Inquisitive Semantics and Logic

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Abstract

In this paper I will present an overview of inquisitive semantics and inquisitive logic (InqL) as a system designed to model an intuitive (and useful) notion of disjunction and of questions. I will begin by defining the syntax and semantics, and will then proceed to illustrate what the logic does by giving a number of telling examples. Section 3 discusses technical aspects of the logic InqL, and section 4 concludes with a brief survey of the work in progress.[*]

1 Syntax and Semantics

I will begin by defining the syntax and semantics of an inquisitive propositional logic I will henceforth call InqL. In this preliminary section, I will present a minimal set of relevant definitions, deferring examples of the logic at work and discussion for later sections.

I first recursively define the language(s) of InqL, where the term 'language' is meant in the broad sense, namely, a set of well-formed sentences.

Definition 1 (Inquisitive syntax). A language L_P of InqL, indexed to a finite set of propositional atoms P, is the smallest set such that

$$\begin{aligned} p &\in L_P \text{ for all propositional atoms } p \in P \\ \bot &\in L_P \\ \text{if } \varphi, \psi \in L_P, \text{ then } \varphi \wedge \psi &\in L_P \\ \varphi \vee \psi &\in L_P \\ \varphi \to \psi &\in L_P \end{aligned}$$

Following standard practice, we will use the abbreviations $\neg\varphi := \varphi \to \bot$ and $\top := \neg\bot$. $\dashv$

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[*] The inquisitive semantics and inquisitive logic defined in Section 1 and exemplified in the beginning of Section 2 will be part of a joint paper with Jeroen Groenendijk (Groenendijk & Mascarenhas 2008). The remaining portion of Section 2 and all of Section 3 constitute an initial, programmatic draft of my master's thesis at the ILLC, under the supervision of Jeroen Groenendijk. I am also indebted to Anna Szabolcsi and Chris Barker for comments on an earlier version of this draft.

Throughout this paper, definitions, lemmas and theorems will be numbered sequentially, by counters distinct from those of natural language examples.

Notice that we are constraining the set of propositional atoms for any given inquisitive language to be a finite set. Indeed, this restriction is what forces us to consider what's essentially an infinite number of inquisitive languages, one for each finite set of propositional atoms. I won't discuss the issue at length in this paper, so suffice it to say that it serves the purpose of keeping the semantics manageable. Most notably, finiteness of P gives an important functional completeness result, which becomes unavailable once we allow the language (and hence the models thereof) to have proposition sets of arbitrarily infinite cardinalities. In any event, the restriction is of no consequence to the expressive power of the logic and its applications to natural language semantics.

The syntax of an inquisitive language for InqL is otherwise perfectly standard, so I move on to the inquisitive semantics, by first defining the relevant class of models.

Definition 2 (Inquisitive models). Models for InqL are relations $\sigma \subseteq W \times W$, where W is the set of total valuations (worlds) on the set of propositional atoms P of an inquisitive language L_P. σs are in addition required to be reflexive and symmetric (NB, not necessarily transitive). We will call the set of all such states Σ_P, for each language L_P. $\dashv$

Each σ (for 'information *s*tate') is to be interpreted as a relation of indifference on the underlying set of worlds W: if w_1 is connected by σ to w_2, this means intuitively that the difference between w_1 and w_2 is *not* an issue. Conversely, if w_1 and w_2 are not connected, the difference between them *is* an issue. The relation of indifference is at the core of the power of InqL as an erotetic logic, for we will model the meaning of a question as an utterance that creates issues. The next section will be dedicated to spelling out how all these meanings are modeled so, for now, let us just define the update semantics[1] for InqL ($\sigma[\varphi]$ stands for "σ after the update with φ"):

Definition 3 (Inquisitive semantics). The update of a state $\sigma \in \Sigma_P$ with a formula φ of a language L_P of InqL is inductively defined as follows:

$$\begin{aligned}
\sigma[p] &= \{(i,j) \in \sigma : i(p) = j(p) = 1\} \\
\sigma[\bot] &= \emptyset \\
\sigma[\varphi \wedge \psi] &= \sigma[\varphi] \cap \sigma[\psi] \\
\sigma[\varphi \vee \psi] &= \sigma[\varphi] \cup \sigma[\psi] \\
\sigma[\varphi \to \psi] &= \{(i,j) \in \sigma : (\forall \iota \in \{i,j\}^2)\ \iota \in \sigma[\varphi] \implies \iota \in \sigma[\varphi][\psi]\}
\end{aligned}$$

And, as mentioned before, let $\sigma[\neg\varphi] := \sigma[\varphi \to \bot]$ $\dashv$

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[1] I think the update semantics version is the most perspicuous, but the fact of the matter is that, for now, there is no true dynamic flavor to it, so a classical, static formulation can also be given and, in fact, it is the one I will use for the remarks in section 3. Consider states σ as defined above, the following is a point-wise (where points of course are pairs of worlds in σ) truth definition of the connectives that is equivalent to the update version:

$$\begin{gathered}
(i,j) \vDash p \text{ iff } i(p) = j(p) = 1 \\
(i,j) \vDash \neg\varphi \text{ iff } (i,i) \nvDash \varphi \ \&\ (j,j) \nvDash \varphi \\
(i,j) \vDash \varphi \vee \psi \text{ iff } (i,j) \vDash \varphi \text{ or } (i,j) \vDash \psi \\
(i,j) \vDash \varphi \wedge \psi \text{ iff } (i,j) \vDash \varphi \ \&\ (i,j) \vDash \psi \\
(i,j) \vDash \varphi \to \psi \text{ iff for all } \iota \in \{i,j\}^2 : \iota \vDash \varphi \implies \iota \vDash \psi
\end{gathered}$$

Notice, once more, how the finiteness of P constraint forces us to index the models in definitions 2 and 3 to the relevant set P. Again, this is no significant shortcoming of the logic, and it is safe to forget about it in general, as I will for the remainder of this paper, by omitting these indexes

Save for the implication clause, which, granted, might look a little odd, the update definitions seem perfectly standard. The expressive power of the logic and semantics thus defined will become clear once we move on to some examples of how it works.

2 InqL in action

In this section I will give several (hopefully) telling examples of how the inquisitive semantics defined in the previous section works. I will then discuss, from a somewhat programmatic point of view, the advantages and disadvantages of an inquisitive system like InqL.

2.1 Disjunctions

Inquisitive models can be represented pictorially. Without loss of generality, let's consider only two propositional letters (otherwise the pictures become too messy). In the pictures that follow, each node represents a world and the first value at each node is the truth value of p, and the second that of q. Figure 1 represents a state (call it σ) of total indifference, i.e., all worlds are connected to each other, and of total ignorance, for all possible valuations on atomic propositions are present[2].

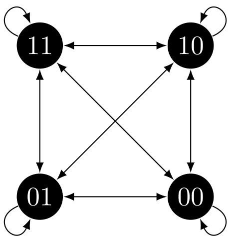

Figure 1: The indifferent, ignorant state σ

After an update with p, the lower part of the picture gets completely disconnected, and we get the state represented in Figure 2 (see below).

White nodes represent worlds that are absent from the relation (because the identity pairs that correspond to them have been eliminated)[3]. An update with q will of course give

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[2] Notice that sigma is, strictly speaking, only the relation, the arrows in this representation, that is, σ is the reflexive, symmetric closure of

$$\{(w_{11}, w_{10}), (w_{11}, w_{01}), (w_{11}, w_{00}), (w_{10}, w_{01}), (w_{10}, w_{00}), (w_{01}, w_{00})\}$$

[3] Just for the sake of clarity, notice that the state in Figure 2 is

$$\sigma[p] = \{(w_{11}, w_{11}), (w_{11}, w_{10}), (w_{10}, w_{10}), (w_{10}, w_{11})\}$$

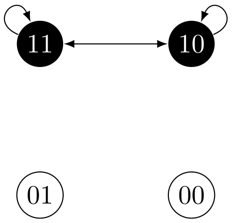

Figure 2: $\sigma[p]$

us the left hand side of the state in Figure 1, that is, the state represented in Figure 3, while an update with $p \wedge q$ will yield the single reflexive point (w_{11}, w_{11}).

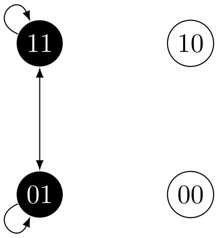

Figure 3: $\sigma[q]$

Although the disjunction clause may look perfectly innocent[4], it is directly responsible for the expressive power of this logic. Consider our minimal and indifferent state σ, as defined by Figure 1, and an update with $p \vee q$. We get the union of the states in Figures 2, and 3. This means that we disconnect from the model the world w_{00}, as one would expect, but that is not all the update with $p \vee q$ does. If we look at the representation of $\sigma[p \vee q]$, in Figure 4, we see that w_{01} and w_{10} are no longer connected to each other by σ. In InqL, as I mentioned in the previous section, this means that the difference between worlds w_{01} and w_{10} interests us. Equivalently, the difference between these two worlds *is an issue*; σ is not an indifferent state.

To make proper use of the relation of indifference in modeling the existence of an issue in our models, we need an accessory semantic concept: that of alternatives.

Definition 4 (Alternatives). For σ a model of InqL, the set A_σ is the set of alternatives in σ, where $\alpha \in A_\sigma$ iff

1. $\alpha \subseteq \sigma$;
2. α is a total relation; and

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[4] That is, disjunction is straightforwardly defined as the union of two update potentials. Interestingly, it is because we're dealing with states that are relations on possible worlds, not just sets of possible worlds, that this simple definition of disjunction manages to confer to the logic the expressive power I'll introduce in this overview.

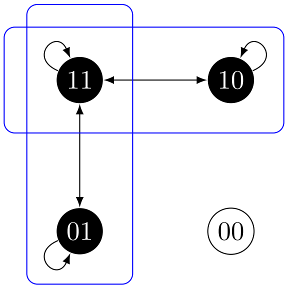

Figure 4: $\sigma[p \vee q]$

3. there is no $\beta \supset \alpha$ such that β is a total relation. ⊣

In other words, the set of alternatives in σ is the set of all maximally connected subsets of σ.

Since indifferent states are by definition total relations, we can equivalently define the set of alternatives for a state σ as the set of maximal indifferent substates of σ. In addition, we can also see that, for indifferent states, such as the ones in Figures 1 to 3, the set of alternatives is the singleton set containing the state itself. Thus, σ is indifferent iff $A_\sigma = \{\sigma\}$.

The state represented in Figure 4, however, is not indifferent, for $A_{\sigma[p \vee q]} \neq \{\sigma[p \vee q]\}$. In fact, $A_{\sigma[p \vee q]} = \{\{(w_{11}, w_{11}), (w_{11}, w_{01}), (w_{01}, w_{11}), (w_{01}, w_{01})\}, \{(w_{11}, w_{11}), (w_{11}, w_{10}), (w_{10}, w_{11}), (w_{10}, w_{10})\}\}$.

These two alternatives for $\sigma[p \vee q]$, represented in blue in Figure 4, correspond to two propositions, namely, p and q. Thus, the disjunction $p \vee q$, besides having brought the information that at least one of p and q must be the case (by eliminating the $\neg p, \neg q$-world), has also created an issue between the p-worlds and the q-worlds, that is, it has brought about an issue concerning which one of the two propositions is in fact the case.

As can immediately be seen, this is a non-classical view of disjunction. Whereas conjunction, implication and negation are purely informative connectives, incapable of introducing issues by themselves, disjunction is seen as something of a potential hybrid, bringing about both data (eliminating worlds) and issues (disconnecting worlds from one another). From an intuitive point of view, this models the idea that, when a speaker utters $p \vee q$, he or she is not merely informing the hearer that $\neg(\neg p \wedge \neg q)$, but also raising the issue of which one of the two is indeed the case. We will return to why it is desirable to have this meaning for disjunctions in the next sections. For now, let us proceed with a few more illustrations of how the semantics works.

An update with a negated atomic formula has unsurprising effects: $\neg p$ eliminates all the p-worlds from an information state. Interestingly however, the effect of double negation on disjunction isn't vacuous as one might have expected. An update with $\neg(p \vee q)$ yields the single reflexive point (w_{00}, w_{00}), and an update with $\neg\neg(p \vee q)$ gives us the complement[5] of the latter information state. Figure 5 is a picture for $\sigma[\neg\neg(p \vee q)]$.

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[5] Strictly speaking, it isn't accurate to see the update with a negation as the complement of the non-negated subformula, because problems would arise from such a definition when considering updates with conditionals or once-negated disjunctions. Consider the case $\neg(p \vee q)$, if this were to yield the complement of $\sigma[p \vee q]$ then, since the pair (w_{10}, w_{01}) is *not* a member of $\sigma[p \vee q]$, it would be present in $\sigma[\neg(p \vee q)]$.

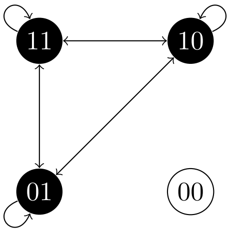

Figure 5: $\sigma[\neg\neg(p \vee q)]$

As one can see in Figure 5, the state $\sigma[\neg\neg(p \vee q)]$ has the same amount of data as $\sigma[p \vee q]$. That is, if we abstract away from the relation and consider solely the subsets of W that $\sigma[\neg\neg(p \vee q)]$ and $\sigma[p \vee q]$ are relations on, we observe that they are identical: the same reflexive pairs have been eliminated by the two updates. However, while the latter has two alternatives, the former has only one. Double negation has transformed an issue-raising disjunction into a purely informative (no issues) update.

The obvious way to consider this effect is to interpret double negation as canceling the inquisitive potential of an update and thereby performing an assertive closure. Accordingly, we define the following abbreviation for an assertive operator '!':

$$!\varphi := \neg\neg\varphi$$

We have now seen the simplest examples of how the inquisitive semantics defined for InqL works. In particular, I have illustrated how an update with a disjunction can potentially introduce both data and issues, whereas conjunction and implication cannot do so by themselves. Furthermore, I have shown how negation has the property of destroying the issue-generating potential of disjunction, and how this property allows us to express the generalized connective 'disjunction' in two semantically different flavors, simplex and double negated.

In the next section I will show how questions come about in InqL. I will complete this (introductory) picture of the logic with more examples and a discussion of the strong and weak points of InqL.

2.2 Questions

The most consensual view on the semantics of questions identifies the meaning of a question with the set of possible answers to it, assuming that these correspond to a partition of the logical space. Our notion of alternatives calculated as a function of the issues in a state is primarily designed to model just that.

Suppose now we had an operator '?'; an update with an atomic question $?p$ ought to yield the state represented in Figure 6 below.

This would of course give rise to a non-standard model, for that information state would not be a reflexive relation (there would be a connection between two worlds whose reflexive pairs had been eliminated).

These problems prompted the slightly more complex definition we gave for the implication clause (remember that $\neg\varphi = \varphi \to \bot$). The details of how it works are tedious to show, so I will ask the reader to trust it does what we intend it to do.

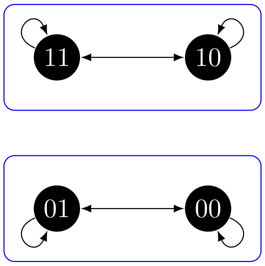

Figure 6: $\sigma[?p]$

An update with $?\varphi$ should eliminate from the state the connections between worlds that have opposing values regarding φ, thus creating two alternatives, namely, φ and $\neg\varphi$. Interestingly, this is something we can easily achieve with our semantics for disjunction. To clarify what I mean, I introduce the abbreviation

$$?\varphi = \varphi \vee \neg\varphi$$

Clearly, the state in Figure 6 corresponds precisely with $\sigma[p \vee \neg p]$: no new information is added (that is the reflexion of the 'tautological[6]' quality of a formula like $p \vee \neg p$), but two alternatives arise that correspond to the two propositions p and $\neg p$.

Since InqL has a free, standard syntax, the language includes formulas such as $?p \wedge ?q$, $p \wedge ?q$, and $p \rightarrow ?q$. The conjunctive formulas do exactly what one would expect: $?p \wedge ?q$ raises two issues, generating four alternatives (partitions in this case), while $p \wedge ?q$ introduces both the assertion that p and the question whether q is the case.

The formula $p \rightarrow ?q$ is the InqL formalization of a conditional question, such as (1), with the possible answers[7] in (2).

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[6] But such formulas are *not* strictly speaking, tautological in InqL (hence the scare quotes); for, although they introduce no new data, they create issues, and thus an update with them is all but vacuous. I will return to this in section 3, when discussing the logical properties of InqL.

[7] From a purely syntactic point of view, interrogative conditionals should theoretically be capable of getting one of two readings (or be ambiguous between them), one where the interrogation operator takes narrow scope over the consequent, and one where it takes wide scope, over the whole conditional. Although the situation with interrogation might be quite close to that of negation, inasmuch as narrow scope is probably the preferred reading for both negated and interrogative conditionals, I find that the wide scope reading is often available, and sometimes quite prominent. A good way to discern between the two readings is to check the meaning of a negative answer to an interrogative conditional. Compare (1) and the meaning of its negative answer to the example below:

(i) If I learn to play the violin, will I get a job at the BSO?
No (it's possible that you'll learn the violin and still not get the job at the BSO).

If the negative answer to (i) means what I think it does, the assertion between parentheses, then the question wasn't of the type in (1), but rather something that ought to be formalized, in a modal InqL, as $?\Box(p \rightarrow q)$, a question about the necessity of a material implication, roughly, a question about the (possibly causal) relation between the two events in the conditional. Such a formula would generate the alternatives $\Box(p \rightarrow q)$ and $\Diamond(p \wedge \neg q)$ (for this is equivalent to $\neg\Box(p \rightarrow q)$), as intended. I will disregard this reading of interrogative conditionals, on the grounds that it can only be dealt with in an inquisitive modal logic, which, although it's already in the making, is not the object of discussion in this paper. In any event, it is I believe clear that $?(p \rightarrow q)$ cannot be the correct way to model this meaning.

(1) If Jake goes to the party, will Mary also go?
$p \to ?q$

(2) a. Yes. (If Jake goes, then so does Mary.)
$p \to q$
b. No. (If Jake goes, then Mary won't go.)
$p \to \neg q$

An update with $p \to ?q$ on the minimal information state I've been using for expository purposes in this section yields the state depicted in Figure 7. As the picture shows, the update contributes no data to the information state; instead, it creates an issue with two alternatives that correspond to the propositions $p \to q$ and $p \to \neg q$, as is our intended semantics for the conditional question. Notice that this semantics for the conditional question can only be given if we drop the assumption that questions always generate partitions, and indeed we have in InqL, by specifying that the relation of indifference that is the crux of InqL models needn't be transitive. Indifference is *not* an equivalence relation, and therefore questions do not always give rise to partitioned logical spaces[8].

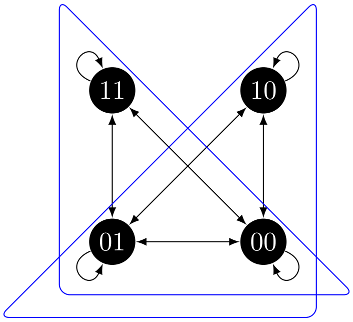

Figure 7: $\sigma[p \to ?q]$

2.3 Pros

Hopefully, I have managed to give a clear view of how InqL works at the most basic level. In particular, I have shown that InqL assigns an intuitive semantics for questions while dispensing with primitively defined question and assertion operators. By defining questions in terms of disjunction, InqL has a perfectly standard syntax and the full expressive power

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In conclusion to this short excursus, I am convinced that interrogative conditionals deserve further investigation, but I will assume here that the reading spelled out in (1) is the preferred reading for conditional questions. In particular, I will considering that answering (1) by uttering "Jake isn't going to the party" is doing more than just addressing the issue at hand, since it gives more information than what was requested of the responder. As a consequence, negating the antecedent ought not to be an alternative generated by this kind of conditional questions (see Velissaratou, 2000, for a discussion).

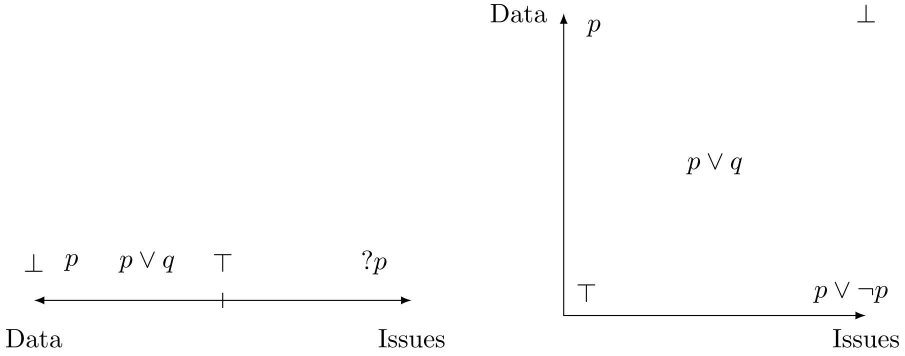
[8] The idea that questions needn't necessarily partition the logical space was the guiding premise for the unpublished Groenendijk (2007), which gave a semantics with the same results as this inquisitive semantics for conditional questions. That system had a somewhat restricted syntax, two categories of formulas and several semantic clauses, for the question operator was defined as a primitive and disjunction was classical. That work was superseded by our discovery of inquisitive semantics and InqL.

of an erotetic logic. I have also illustrated how questions that (arguably) give rise to non-partitioned states, such as the conditional question, can be straightforwardly modeled in this system, which rejects the stipulation that questions must correspond to partitions of the logical space.

InqL has, I believe, many strengths. I will address some in the remarks that follow.

2.3.1 Two dimensions

Standard erotetic logics, as I mentioned already, typically use primitively defined question operators. This, I believe, gives rise to a flat, one-dimensional view of questions and assertions for, in those approaches, the building blocks of formulas are exclusively informative and exclusively inquisitive subformulas. Data and issues are kept separate at the level of semantic definitions, only arising in some of those logics in the form of hybrid formulas, which are typically restricted by the syntax.



(a) Classical view (b) InqL view

Figure 8: Two views on data and issues

In InqL, however, we achieve the intended expressive power by choosing the hybrid quality of natural language disjunction as our starting point and defining questions in terms of it. This gives us a two-dimensional view on informativeness and inquisitiveness. The dichotomy between these two views is pictorially represented in Figure 8 (which is not to be taken too seriously...).

This two-dimensional perspective on the informative and inquisitive power of InqL formulas confers to it an important property: there is no primitive clear distinction, syntactic or semantic, between assertions and questions. Naturally, a distinction can be defined over the syntactic and semantic primitives we gave; for example, one can define a question as a formula whose update potential raises issues but eliminates no worlds. Under this definition, $?p$ (and remember that this is merely an abbreviation for $p \vee \neg p$), $p \rightarrow ?q$, $?p \wedge ?q$ would all be questions, whereas p, $p \wedge ?q$, $p \vee q$ would not.

But is such a unifying approach to questions and assertions desirable? Aren't there clear differences, in natural language, between the two speech act types, visible also at the syntactic level? If so, ought we not to mirror those differences in our semantics? Yes and no.

It is important to notice that InqL as it stands at the moment is *not* a logic to be used out-of-the-box for model-theoretic translation of natural language *sentences*. The hypothesis is, rather, that the general guiding premises of InqL, the assumptions it incorporates, are interesting suggestions concerning what's behind certain natural language *meanings*. In particular, I am not claiming that, at a surface syntactic level, interrogation can be straightforwardly identified with disjunction[9]. That being said, I am convinced that the hypotheses I am presenting here are indeed worth investigating.

The idea that the meaning of a natural language disjunction isn't merely propositional is by no means new to inquisitive semantics. In so-called Hamblin semantics for disjunction, the *or* operator is not identified with the Boolean join, which produces a simple union of propositions; rather, it takes a number of propositions and yields an alternative set, a set of propositions, much like the interpretation InqL assigns to a formula like $p \vee q$. Interestingly, this kind of meaning is rather close to that of questions, since the classical view takes them to be alternative sets as well, although with different characteristics (more about this in the next subsection). The inquisitive semantics program suggests that this is not a coincidence, that the alternative sets generated by disjunctions and questions are actually two instances of application of the same basic mechanism, which reduces to disjunction.

Moreover, inquisitive semantics, with its two-dimensional perspective on questions and assertions, avoids the type-theoretical problems that arose from operations on questions, in the classical view. In an inquisitive semantics, there is no difference in type between an assertive and a question meaning: the same types of structures model both kinds of meanings.

Finally, as a corollary of this idea, inquisitive semantics as a semantics for natural language disjunction solves a long standing type-theoretical problem in Hamblin semantics for disjunction, a point I will return to at the of the next subsection.

2.3.2 The issue raised by disjunction

At the crux of the expressive power of InqL lies, as I've stressed before, the issue-raising potential of disjunction. This idea is intuitive, I believe, in more than one sense.

(3) A: John or Mary are coming over tonight.
B: Well, which one?

The dialog in (3) sounds quite natural to my ears, and it illustrates an instance where a use of disjunction in an indicative sentence seems to create an issue. Even if B weren't to ask the question "which one?" directly, a natural way for this dialog to proceed would be to address that underlying question. A reading where (3-A) is purely informative seems rather unnatural, at least with a neutral intonation pattern.

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[9] But the issue deserves a lot of attention. Anna Szabolcsi pointed out to me that a number of East Asian, South Asian and Slavic languages use the same morpheme for disjunction, interrogation and existential quantification (see the remark concerning predicate logic in section 4). For example, in Korean, *na* is a disjunctive suffix and it occurs as part of the interrogative complementizer and of the existential quantifier; similarly for Japanese *ka* (Szabolcsi, p.c.). These morphosyntactic facts are quite exciting for the inquisitive semantics program. The claim I am making is, for the time being, that at some abstract semantic level these three natural language operations share a common mechanism, that of introducing alternatives, and this empirical syntactic and morphological data that corroborate that connection are of the utmost interest. For now, I can only note these facts and the importance of further investigation.

If (3-A) is to be formalized as $p \vee q$, its purely informative cousin is, as we saw above, $!(p \vee q)$. A more natural example of these two flavors of disjunction can be seen in the following:

(4) A: Do you want **coffee** or **tea**?
B: Coffee, please.
B: Tea, please.
B: Neither(, actually).
B: % Yes.
B: % Yes, coffee.

(5) A: Do you want **coffee or tea**?
B: Yes / Sure.
B: (%) Yes, coffee.
B: % Coffee, please.

In (4) and (5), A is asking two different questions (a fact reflected in the intonation patterns): (4) is an alternative question, and (5) a polar, *yes-no* question. Notice that, when B answers A's question in (5) by saying "Yes, coffee", B is giving more information than required. In particular, a *yes* answer is required first, as the contrast with "Coffee, please" shows (without B nodding or otherwise explicitly indicating a simple affirmative answer).

Consider now the following two InqL formulas and the alternative sets they generate:

(6) $?(p \vee q)$
Alternative set $= \{p, q, \neg(p \vee q)\}$

(7) $?!(p \vee q)$
Alternative set $= \{!(p \vee q), \neg(p \vee q)\}$

The two updates are questions, in the sense that they are purely inquisitive, and they seem to correspond quite intuitively to the natural language questions in (4) and (5), respectively. In particular, they generate the appropriate alternative sets[10].

The examples in (6) and (7) further illustrate how the alternatives generated by a simple disjunction contribute to the meaning of an alternative question like the one in (4). The three possible answers could only be generated because of the inquisitive quality present in simple disjunction. If we override disjunctive inquisitiveness, as in (7), by double=negating[11] a disjunction, we get the often unnatural, 'logician's' interpretation of a disjunction ("Shall

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[10] It is of course an open issue whether the *neither* answer to the question in (4) has the same status as the *coffee* and *tea* answers. Many people find it unnatural without 'actually', which might indicate that it is not a complete answer *per se*, in that it rejects a presupposition. I am not at all convinced that the presupposition *that* $p \vee q$ is part of the meaning of the alternative question, but I certainly understand and respect the arguments of those who do. In any event, I won't discuss the issue here. [Incidentally, Jeroen Groenendijk is giving a talk at Rutgers in March, I believe, where, among many other things, he derives the result that the *neither* answer is to be excluded as an implicature, arising from Gricean reasoning over alternative inquisitive candidates.]

[11] Notice that, as Han and Romero (2001) point out, simple negation, when it takes wide scope over disjunction, also bars the alternative question interpretation:

(i) Don't you want coffee or tea?

we go to the movies or the theater? Yes.")[12].

In addition, the alternative-generating potential we see in disjunctions can be seen as an implementation of so-called Hamblin semantics for disjunction (see, to give but one, recent reference, Alonso-Ovalle, 2006), as I mentioned in the previous subsection. The intuitive idea behind these is that natural language disjunction isn't the simple Boolean join of two propositions, rather, it yields a set of alternative propositions. This idea has proven both interesting and fruitful in dealing with the free choice problem, the exclusiveness implicature and other non-standard inferences legitimized by NL disjunction.

InqL is something of an alternative semantics in this sense as well, in that the essential premise of Hamblin semantics, namely, that disjuncts be accessible to higher operators, is to some extent implemented in InqL[13]. Interestingly, Hamblin semantics for disjunction all face a tough problem having to do with type-lifting that InqL as a semantics for disjunction completely avoids.

The purpose of having disjunctions generate alternative sets, in the Hamblin semantics I'm considering here, is to make sure each disjunct is accessible for further semantic and pragmatic computations. This is not only defended for contexts of modal subordination, counterfactual antecedents and so on, but also for the simplest indicative cases, for reasons of systematicity, of course, but also because deriving exclusiveness implicatures in the semantic component is often a part of these Hamblin semantics enterprises. Naturally, a problem arises with a sentence as simple as (8), when compared to (9).

(8) Stan or Dan bought a house.
$\{B(s), B(d)\}$

(9) Stan and Dan bought a house.
$B(s) \wedge B(d)$

Let me be conveniently informal and represent "a bought a house" as $B(a)$, also abstracting away from the contrast between collective and distributive reading. The point I want to make is that, in Hamblin semantics for disjunction, (8) is a set of propositions, whereas (9) is a proposition. This distinction in type forces the proponents of similar analyses to

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Clearly, this is a *yes-no* question, as is indeed predicted by InqL, for $?\neg(p \vee q)$ has the same possible answers as $?!(p \vee q)$, and not those of $?(p \vee q)$.

[12] I have recently learned that the joke works the other way around too. I hope the reader familiar with the original anecdote will forgive me, since I can't recall who exactly its protagonist was, but there is the story of an important intellectual being interrogated during the McCarthy era: when asked "do you believe that the American people ought to be allowed to advocate overthrowing the US government by force or other violent means?", the person under interrogation thought for a while, and then replied "By force."

[13] These reticent provisos ('something of an alternative semantics', 'to some extend implemented') are justified by the fact that InqL has a weaker expressive power than that which is usually assumed for Hamblin semantics. More precisely, InqL generates disjunctive alternatives up to logical entailment, which is not a restriction implemented upon any of the Hamblin semantics I am familiar with. Consider the sentence below:

(i) Phillip lives in Massachusetts or Boston.

The alternative semantics I am referring to would assign two alternatives to (i), while in InqL we would have only one, namely the weaker "Phillip lives in Massachusetts", for the other proposition asymmetrically entails this one. This is a consequence of the InqL system which, at least in this case, seems to be an advantage over other alternative semantics for disjunction, insofar as the sentence in (i) is not a felicitous use of NL disjunction — InqL correctly prevents it from generating the alternatives it intends to.

postulate an existential operator scoping over the alternative set in (8)— meaning something like "one of the propositions in this set is true"—, which can either be done by arguing that it is syntactically invisible, or stipulating a principle of existential closure on bare alternative sets. In any event, the type distinction between (8) and (9) is at the very least hard to motivate from a purely intuitive point of view and, interestingly, it quite simply doesn't arise in an inquisitive semantics. The update potentials of these two sentences, in InqL, are no different in terms of their types, and the states they yield are perfectly compatible and comparable: no principle of existential closure is necessary. Crucially, however, the disjuncts in (8) are still accessible to an inquisitive semantics, since disjunction raises issues.

Research on the viability of InqL as an alternative semantics for disjunction is part of my masters thesis, but the work is at too early a stage for discussion.

2.4 Cons?

At this early stage of development, and assuming that the points discussed in the previous paragraphs count as virtues of InqL, rather than fundamentally flawed hypotheses, it is hard to discern between substantial problems of InqL and simply issues that haven't been addressed yet, but can eventually be solved. I briefly mention here only one issue of InqL that may turn out to be an unsolvable problem, that of overgeneration.

Its free syntax is what allows InqL to be such a simple and elegant erotetic logic, and yet powerful enough to account for non-partitioning questions, issue-raising disjunctions, and so on. However, this free syntax also gives rise to such oddities as the following, which I introduce roughly in ascending order of awkwardness:

1. $\neg ?p$
2. $!?p$
3. $??p$
4. $?p \vee ?q$
5. $?p \rightarrow q$
6. $?p \rightarrow ?q$

Naturally, the odd quality of these formulas only arises under the interpretation of $\varphi \vee \neg\varphi$ as a question. That interpretation, although it is not a primitive in our semantics, is of course essential to establish the viability of InqL as an erotetic logic, so it cannot be disregarded at will.

The first three formulas are instances of negation of questions and of iteration of the question operator. Now, since the question operator is not a primitive, these boil down to, respectively, $\neg(p \vee \neg p)$, $\neg\neg(p \vee \neg p)$ and $p \vee \neg p$. Thus, the negation of an atomic question is equivalent to $\bot$, the double negation of an atomic question to $\top$, and the iteration of questions has no effect. Notice that, while these formulas all get a semantic interpretation, it not at all clear how that interpretation is intuitive. The negation of a question is completely absent from natural language, and, granted, assigning to it the interpretation of the contradiction may be considered acceptable. But how is the assertion of a question a tautology? Intuitively, it ought to be just as contradictory as the simple negation of a question. The same remark applies to the iteration of questions in 3.

The last three formulas above are combinations of questions via disjunction and implication[14]. Their intuitive readings are far from obvious, in some cases, and downright unimaginable in others. Case 4. above, however, deserves special attention.

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[14] Although this is a highly programmatic draft, I hope I am succeeding in keeping purely speculative remarks out of it. However, I ask the reader to allow me to indulge in a short digression about a formula in this list that I find particularly puzzling.

Szabolcsi (1997) argues that disjunctions of questions in the strict sense are in general absent from natural languages. In English, for example, sentences like "Whom did you marry or where do you live," where *or* is meant to operate at the inter-sentential level, are extremely odd, to say the least. Moreover, Hungarian requires that one add a word to the effect of 'rather' or 'instead', suggesting that disjunctions of questions aren't really choice questions, as one might have predicted, but rather "an idiomatic device that allows one to cancel the first question and replace it with the second" (*ibid*). Things are quite different concerning conjunctions of questions, which are predicted to be available in all languages, and certainly seem to operate at an inter-sentential level.

Now, the formula in 4., an InqL disjunction of questions, corresponds to a true choice question, in the sense that, if we again consider a minimal state with two propositional atoms, $\sigma[?p \vee ?q]$ corresponds to the state represented in Figure 9. This state has alternatives p, q, $\neg p$ and $\neg q$, as one would expect from a choice question. The conclusion to draw here is that InqL, in that it allows disjunctions of questions,is prey to the same comments Szabolcsi (1997) makes about Groenendijk and Stokhof (1984), namely, that these systems give an intuitive semantics for the notion of a disjunction of questions, but, very likely, there is no natural language construction that directly encodes that meaning. Clearly, Szabolcsi doesn't want to exclude that meaning from a semantic of questions, just the idea that it is to be assigned to sentences like "Whom did you marry or where do you live."

I have discussed in this sections some examples of what might be overgeneration in InqL. Although I have argued that we can make some sense of these strange formulas, this

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The formula $?p \to q$ is *prima facie* quite hard to make sense of. What might an interrogative antecedent possibly mean? Although I will of course agree that syntactically interrogative antecedents are impossible in natural languages, I find it a pertinent question whether we can be sure that a semantically interrogative antecedent is also completely barred. I will make the (outlandish?) suggestion that perhaps it is not. Consider an Austin conditional (aka biscuit conditionals) like (i):

(i) If you're hungry, there's pizza in the fridge.

The standard view on (i) is that the antecedent establishes a condition on relevance: if you are indeed hungry, then the following is a relevant true statement: there's pizza in the fridge. Now, relevance is of course tied to relatedness, which according to most accounts ought to be analyzed with mechanisms that compare sentences to *questions under discussion*. Thus, at a relevant heuristic level, (i) can be seen to correspond to an InqL formula such as $?p \to q$, meaning something like "If p is an issue, then q."

Notice also that,, while it might be tempting to argue that relatedness is not the relevant notion here, and that (i) incorporates a relevance condition that is exclusively connected to the proposition "you are hungry," and not to the question thereof, this is by no means obviously true. Granted, should the situation be such that the hearer has explicitly stated that he or she is *not* hungry, the consequent in (i) is no longer relevant. However, it seems to me that the mere presence of the issue "whether you are hungry" is enough to make the consequent, besides of course the whole conditional, quite felicitous.

(ii) A: I don't quite know if I'll be hungry before you arrive, I may be.
B: Well, (if you get hungry,) there's pizza in the fridge.

Now, for the pièce de résistence: $?p \to q$, in InqL, is actually equivalent to just q, for the obvious reason that $?p = p \vee \neg p$ and thus, at the relevant level of analysis, we have a tautological antecedent.

I am of course being facetious when I refer to this quirky property of $?p \to q$ as a strong point in favor of this analysis for Austin conditionals. The equivalence with q is a mere consequence of the idea that questions reduce to disjunction, and it is certainly not insightful as to what's going on with actual natural language mechanisms, at least not *per se*. At any rate, if my suggestion that Austin conditionals might have interrogative antecedents isn't completely ludicrous, it is at the very least interesting that InqL can, at a low semantic level, assign to them their intuitive meaning.

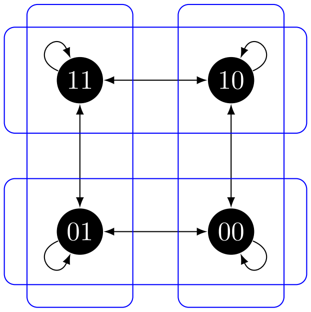

Figure 9: $\sigma[?p \vee ?q]$

discussion certainly begs the question whether we should impose some restrictions on our syntax, or rather there should be some other kind of non-semantic filter on what counts as a NL question. For the time being, all I can say is that I am strongly inclined towards the latter hypothesis. The logical elegance of InqL is at the core of its interest as an erotetic logic with many applications; restricting its syntax would entail, among other things, the loss of many of the properties we will consider in the next section.

3 Some remarks on the logic InqL

So far we have been concentrating on the expressive power of the Inquisitive Semantics defined in Section 1; we have seen that this semantics gives a meaning to disjunction that is close in spirit to the one defended by alternative semantics *à la* Hamblin, and we have taken that premise a step further, by defining the question operator in terms of disjunction. We will turn now to some logical properties of InqL as a logic in the technical sense, that is, a set of sentences closed under logical consequence and uniform substitution[15]. In what follows, I will use the classical, non-update semantics for InqL given in footnote 1. Please refer back to it when checking the proofs and so on.

First of all, we must define 'logical consequence', via the notion of entailment. In InqL, $\Gamma \vDash_{\mathsf{InqL}} \varphi$ iff, for all (i, j) that model all formulas in Γ, $(i, j) \vDash_{\mathsf{InqL}} \varphi$. In the case where $\Gamma = \emptyset$, I write $\vDash_{\mathsf{InqL}} \varphi$ (read φ is a tautology of InqL), and mean simply that all (i, j) model φ. I will omit subscripts on the entailment relation when the logic I'm referring to is obvious from the context.

In a sense, InqL is more standard than most logics that intend to model assertions and questions, for it has a deduction theorem:

Theorem 5 (Deduction theorem for InqL). *$\varphi \vDash \psi$ iff $\vDash \varphi \to \psi$.*

Proof. Immediate from the definitions of entailment and implication in the object language (cf. footnote 1). ☐

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[15] The discussion in this section is more than just an exercise in pure logic for one's own private gratification. Indeed, if InqL turns out to be effective in modeling questions and assertions, then it has potential applications in other fields, most notably, in the realm of computer science (e.g., as a logic for reasoning about database queries). If that is the case, then a knowledge of the more logical properties of InqL will be extremely important.

This important result saves us the trouble of having to define entailment under specific axioms[16], and it is, of course, a consequence of the fact that InqL makes no syntactic distinction between assertive and interrogative sentences.

Since we now have a notion of logical consequence, we can consider the logic InqL, as said above, as the set of all sentences that are tautologies of the inquisitive semantics defined in the beginning of this paper, closed under consequence and uniform substitution. But what logic is that?

Clearly, InqL is *not* classical propositional logic (**CPL**). This fact can be shown quite simply by pointing out a counterexample to the **CPL** tautology $\neg\neg\varphi \to \varphi$: if we consider a disjunction like $p \vee q$, we see that the model in Figure 5 (page 6) supports $\neg\neg(p \vee q)$ but not $p \vee q$, and thus we have shown that, in InqL, the rule of double negation elimination is not in general valid. Equivalently, we may also say that the principle of excluded middle is not valid in InqL, that is, $\vDash_{\mathbf{CPC}} \varphi \vee \neg\varphi$ but $\nvDash_{\mathsf{InqL}} \varphi \vee \neg\varphi$. A counterexample to the latter can be seen in the model I introduced for $p \vee \neg p$, in Figure 6, where it is clear that an update with this formula is by no means vacuous.

We might now conjecture that InqL is intuitionistic propositional logic (**IPL**). Even though the semantics for InqL is not at all similar to that of **IPL**, its tautologies might at first glance seem to coincide. However, that is also false, as can be easily shown by giving a tautology of InqL that is not a tautology of **IPL**. An example of such a formula is $\neg\neg p \to p$, for any propositional letter p. We first show that this is a tautology of InqL, by giving a semantic proof.

Lemma 6 (Atomic double negation). *For all propositional letters p, $\neg\neg p \to p$ is a tautology of* InqL.

Proof. It suffices to show that, for any pair of worlds (i, j) and an arbitrary propositional letter p, $(i, j) \vDash \neg\neg p \to p$. By definition of implication, this is equivalent to the conjunction of the following:

$$\begin{aligned} (i,i) \vDash \neg\neg p &\implies (i,i) \vDash p \\ (j,j) \vDash \neg\neg p &\implies (j,j) \vDash p \\ (i,j) \vDash \neg\neg p &\implies (i,j) \vDash p \end{aligned}$$

We begin with the third conjunct. $(i, j) \vDash \neg\neg p$ iff $(i, i) \vDash p$ and $(j, j) \vDash p$, which in turn is equivalent to $i(p) = j(p) = 1$. By definition, this means $(i, j) \vDash p$, and we're done. The two reflexive points (conjuncts one and two above) follow immediately from the definitions. □

That $\neg\neg p \to p$ is not a tautology of **IPL** is more than obvious: consider a Kripke model with two nodes, w_0 and w_1, such that $w_0 \nVdash p$, $w_1 \Vdash p$, and $w_0 R w_1$. Clearly, $w_0 \Vdash \neg\neg p$, but $w_0 \nVdash p$, and we have shown the existence of a countermodel. Thus, $\mathsf{InqL} \neq \mathbf{IPL}$.

It is extremely tedious, although trivial, to show that $\mathbf{IPL} \subseteq \mathsf{InqL}$: we have to take a Hilbert-style axiomatization of **IPL** and show that all axioms are true in InqL, or, alternatively, check that all the rules in a Gentzen natural deduction system for **IPL** preserve truth in InqL. I won't reproduce this tedious procedure here, but it should be intuitively clear that indeed **IPL** is included in InqL.

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[16] As, incidentally, had to be done for the system in Groenendijk (1999), by ten Cate and Shan (2007), since that logic lacked a deduction theorem.

These few but important results establish the following inclusion relations (NB, these are proper inclusions):

$$\mathbf{IPL} \subset \mathsf{InqL} \subset \mathbf{CPL}$$

that is, InqL is an intermediate logic[17] (also known as superintuitionistic logics), properly extending **IPL**, and properly included in **CPL**.

Intermediate logics can be complete for a relevant class of Kripke models[18]. As for finding out which class of frames or models InqL might be complete for, that is an extremely difficult question, very likely an impossibly difficult problem.Jankov has shown that there are continuum many logics between **IPL** and **CPL**, and we know precious little about where InqL is in that lattice. I have inspected a number of famous intermediate logics, and InqL turned out not to correspond to any of them. Indeed, in all likelihood, the logic is brand new, which, given the sheer amount of intermediate logics, is not at all surprising.

This is not to say it is pointless to continue our logical investigation of InqL. Finding out what class of models InqL is complete for would be a pretty result, but ultimately useless, both for the semanticist and the computer scientist.

Aside from the validity of atomic double negation (Lemma 6), there is another non-intuitionistic tautology of InqL that plays an important role, namely, the independence of premise axiom (IP), also known as the Kreisel-Putnam axiom[19]. The axiom (IP) is as follows (the right-to-left direction of this axiom is actually an **IPL** tautology, so the interesting part is the direction in (IP) below):

$$\text{(IP)} \qquad (\neg\varphi \to \psi \vee \theta) \to (\neg\varphi \to \psi) \vee (\neg\varphi \to \theta)$$

In InqL, we can read a disjunction as a request for a choice to be made between the disjuncts. As such, (IP) intuitively says that, if a choice between ψ and θ depends on the satisfaction of some assertive premise, then the choice can be pulled up a level. In other words, if I am asked to assume a premise $\neg\varphi$ and *then* choose between ψ and θ, I might as well choose *now* between whether the premise will lead me to ψ or θ. Time and space prevent me from giving a proof of (IP) in InqL (another long and tedious one), so I will ask the reader to grant me that it is indeed InqL-valid. This result prompts two important remarks.

Firstly, it should be noted that the negation in the subformula $\neg\varphi$ in (IP) is absolutely crucial. If I may briefly return to the semantics behind InqL in order to illustrate why, let me remind the reader that negated formulas are necessarily assertions. Thus, (IP) states that a request for a choice inside an implication can be pulled up a level under the condition that the premise of the implication is assertive. The case where the premise is not purely assertive (i.e., it is inquisitive as well, or perhaps purely inquisitive) is not contemplated in

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[17] For a (readable) discussion of several intermediate logics, see Chagrov and Zakharyaschev (1997). Concerning **IPL**, the Kripke semantics for **IPL** and also a short discussion of intermediate logics, van Dalen (1986)

[18] **IPL** is complete for finite Kripke models over trees and **CPL** for the single reflexive point.

[19] The intermediate logic **KP** is axiomatized by precisely this axiom (IP). InqL is not **KP**, in fact, $\mathbf{KP} \subset \mathsf{InqL}$, for the InqL tautologies established in Lemma 6 aren't valid in **KP**. In any event, it is interesting to remark, in connection to my conjecture about the high complexity of the problem of what class of Kripke models InqL might be complete for, that **KP** is complete for a class of models satisfying a rather complex condition on the accessibility relation (result due to Gabbay).

(IP). In particular, the following are *not* tautologies of InqL:

$$(p \vee q \to r \vee s) \to (p \vee q \to r) \vee (p \vee q \to s)$$
$$(?p \to ?q) \to (?p \to q) \vee (?p \to \neg q)$$

Countermodels to these formulas are easy to find, but I will omit them from this discussion for the sake of brevity.

Second, (IP) is actually an admissible rule of **IPL**, that is, when lifted to the meta-level, it expresses a true statement about **IPL**, namely, that whenever we have $\neg\varphi \to \psi \vee \theta$, we must always have one of $\neg\varphi \to \psi$ or $\neg\varphi \to \theta$. This **IPL** rule, however, doesn't correspond to an **IPL** theorem, for (IP) is not a tautology of **IPL**. The existence of such admissible rules is a property of intuitionistic logic and intermediate systems. It is important to notice, however, that (IP) is actually a theorem (a tautology) of InqL, not just an admissible rule.

Let us sum up what we have so far. I have shown that InqL is an intermediate logic, strictly between **CPL** and **IPL**, and I have pointed out two theorems of InqL that are not theorems of **IPL**, namely, atomic double negation (ADN) and (IP), repeated below. I call these InqL theorems axioms of InqL, for they (at least partially) characterize[20] the more than intuitionistic status of InqL.

(ADN) $\neg\neg p \to p$ for all propositional atoms p
(IP) $(\neg\varphi \to \psi \vee \theta) \to (\neg\varphi \to \psi) \vee (\neg\varphi \to \theta)$

These two axioms are instrumental in proving two important results, given as Theorems 7 and 9.

Theorem 7. *For all φ free of disjunction, $\vDash \neg\neg\varphi \to \varphi$ (that is, double negation elimination is valid in the disjunction-free fragment of* InqL*)*

Proof. By induction on φ. The base case follows immediately from axiom (ADN), and it suffices to do the induction step for conjunction, implication and negation.

For $\varphi = \psi \wedge \theta$, assume we have $\vDash \neg\neg(\psi \wedge \theta)$, to show that $\vDash \psi \wedge \theta$. It can be shown by a simple induction that double negation can be pushed inside conjunction (i.e., $\neg\neg(\varphi \wedge \psi) \leftrightarrow \neg\neg\varphi \wedge \neg\neg\psi$; this is actually a property of **IPL** as well), and we get $\vDash \neg\neg\psi \wedge \neg\neg\theta$. We now apply the induction hypothesis to the two conjuncts, and we're done. Implication, for which double negation can also be pushed in and out, follows similarly.

Finally, the negation step is immediate from the theorem (also valid in **IPL**) $\neg\neg\neg\varphi \to \neg\varphi$. ☐

Theorem 7 gives us a very simple characterization of assertiveness in the logic, namely, a formula φ is purely assertive iff $\vDash \varphi \leftrightarrow \neg\neg\varphi$. Moreover, it has as one of its corollaries that the assertive fragment of InqL is classical (a trivial proof since, by definition, the assertive fragment if InqL is that in which double negation elimination is in general valid).

Finally, I will conclude with a brief discussion of alternative disjunctive normal forms (ADNFs).

IPL doesn't have a useful notion of normal forms as does **CPL**, due to the fact that **IPL** connectives are strongly independent. In InqL, independence of connectives is somewhat

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[20] The exact axiomatization of InqL as an extension of **IPL** is work in progress. The axioms (ADN) and (IP) are clearly essential in the process, but I am yet to finish the completeness proof.

weaker than in **IPL**, and there is a meaningful sense in which connectives can be interdefined, under certain restrictions[21] which I won't go into here. Very importantly, we can prove the existence of a kind of normal form, call it the alternative disjunctive normal form (ADNF), as defined below.

Definition 8. φ is a ADNF iff $\varphi = \bigvee\!\!\!\bigvee_{i<n} \varphi_i$, where each φ_i is formed exclusively with $\{\wedge, \neg\}$, i.e., each φ_i is an assertion.

That is, an ADNF is a disjunction of assertions. Intuitively, ADNFs are syntactically readable characterizations of what their semantics amounts to, in the sense that the disjuncts in an ADNF correspond to the alternatives they (have the potential to) generate in an information state[22].

Thus, the ADNF of a formula such as $p \wedge q$ is simply $p \wedge q$. This conjunction is not inquisitive and therefore it creates a single alternative $p \wedge q$. $p \to ?q$, however, is to be assigned the ADNF $(p \to q) \vee (p \to \neg q)$, for this disjunction corresponds to the two alternatives created by it. I conclude this section with Theorem 9, which proves the existence of ADNFs as defined in 8 for every formula of InqL (but see footnote 22).

Theorem 9. *For every φ in the language, there is an ADNF $\varphi^{\vee !}$ such that $\varphi \Leftrightarrow \varphi^{\vee !}$.*

Proof. We proceed by induction on the complexity of φ.

1. φ is atomic. Then $\varphi^{\vee !} = \varphi$.

2. $\varphi = \psi \vee \theta$. By induction hypothesis, there are $\psi^{\vee !}$ and $\theta^{\vee !}$, so set $\varphi^{\vee !} = \psi^{\vee !} \vee \theta^{\vee !}$.

3. $\varphi = \psi \wedge \theta$. By IH, we have $\psi^{\vee !}$ and $\theta^{\vee !}$. Since the law of distributivity holds, we get

   $$\varphi \Leftrightarrow \psi \wedge \theta \Leftrightarrow \psi^{\vee !} \wedge \theta^{\vee !} \Leftrightarrow \bigvee\!\!\!\bigvee_{i,j} (\psi_i \wedge \theta_j)$$

   The latter formula is clearly an ADNF, so we're done.

4. $\varphi = \neg\psi$. We assume we have $\psi^{\vee !} = \bigvee\!\!\!\bigvee_{i<n} \psi_i$. The first De Morgan law holds $(\neg(\sigma \vee \chi) \leftrightarrow \neg\sigma \wedge \neg\chi)$. So, we can set

   $$\varphi^{\vee !} = \bigwedge\!\!\!\bigwedge_{i<n} \neg\psi_i$$

   Since each ψ_i only contains atoms, $\neg$ and $\wedge$, we can eliminate double negations (Theorem 7). $\varphi^{\vee !}$ is then a trivial ADNF.

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[21] I can't pursue the issue here, but let me remark that one of the first important results concerning InqL was that of functional completeness of disjunction, negation and conjunction, thus dispensing with implication. I have shown that these connectives (and other combinations) are functionally complete for the inquisitive semantics defined here, which means, intuitively, that all we need to express an inquisitive semantics is an inquisitive connective (disjunction), an assertive one (conjunction) and negation.

[22] Strictly speaking, the algorithm I introduce in Theorem 9 is not meant to accomplish exactly this. Theorem 9 merely states the existence of ADNFs in general, where some disjuncts may possibly be redundant. It is however quite simple to refine the algorithm so as to assure no disjunct appears in an ADNF that is entailed by another disjunct in the same ADNF, which gives us the intended result. For simplicity's sake, I will omit this refinement from the proof.

5. $\varphi = \psi \to \theta$. By the IH, we have $\psi^{\vee !}$ and $\theta^{\vee !}$. Now, $(\varphi \vee \psi \to \theta) \leftrightarrow ((\varphi \to \theta) \wedge (\psi \to \theta))$ is an **IPL** tautology, and thus it must also hold in InqL, so we get the equivalence below.

$$\psi^{\vee !} \to \theta^{\vee !} \Leftrightarrow \bigwedge_{i<n} (\psi_i \to \bigvee_{j<m} \theta_j)$$

Since all ψ_i are assertions, they are equivalent with a double negated formula $\neg\neg\psi_i$ (Theorem 7), so we can apply the axiom (IP) to pull out disjunctions.

$$\bigwedge_{i<n} (\psi_i \to \bigvee_{j<m} \theta_j) \Leftrightarrow \bigwedge_{i<n} \left(\bigvee_{j<m} (\psi_i \to \theta_j) \right)$$

Each implication is purely assertive, so we can eliminate it in terms of conjunction, via $(\varphi \to \psi) \leftrightarrow \neg(\varphi \wedge \neg\psi)$ and get

$$\bigwedge_{i<n} \left(\bigvee_{j<m} (\psi_i \to \theta_j) \right) \Leftrightarrow \bigwedge_{i<n} \left(\bigvee_{j<m} \neg(\psi_i \wedge \neg\theta_j) \right)$$

$\bigwedge_{i<n} (\bigvee_{j<m} \neg(\psi_i \wedge \neg\theta_j))$ is clearly a conjunction of ADNFs, so we can apply the law of distributivity, as in 3. above, to each pair of conjuncts. This concludes the → step. □

4 Concluding remarks

This paper has served, I hope, as a clear overview of the most basic aspects of InqL, the way it functions, its advantages, and the most important hypotheses behind it. InqL is, as I mentioned several times, work in progress, and I will conclude this paper with a brief note on three of the aspects of InqL that I am concentrating on at this moment.

Logic A sound and complete axiomatization for InqL is an important achievement in the process of making it more widely available for use as an erotetic logic. Without having worked out this aspect of the proof-theory, applications in computer science are a mere promise.

Cross-linguistic evidence A crucial part of the inquisitive program outlined here is to investigate the linguistic consequences of the inquisitive triad *disjunction*, *interrogation*, *existential quantification*, as I briefly discussed in footnote 9. I have only just begun to pursue this line of research, so for now I can add nothing more to the short discussion above.

Alternative semantics for disjunction As I mentioned in footnote 13, the expressive power of InqL as an alternative semantics for disjunction is weaker than that which is usually assumed (see for example Alonso-Ovalle, 2006). This of course begs the question whether InqL can do the trick as it stands, or whether expressive power ought to be added. The latter might be undesirable, for it would carry through to our semantics of questions as well[23].

Predicate logic Evidently, InqL, as a propositional logic, is much too coarse to deal with many of the most interesting facets of interrogation; an inquisitive predicate logic is needed. The first drafts of the semantics are very promising: the definition of the question operator in terms of disjunction carries through to the predicate logic, and, in addition, the existential quantifier is straightforwardly imbued with an inquisitive flavor, much like disjunction. This gives us an inquisitive reading of existential statements, which assigns to a sentence like $\exists x K(x,f)$, ("There is a king of France") an interpretation that not only asserts the existence of an entity, it also generates an issue as to which x is indeed meant. This can be seen to correspond to a specific reading of the quantifier, as opposed to its double-negated, non-inquisitive, non-specific reading.

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[23] Aloni (2007) began to design a system that is meant to deal with alternative semantics and erotetic logics. Aloni's logic was an independent discovery, and indeed her enterprise has different objectives from mine and Jeroen Groenendijk's. For example, she doesn't intend to unify disjunctive and interrogative alternatives, maintains the stipulation that the former must partition the logical space, and proposes an extremely complex logical language, with quantification over propositions in the object language and a dynamic character that is absent from InqL.

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