Segregation: Exogenous Responses

ρ	λ	w_s	w_u	$std \log y$	std logfy	V_s	V_u	U
.5	0.49161	54000	30000	0.29385	0.25536	11.509	11.082	11.292
.6	0.48823	54179	29828	0.29834	0.26765	11.524	11.061	11.287
$\Delta_s = 0.0149$			$\Delta_u = -0.0208$				$\Delta = -0.0049$	

To understand the results above, it is best to recall from the theory discussed in section 3 that an increase in segregation will necessarily decrease the steady-state value of λ if $f_{ss}\Gamma_{ss} - 2f_{su}\Gamma_{su} + f_{uu}\Gamma_{uu}$ is negative and $f_{ss} - 2f_{su} + f_{uu}$ is positive. Using the UK numbers discussed previously for these parameters we

[19] In Fernández and Rogerson (2001) individuals either have or do not have ability to become skilled (e.g., $\gamma \in \{0, 1\}$). Thus a child's γ can be considered a draw from a binomial distribution G_{ij}.

obtain $-.1198$ and 0.3 respectively. Thus, an increase in θ will unambiguously decrease the proportion of skilled people in the new steady state.

As shown in row 2 of Table 2, the effect of an increase in the correlation of spouses is to decrease λ from .492 to .488 (around 0.69%). This increases the skilled wage by .33%, decreases the unskilled wage by .57%, and serves to increase the inequality in the personal income distribution as measured by the standard deviation of $\log y$ (as shown in column 4) and in the distribution of family income (as shown in column 5 by $std \log f$y). The standard deviation of personal log income increases by 1.53% and that of family income increases by 4.81%. Skilled workers are made better off (they would require a 1.5% increase in household incomes to be as well off in the original steady state); unskilled workers are made worse off (they would require a 2% decrease in the original steady-state household incomes to be indifferent). Ex-ante welfare falls and a .5% decrease in the original steady-state household incomes would be required to create indifference.[20] A similar exercise for an increase in θ to .7 instead results in roughly double the reported changes in distributions of individual and family incomes and in the compensatory incomes required by each group (and in the ex ante welfare calculation) for this change.

Of course, a comparison of steady-state welfares is not necessarily a meaningful exercise as it ignores the transition path to the steady state. We next turn to an examination of the transition to the steady state. We consider the following timing for the correlation increase. In period 0 the economy is in the original steady state with $\lambda^0 = \lambda^*$. Before matching, however, there is an increase in segregation that increases the correlation of spouses' education immediately from .5 to .6. Note that an increase in correlation in period 0 affects utility in that period (since it affects the probability with which different matches are formed), but not wages as these are determined by λ^*. In the following period, λ does change (since the fraction of matches of each type have changed) and hence wages are affected as well.

As shown in Table 4 below, the transition to the new steady state takes approximately 10 periods. The first column shows the value of the variables in

[20]It is interesting to note that these numbers are significantly smaller than those obtained by Fernández and Rogerson (2001) in a similar exercise for the US. As explained in Fernández (2001b), this is primarily due to the greater concavity in the US in the relationship between average parental human capital and the production of skilled children (or, more generally, to the greater absolute value of the cross-partial in the production of next period's human capital as a function of maternal and paternal human capital).

the original steady state. The values in period 0 are those with the original steady-state λ but with the new correlation. As shown below, the transition to the new steady state is monotonic in λ and welfare. Over half the transition to the new steady-state value of λ though is completed in period 2 and over 80% of the welfare transition is completed in period 0 for skilled workers and over 65% for unskilled workers. The reason for the very large immediate welfare changes is that these are mostly a result of the immediate increase in correlation and only to a smaller extent a result of the changes in wages and probabilities that follow.

Table 4
Transition to New Steady State: Exogenous Responses

Period	λ	V^s	V^u	U
s.s.	.49160	11.509	11.082	11.292
0	.49160	11.521	11.068	11.291
1	.49002	11.523	11.066	11.290
2	.48918	11.523	11.065	11.289
4	.48850	11.524	11.064	11.289
6	.48831	11.524	11.064	11.288
8	.48825	11.524	11.064	11.288
9	.48824	11.524	11.064	11.288

5.2. Endogenous Sorting

The second analysis allows sorting to be endogenous but maintains the supply response constant. In order to examine the effect of an increase in segregation when sorting is endogenous, as explained before, we arbitrarily set $\theta = .25$ resulting in $Q(q^*) = \frac{1}{3}$ and thus in a correlation of spouses in years of education equal to .5. We increase θ from .25 to .4. This is the value of θ such that, if there were no endogenous response in sorting, would produce a correlation of spouses of .6. Table 5 below shows the result of the increase in segregation in the new steady state.

The main difference between Tables 3 and 5 is that in the latter there is an additional (endogenous) response to the increase in segregation as the increase in wage differentials causes skilled individuals to become pickier (i.e., to require a higher q^*) about matching with unskilled individuals. This is shown in the value of $Q(q^*)$ which before implied that skilled individuals rejected 1/3rd of the matches with unskilled individuals whereas now they reject almost 40% of these matches. This in turn implies that the correlation of spouses in the new steady state now slightly exceeds .6 and thus causes λ to fall by an additional quarter of 1% more than in the first case as even fewer matches between skilled and unskilled individuals are formed leading to even lower levels of aggregate human capital.

As shown in the table below, unskilled individuals would be willing to give up around 1.95% of household income to return to the original steady state, whereas skilled individuals would be willing to give up around 1.75% of income to maintain the new steady state. Ex ante utility is lower in the new steady state.

Table 5
Increase in Segregation: Endogenous Sorting

	$\theta = .25$	$\theta = .4$
λ	0.4916	0.48811
w_s	54000	54185
w_u	30000	29822
$std \log y$	0.29385	0.29850
std logfy	0.25536	0.26808
q^*	4.261	4.265
$Q(q^*)$	0.333	0.339
ρ	0.5000	0.604
V_s	15.836	15.853
V_u	15.409	15.389
U	15.552	15.616
$\Delta_s = 0.0175$ $\Delta_u = -0.0195$ $\Delta = -0.0031$		

Contrasting the welfare numbers above with the ones we obtained in the all exogenous case, it is interesting to note that unskilled workers need a smaller fraction of income as compensation when sorting is endogenous. This is puzzling, at first glance, since λ has fallen by more than before, implying that unskilled wages are lower than under the exogenous sorting scenario. This mystery is resolved if we decompose the change in welfare into its two component parts–the part due to changes in expected incomes and the part due to changes in expected quality of the match. If we do this, we obtain that the income component would require a compensation of 2.15% of household income whereas the quality component actually increases and requires a negative compensation of $-.21\%$. It is the sum of these two components that gives us the overall lower compensation required. Why has the quality component of matching improved? The reason for this is the increase in the exogenous component of sorting, which implies that unskilled individuals have a lower chance of being matched with a skilled individual in the first round.[21] This means, as explained in section 3, that they now have an increased chance of having two opportunities in which to obtain a high quality match, and a correspondingly decreased probability of meeting a skilled individual in the first round in which case they had a high chance of being rejected and hence the expected value of the quality of their match was relatively low.

5.3. Endogenous Sorting and Variable Family Supply

The last analysis we perform is to allow both sorting and the Γ_{ij}'s to respond to the change in segregation. Note that incorporating an endogenous supply response at the family level does not require us to recalibrate the Q distribution. As described in section 3.1, it requires us to calibrate the G_{ij}'s so that when evaluated at γ^* they match the parameter values given by the data (the Γ_{ij}'s) and the aggregate supply response. Throughout, as in the previous two sections, we will assume that the changes observed in wages do not change the bindingness of borrowing constraints (i.e., the m_{nij} remain as specified in column 2 of Table 1).

The introduction of an endogenous supply response to the change in incentives alters in an important fashion the results of our first two analyses. Table 6 below shows the effect of an increase in segregation from its initial steady state value of .25 to .4 which, as before, is the value of θ that would be required to produce a correlation of spouses of .6 if there were no reaction of individuals to the change in segregation.

The first important result to note is that λ now increases by almost 0.7% rather than decreasing by around the same percentage as it did in the exercise of the previous section. There are two effects at work here: the first is the one we have analyzed previously, that is, ceteris paribus, an increase of correlation of types causes a destruction of mixed marriages which leads to a lower average production of skilled children as a fraction of the population. The second effect is a new one: ceteris paribus, the increase in segregation makes it relatively more attractive to become skilled (i.e., $\gamma^*(\lambda;\theta)$ increases for a given λ) as now a skilled person will be more sure of being matched with another skilled individual (whereas this probability decreases for an unskilled individual). Thus, the first effect is to decrease λ whereas the second effect is to increase it.

[21] The probability that an unskilled individual meets another unskilled invidual in the first round is given by $\theta + (1-\theta)(1-\lambda)$. This has increased from .63 in the old steady state to .71 in the new steady state both because of the increase in segregation and because λ is lower.

The two effects are shown in Figure 2. The first effect is seen in the upward shift of the Ψ curve since a greater value of γ is required to produce a given λ given that $f_{ss}\Gamma_{ss} - 2f_{su}\Gamma_{su} + f_{uu}\Gamma_{uu}$ is negative and $f_{ss} - 2f_{su} + f_{uu}$ is positive. The shift of this curve shows the negative effect of replacing 2 us couples with one uu and one ss couple on the production of skilled individuals on aggregate in the population. The second effect is shown in the upward shift of the $\gamma^* = V^s - V^u$ curve, indicating the increase in the relative attractiveness of becoming skilled for a given λ. As can be seen in the figure, the shifts of these two curves necessarily increases γ^* (i.e., the differential in utilities between a skilled and an unskilled person), but the net effect on λ depends on which curve shifts more. As is clear from the result of our analysis above the second effect dominates.

Table 6
Increase in Segregation: Endogenous Sorting, Variable Family Supply

	$\theta = .25$	$\theta = .4$
λ	0.4916	0.4950
w_s	54000	53821
w_u	30000	30174
$std \log y$	0.29385	0.28934
std logfy	0.25536	0.25913
q^*	4.261	4.258
$Q(q^*)$	0.333	0.327
ρ	0.5000	0.5965
γ^*	0.427	0.447
V_s	15.836	15.848
V_u	15.409	15.401
W	0.059	0.061
U	15.560	15.562
$\Delta_s = 0.013$ $\Delta_u = -0.007$ $\Delta = 0.003$		

As a result of the λ increase, the skilled wage drops by .33% whereas the unskilled wage increases by .58%. This makes the personal income distribution more equal, as shown by the 1.5% fall in the standard deviation of log income. The utility differential between skilled and unskilled individuals increases, however, as shown by the increase in γ^*. The narrowing of the skill premium also serves to make skilled individuals less picky (i.e., q^* falls) so that the correlation of spouses rises from .5 to .596 rather than to the .6 that would obtain under exogenous sorting. The increase in the final correlation still implies though that the family income distribution becomes more unequal, with the standard deviation increasing by about 1.5%.

Skilled individuals are made better off in the new steady state despite the fall in skilled wages as they are now more sure of matching with another skilled individual (they need 1.3% increase in household income to be indifferent between steady states).[22] Although the welfare lost due to effort (i.e., W) is higher in the new steady state, this is due to the fact that more people find it more attractive to become skilled. What is interesting to note, however, is that despite their wage increase unskilled individuals are on the whole left worse off. Although they earn more, they are also less likely to marry a skilled person, and the welfare consequences of this second effect outweighs the first; unskilled individuals would be willing to give up .75% of their original steady-state household income in order to remain in that steady state. Ex-ante utility, on the other hand, increases for the first time as can be seen by the fact that Δ is positive. Ex ante, individuals would require a .26% increase in all household incomes to be indifferent at the original steady state.

Table 7

Transition to New Steady State: Endogenous Sorting, Variable Family Supply

Period	λ	V^s	V^u	U
s.s.	.49161	15.836	15.409	15.559
0	.49161	15.851	15.395	15.560
1	.49365	15.849	15.399	15.560
2	.49447	15.849	15.400	15.561
3	.49478	15.848	15.401	15.561
4	.49491	15.848	15.401	15.562
5	.49496	15.848	15.401	15.562
6	.49498	15.848	15.401	15.562

The transition to the new steady state is faster than under the exogenous scenario. As shown in Table 7 above, it takes some 7 periods to transit to the new steady state. As before, λ monotonically increases to its new steady-state value. Over 60% of its decline takes place in period 1. Welfare, on the other hand,

[22]Note though that to examine the effect on the representative skilled person from family type ij requires not only examining what happens to V^s but rather to $V^s - \frac{\int_0^{\gamma^*} \gamma dG}{G_{ij}(\gamma^*)}$ since this will take into account the expected effort cost contingent on becoming skilled.

first overshoots its steady-state value (higher for skilled individuals; lower for unskilled individuals) before monotonically transitioning to the new steady-state value. This is due to the fact that λ is unaffected by the change in segregation in period 0 and only reacts in period 1. Hence, welfare in period 0 reflects only the change in segregation, and not the changes to household income induced by changes in λ. Thus, in period 0, skilled individuals do not have any of the negative income effects associated with the decrease in w_s; similarly, unskilled individuals have not yet reaped the positive income effects associated with the increase in w_u. Consequently, welfare overshoots its steady-state value in period 0.

5.4. Robustness

We have checked the robustness of our results to various assumptions. For example, instead of assuming that individuals from uu families are mildly constrained, we have assumed that there are no borrowing constraints for any family types. The results are very similar. We have also assumed that the skill premium is 1.4 rather than 1.8. Recalibrating the model to this value produces similar results.

We also examined the effects of replacing our CES production function with an elasticity of substitution of 1.5 with a Cobb-Douglas production function. In all cases, similar results were obtained but the changes in the standard deviation of the income distributions were larger. For example, in the all exogenous case, the effect of the correlation increase from .5 to .6 increased the standard deviation of the log wage by 2.28% rather than the 1.53% obtained under our specification. The increase in the standard deviation of log family income was also correspondingly larger. The welfare comparisons, however, remained very similar in that not very different percentages of household incomes are required to compensate winners and losers. This same conclusion holds for the case in which sorting is endogenous and there is a variable family supply of skilled labor: the change in personal and family inequality was only slightly larger and the welfare responses were very similar.

The statistic that is most open to question is probably the one used to calibrate the family supply response.[23] Below we show the results of recalibrating the model to double the aggregate supply response that we assumed for our original calibration, i.e. to an elasticity in the relative supply of factors to relative wages of 1.12 rather than .56. As can be seen in Table 8, the results obtained are similar.

[23] The Q distribution can also be questioned, but including it or getting rid of it does not have an important effect as shown previously, providing of course that it is not significantly larger.

Alternatively, one could ask what would be the supply response required to obtain an improvement in the welfare of unskilled individuals. This would require a larger response of λ, thus producing a larger increase in w_u. Recalibrating the model to solve for this response yields an elasticity of 1.88, significantly larger than the response supply we find in the data.

Table 8
Increase in Segregation: Endogenous Sorting, Large Supply Response

	$\theta = .25$	$\theta = .4$
λ	0.4916	0.4976
w_s	54000	53687
w_u	30000	30306
$std \log y$	0.29385	0.28591
std logfy	0.25536	0.25579
q^*	4.261	4.255
$Q(q^*)$	0.333	0.323
ρ	0.5000	0.5938
γ^*	0.427	0.441
V_s	15.836	15.847
V_u	15.409	15.406
W	0.066	0.068
U	15.552	15.557
$\Delta_s = 0.011$	$\Delta_u = -0.003$	$\Delta = 0.004$

6. Conclusion

This paper presents a model of the intergenerational transmission of education and marital sorting where parents matter both because of their household income and because parental human capital determines the expected value of a child's disutility from making an effort to become skilled. We show that an increase in segregation has potentially ambiguous effects on the fraction of individuals that become skilled in the steady state, and hence on marital sorting, the personal and household income distribution, and welfare.

We calibrate the steady-state of our model to UK statistics examine the effects of an increase in segregation. When sorting is exogenous and the relative supply of factors is constant at the family level, an increase in segregation lowers the proportion of skilled individuals, decreases welfare for unskilled individuals, increases the welfare of skilled individuals, and lowers ex ante utility. When sorting is endogenous and the family supply response is variable, we find that an increase in segregation will lead to a higher proportion of skilled individuals, a more compressed personal income distribution, and higher welfare for skilled individuals. The effect on the welfare of unskilled individuals is negative as the increase in their wages is insufficient to compensate for their decreased probability of matching with a skilled individual. Thus the welfare gap between skilled and unskilled individuals increases. Ex ante utility, however, increases.

References

[1] Aiyagari, S. Rao, J. Greenwood, and N. Guner. "On the State of the Union." *Journal of Political Economy*, 108 (2000), 213-244.

[2] Alesina, Alberto, and Eliana La Ferrara, "Participation in Heterogeneous Communities," *Quarterly Journal of Economics*, CXV (2000), 847-904.

[3] Becker, Gary, "The Theory of Marriage, Part 1," *Journal of Political Economy*, 81 (4) (1973), 813-846.

[4] Bénabou, Roland, "Heterogeneity, Stratification and Growth," *American Economic Review,* LXXXVI (1996), 584-609.

[5] Bisin, Alberto and Thierry Verdier, "Beyond the Melting Pot: Cultural Transmission, Marriage, and the Evolution of Ethnic and Religious Traits," *Quarterly Journal of Economics* CXV (2000), 955-87.

[6] Burdett. Ken, and Melvyn Coles, "Marriage and Class," *Quarterly Journal of Economics*, CXII (1997), 141-68.

[7] _ _ _ _ _ _ _ _ _, "Transplants and Implants: The Economics of Self-Improvement," mimeo, (1999).

[8] Cole, Harold, George Mailath, and Andrew Postlewaite, "Social Norms, Savings Behavior and Growth," *Journal of Political Economy*, C (1992), 1092-1125.

[9] Cooper, Suzanne, "A Positive Theory of Income Redistribution," *Journal of Economic Growth*, III (1998), 171-195.

[10] Department for Education and Employment, National Statistics, "First Release: The level of Highest Qualification Held by Young People and Adults: England 2000," 22 February (2001).

[11] Durlauf, Steven, "A Theory of Persistent Income Inequality," *Journal of Economic Growth*, I (1995), 75-93.

[12] Fernández, Raquel, "Sorting, Education and Inequality," in *Advances in Economic Theory*, Dewatripont, Hansen, and Turnovsky (Eds.), Cambridge University Press, forthcoming, 2001a.

[13] ______, "Marital Sorting and Human Capital: Comparing the US and the UK," mimeo, (2001b).

[14] ______, Nezih Guner, and John Knowles, "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality," mimeo, 2001.

[15] ______and Richard Rogerson, "Income Distribution, Communities and the Quality of Public Education," *Quarterly Journal of Economics*, CXI (1996), 135-164.

[16] ____________, "Education Finance Reform: A Dynamic Perspective," *Journal of Policy Analysis and Management*, XVI (1997), 67-84.

[17] ____________, "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education Finance Reform," *American Economic Review*, LXXXVIII (1998), 813-833.

[18] ____________, "Sorting and Long-Run Inequality," *Quarterly Journal of Economics*, 116 (2001), 1305 - 1341.

[19] Greenwood, Jeremy, Nezih Guner and John Knowles, "More on Marriage, Fertility, and the Distribution of Income," University of Rochester, mimeo, (1999).

[20] Jargowsky, Paul, "Take the Money and Run: Economic Segregation in US Metropolitan Areas," *American Sociological Review*, LXI (1996), 984-998.

[21] ______, *Poverty and Place: Ghettos, Barrios, and the American City,* New York: Russell Sage Foundation, (1997).

[22] Katz, Larry and David Autor, "Changes in the Wage Structure and Earnings Inequality," in *Handbook in Labor Economics,* volume 3A, edited by Orley Ashenfelter and David Card, (Amsterdam: North-Holland, 1999).

[23] Kremer, Michael, "How Much Does Sorting Increase Inequality," *Quarterly Journal of Economics*, CXII (1997), 115-139.

[24] ______ and Eric Maskin, "Wage Inequality and Segregation by Skill," Harvard University mimeo, 1996.

[25] Machin, Steve "Wage Inequality in the 1970s, 1980s and 1990s', in P. Gregg and J. Wadsworth (eds.) *The State of Working Britain*, Manchester University Press, 1999.

[26] Mare, Robert, "Five Decades of Educational Assortative Mating," *American Sociological Review*, LVI (1991), 15-32.

[27] McVicar, Duncan and Patricia Rice, "Participation in Further Education in England and Wales: An Analysis of Post-War Trends," *Oxford Economic Papers*, 53 (2001), 47-66.

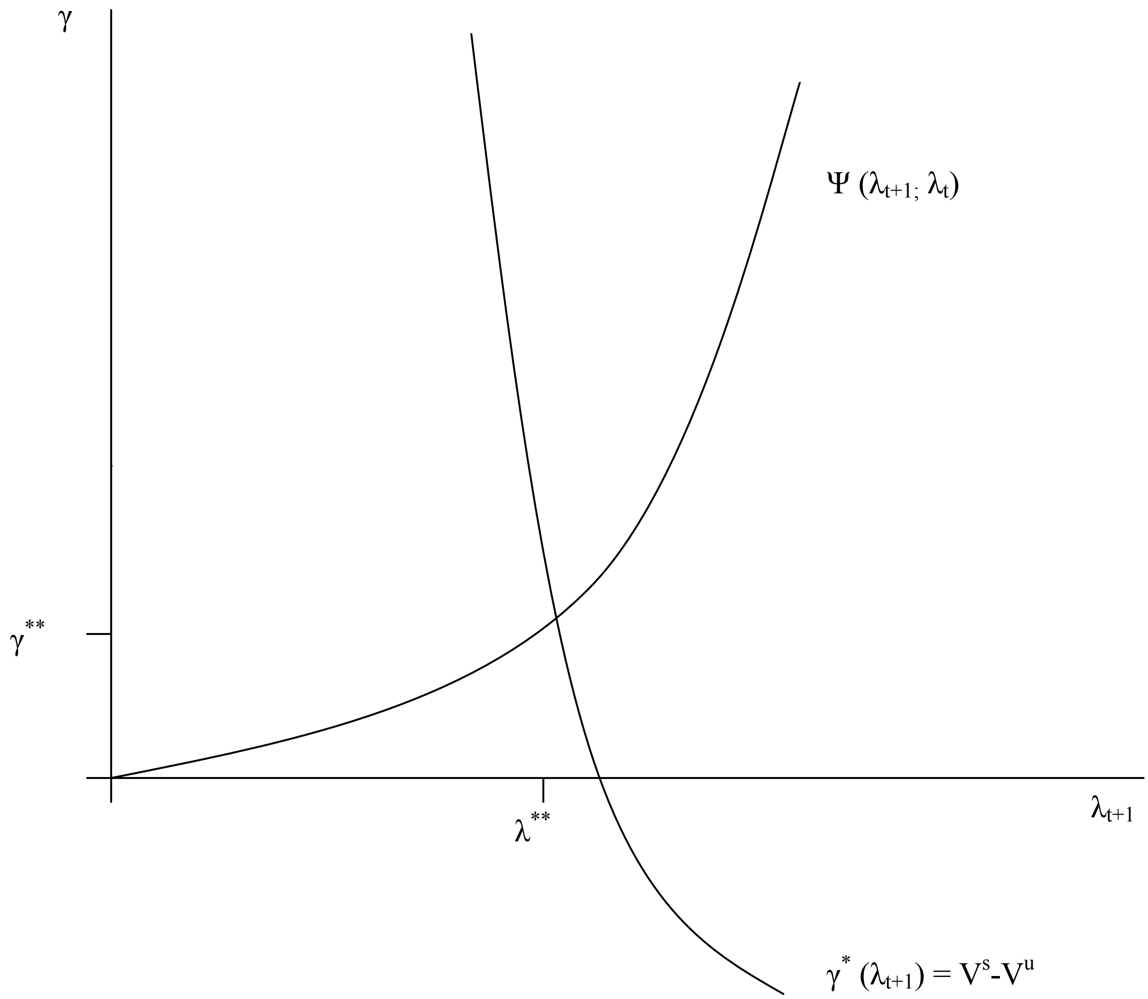
[28] Reich, Robert, "Secession of the Successful," *New York Times Magazine*, Jan. 20 (1991).

[29] Wilson, William, J.,*The Truly Disadvantaged: The Inner City, the Underclass Public Policy*, Chicago, IL., University of Chicago Press, (1987).

Appendix

Correlation of spouses in years of education, $\rho = .5$	BCS sample.
Fertility profiles: $f_{ss} = 2.07$, $f_{su} = 2.11$, $f_{uu} = 2.18$	BHPS (1992)
Fraction of children that become skilled: $\Gamma_{ss} = .76$, $\Gamma_{su} = .52$, $\Gamma_{uu} = .23$	BCS sample
Distribution of effort cost,G_{ij}. Two-step distributions. $a_\gamma = 0.214$, $v_\gamma = 3.004$ $h_{ss} = .740$, $h_{su} = .480$, $h_{uu} = .189$ Supply response: $\frac{\partial \frac{\lambda}{1-\lambda}}{\partial \frac{w_s}{w_u}} \frac{\frac{w_s}{w_u}}{\frac{\lambda}{1-\lambda}} \approx .56$	McVicar and Rice (2001) BCS sample
Distribution of quality,$Q(q)$. Two-step distribution. $a_q = 4.017$, $v_q = 4.640$, $h_q = 1/6$ Assume $\theta = .25 \Longrightarrow Q(q^*) = 1/3$. Correlation response: $\frac{d\rho}{d\frac{w_s}{w_u}} = .27$	Fernández, Guner, and Knowles (2001)
CES Production function parameters $A = 83992.7$, $b = .63768$ elasticity of substitution $= 1.5 \Longrightarrow \delta = 1/3$ skill premium $= 1.8$; $w_u = 30,000$	 Katz and Autor (1999) Machin (1999), Normalization

Figure 1



γ
$\Psi\ (\lambda_{t+1;}\ \lambda_t)$
γ^{**}
λ^{**}
λ_{t+1}
$\gamma^*\ (\lambda_{t+1}) = V^s\text{-}V^u$

Figure 2