

# **Job Turnover and Policy Evaluation: A General Equilibrium Analysis**

Hugo Hopenhayn and Richard Rogerson

Prof. Sargent's Reading Group

# Motivation

- Hopenhayn and Rogerson extend the model of industry dynamics in Hopenhayn (1992) to a general equilibrium setting
- The aim is to study the implications of government policies that *"..make it costly for firms to adjust their employment level"*
- The focus will be on the characterization of a stationary equilibrium for the economy
- The main result of the paper is that it is very costly to distort the job creation/destruction process

# The Model: Technology

- $\pi_t = p_t f(n_t, s_t) - n_t - p_t c_f - g(n_t, n_{t-1})$
- $s$  is a firm specific production shock that follows a first-order markov process described by  $F(s, s')$
- $g(n_t, n_{t-1}) = \tau \max(0, n_{t-1} - n_t)$ ,  $\tau$  is the fixed payment for each job destroyed
- There is an infinite supply of potential entrants at each time  $t$ . They draw the initial  $s$  from a distribution  $v$
- $f(n_t, s_t)$  is continuously differentiable and strictly concave in  $n$  for each  $s$  and  $\lim_{n \rightarrow 0} f_1(n, s) = \infty$
- $F(s, s')$  is continuous and decreasing in its first argument and  $v$  has a continuous C.D.F.

# The Model: Timing

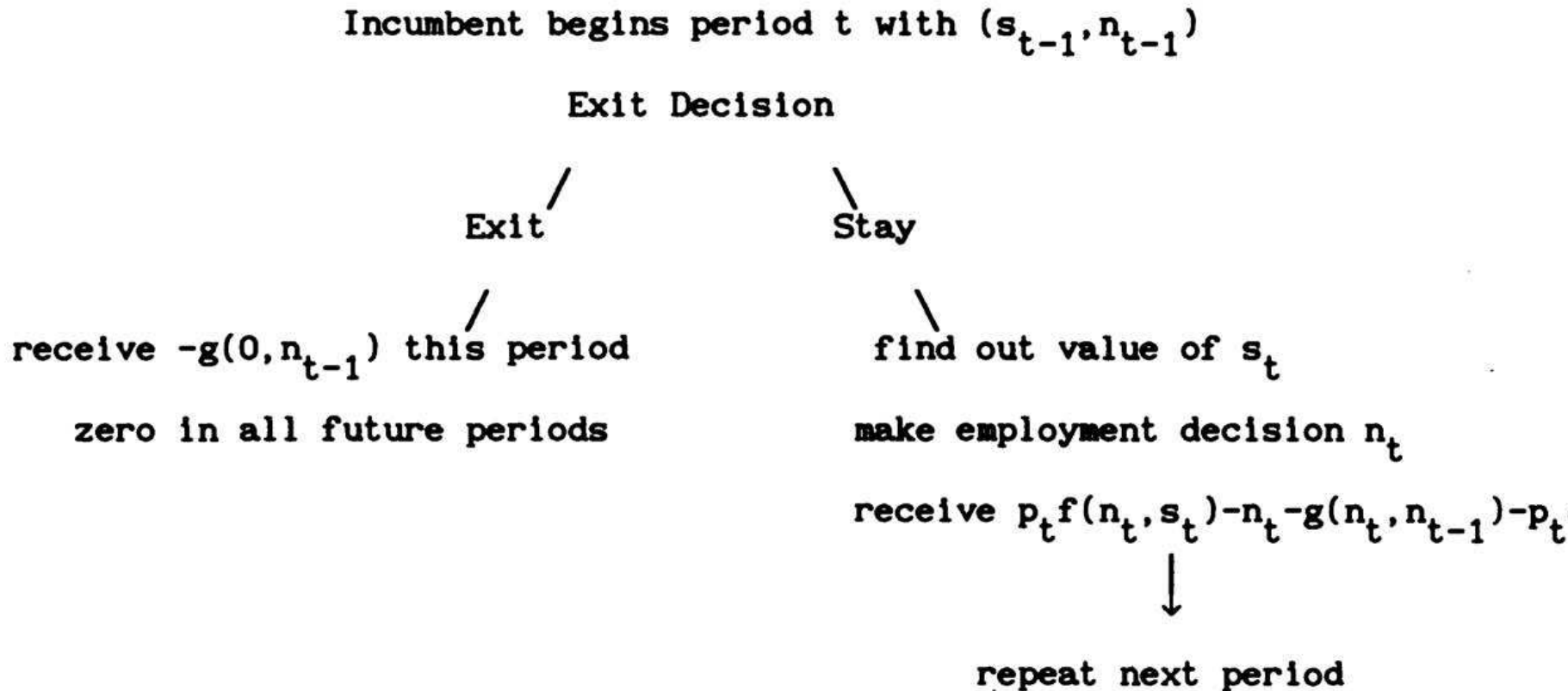


FIG. 1.—Timing of decisions

## The Model: Bellman Equation for the Firm's Problem

$$W(s, n; p) = \max_{n' \geq 0} \{p_t f(n', s) - n' - pc_f - g(n', n) + \quad (1)$$

$$+ \beta \max[E_s W(s', n'; p), -g(0, n')]\} \quad (2)$$

The problem for the potential entrants is simply given by:

$$W^e(p) = \int W(s, 0; p) dv(s) \leq c_e \quad (3)$$

- Let  $(s, n)$  be the state of an individual firm, then the state of the economy is defined as the distribution of the state variables for all individual firms  $\mu(s, n)$
- The transition from  $\mu$  to  $\mu'$  is  $\mu' = T(\mu, M; p)$ . The operator  $T$  has a fixed point:  $\mu^* = T(\mu^*, M; p)$

# The Model: Preferences and Endowments

- There is a continuous of identical agents with utility function:

$$\sum_{t=1}^{\infty} \beta^t [u(c_t) - \kappa(n_t)] \quad (4)$$

- Labor supply  $\in \{0, 1\} \Rightarrow$  individual choose employment lotteries  $\Rightarrow$  representative agent with preferences  $\sum_{t=1}^{\infty} \beta^t [u(c_t) - aN_t]$
- The problem of the household is:

$$\max u(c) - aN \quad s.t. \quad pc \leq N + \Pi + R \quad (5)$$

- $\Pi$  are the profits equally distributed among households and  $R$  is the lump-sum transfer from taxation of job destruction
- $L^s(p, \Pi + R)$  is the labor supply. It is assumed that the income effect on labor supply is negative

# The Model: Equilibrium

$$L^d(\mu, M; p) = \int N(s, n; p) d\mu(s, n) + M \int N(s, 0; p) dv(s)$$

$$Y(\mu, M; p) = \int [f(N(s, n; p), s) - c_f] d\mu(s, n) + M \int f(N(s, 0; p), s) dv(s)$$

$$\Pi(\mu, M; p) = pY(\mu, M; p) - L^d(\mu, M; p) - R(\mu, M; p) - Mpc_e$$

$$(s, n; p) = [1 - X(s, n; p)] \int g(N(s', n'; p), n') dF(s', s) + X(s, n; p)g(0, n')$$

**A stationary equilibrium** consists of an output price  $p^* \geq 0$ , a mass of entrants  $M^* \geq 0$  and a measure of incumbent firms  $\mu^*$  such that

(i)  $L^d(\mu^*, M^*, p^*) = L^s(p^*, \Pi(\mu^*, M^*, p^*) + R(\mu^*, M^*, p^*))$

(ii)  $\mu^* = T(\mu^*, M; p)$

(iii)  $W^e(p^*) \leq p^*c_e$ , with equality if  $M^* > 0$

# Computation of the Equilibrium

- **Step 1:** Use the entry condition (3) with equality to pin down an equilibrium price  $p^*$

$$W^e(p) = \int W(s, 0; p) dv(s) = c_e$$

- **Step 2:** Use the fact that the operator  $T$  has a fixed point to find the stationary distribution  $\mu$ , up to a scale factor  $M$
- **Step 3:** Use the labor market clearing condition for labor to pin down  $M^*$ :

$$L^d(M\mu, M, p^*) = L^s(p^*, M(\Pi(\mu, 1, p^*) + R(\mu, 1, p^*)))$$

$L^d$  is linearly homogeneous in  $M$  and  $L^s$  is strictly decreasing in  $M$   
 $\Rightarrow$  there is a unique value of  $M$  that satisfies the above equation

# Parametrization

- $f(n, s) = sn^\theta$  with  $\theta \in [0, 1]$
- $g(n_t, n_{t-1}) = 0$  in the benchmark model  
otherwise  $g(n_t, n_{t-1}) = \tau \max(0, n_{t-1} - n_t)$
- $\log(s_t) = a + \rho \log(s_{t-1}) + \varepsilon_t$  with  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$   $a \geq 0$  and  $\rho \in [0, 1)$
- $u(c) = \log(c)$ ,  $\kappa(n) = An$  with  $A > 0$
- in the benchmark model the problem of the firm is static and it implies:

$$\log(n_t) = \frac{1 - \rho}{1 - \theta} \left( \log \theta + \log \rho + \frac{a}{1 - \rho} \right) + \rho \log(n_{t-1}) + \left( \frac{1}{1 - \theta} \right) \varepsilon_t$$

## Calibration with 5 years as a unit of time using LRD

- $p^* = 1 \Rightarrow c_e$  is pinned down by the entry condition
- $\theta = 0.64$ ,  $\beta = 0.8$  and  $A$  s.t.  $\frac{\text{employment}}{\text{population}} = 0.6$
- $\rho$  and  $\sigma_\varepsilon^2$  are recovered from the regression of  $\log(n_t)$  on a constant and  $\log(n_{t-1})$
- $c_f$  and  $a$  are chosen to match the cross-sectional average of log employment and the 5-years exit rate
- The distribution of  $v$  is chosen to match the actual size distribution of firms aged 0-6 years in their first and second periods

# LRD Statistics

## A. ESTIMATES DERIVED FROM THE LRD

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Serial correlation in log employment (5-year interval, survivors)	.93
Variance in growth rates (log difference, 5-year interval, survivors)	.53
Mean employment	61.7
Exit rate (5-year interval)	37%

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## B. SIZE DISTRIBUTION FOR FIRMS AGED 0–6 YEARS

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Employees	Share of Total Firms
1–19	.74
20–99	.18
100–499	.08
500+	.01

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# Statistics from Benchmark Model

## A. SUMMARY STATISTICS FOR BENCHMARK MODEL

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Average firm size	61.2
Co-worker mean	747
Variance of growth rates (survivors)	.55
Serial correlation in log $n$ (survivors)	.92
Exit rate of firms	.39
Turnover rate of jobs	.30
Fraction of hiring by new firms	.15
Average size of new firm	7.5
Average size of existing firm	4.9

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## B. SIZE DISTRIBUTION

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	1–19	20–99	100–499	500+
Firms	.52	.37	.10	.01

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# The effect of a tax on job destruction

EFFECT OF CHANGES IN  $\tau$  (Benchmark Model)

	$\tau = 0$	$\tau = .1$	$\tau = .2$
Price	1.00	1.026	1.048
Consumption (output)	100	97.5	95.4
Average productivity	100	99.2	97.9
Total employment	100	98.3	97.5
Utility-adjusted consumption	100	98.7	97.2
Average firm size	61.2	61.8	65.1
Layoff costs/wage bill	0	.026	.044
Job turnover rate	.30	.26	.22
Serial correlation in $\log(n)$	.92	.94	.94
Variance in growth rates	.55	.45	.39

- A tax on job destruction reduces long-run employment, reduces average productivity and, as a consequence of this reduction, produces welfare losses