Curvature of Earnings Profile and Career Length

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Abstract

A finitely lived worker confronts a labor supply indivisibility, chooses when to work, and smooths consumption by trading an interest bearing security. The worker faces an exogenously given increasing schedule that maps accumulated time on the job into an earnings level. With a specification of the worker's preferences that macroeconomists commonly use to assure balanced growth paths, the more elastic are earnings to accumulated working time, the longer is a worker's career.

Key words: Indivisible labor, labor supply, earnings profile, career length.

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1 That high labor supply elasticity

Hansen (1985) postulated indivisible labor and calibrated a high disutility of labor to get a high aggregate labor supply elasticity in a real business cycle model with a Rogerson (1988) employment lottery and complete markets for consumption claims. Abstracting from productivity shocks, Ljungqvist and Sargent (2006) pointed out that, holding the worker's preferences fixed, the same high aggregate labor supply elasticity would emerge if the employment lotteries and complete markets were withdrawn and instead individual workers were forced to confront the indivisible labor choice by choosing fractions of their lifetimes to work while trading a single risk-free asset to smooth their consumption over time. That analysis pinpoints the source of the high labor supply elasticity to be the high disutility of labor, rather than the Rogerson aggregation theory stressed by Prescott (2005).[1,2] The finding refocuses attention on how individual workers' decisions about career lengths affect aggregate labor supply outcomes.

Prescott et al. (2009) extended the Ljungqvist and Sargent (2006) time-averaging model by adding an intensive margin to the worker's labor supply decision.[3] But when they revisited the tax analysis of Ljungqvist and Sargent (2006), Prescott et al. (2009, p. 31) found that the labor supply response to taxes remained exactly the same, because all of the adjustment "takes place along the extensive margin, i.e., the fraction of life devoted to work."[4] To resurrect effects from the intensive margin, Prescott et al. (2009) suggested that governments might impose quantity constraints on either the intensive margin ('constraints on length of workweek') or the extensive margin ('constraints on working life') that would cause the worker to adjust the unconstrained margin to offset the impact on total hours worked over the lifetime.

The change of focus from the fraction of the labor force sent to work in a Rogerson (1988) employment-lottery model to the fraction of an individual's lifetime devoted to work

[1] Prescott (2006a) advocated the Ljungqvist and Sargent (2006) life-cycle framework with indivisible labor as "the initiation of an important research program ... to derive the implications of labor indivisibility for lifetime labor supply." While Prescott's (2005) original Nobel lecture was devoted to the complete-market representative-agent framework, a subsequent version (Prescott 2006b) contains an added section on "The Life Cycle and Labor Indivisibility."

[2] After showing that a particular distribution function of (insurable) ex-ante heterogeneity in an indivisible-labor complete-market model can render the aggregate labor supply isomorphic to that of a representative-agent model with divisible labor, Mulligan (2001, p. 13) suggested that the elimination of employment lotteries and complete markets for consumption claims from the former model might not make much of a quantitative difference, based on the following reasoning: "The smallest labor supply decision has an infinitesimal effect on lifetime consumption and the marginal utility of wealth in the [divisible-labor] model, and a small-but-larger-than-infinitesimal effect on the marginal utility of wealth in the [indivisible-labor] model – as long as the effect on lifetime consumption is a small fraction of lifetime income *or* the marginal utility of wealth does not diminish too rapidly." Abstracting from ex-ante heterogeneity, Ljungqvist and Sargent (2006) offered the first equivalence result for indivisible-labor models with and without employment lotteries in continuous time, and pursued the substantive implications for how life-cycle career decisions affect aggregate labor supply and its response to flat rate income taxes.

[3] Compare section 3 of Prescott et al. (2009) with section 3 of Ljungqvist and Sargent (2006).

[4] Compare section 4.1 of Prescott et al. (2009) with section 4.1 of Ljungqvist and Sargent (2006).

in a Ljungqvist and Sargent (2006) time-averaging model naturally redirects attention to heterogeneity in the situations of individual workers. For example, in their extended model with an intensive margin, Prescott et al. (2009) postulate that occupations differ with respect to the set-up costs that they represent in terms of a mapping from lengths of workweek to labor services. They find that a larger set-up cost results in a longer optimal workweek length, which they suggest can rationalize observed variations on weekly hours across occupations.

In this paper, we revert to Ljungqvist and Sargent's (2006) exclusive focus on the extensive margin and study the consequences of differences in exogenous schedules for converting accumulated working time into an increasing earnings level. We isolate a force through which, under a standard specification of an instantaneous utility function, the *curvature* of the earnings profile affects a worker's choice of career length. The more elastic are earnings to accumulated working time, the longer is a worker's career. Obscured though it is by confounding forces, this effect is also present in the richer model of Kitao et al. (2008). The streamlined choice problem presented here isolates the effect.

2 A life-time problem

A worker's preferences are ordered by

$$\int_0^1 \Big[\log(c_t) - Bn_t\Big]dt, \qquad B > 0, \tag{1}$$

where $c_t \geq 0$ and $n_t \in \{0,1\}$ are consumption and indivisible labor supply at time t, respectively. The worker can freely borrow and lend at a zero interest rate.[5] A worker with past employment spells totaling $h_t = \int_0^t n_s ds$ has the opportunity to work at wage

$$w_t = \Phi h_t^{\phi}, \qquad \Phi > 0, \;\; \phi \in [0,1]. \tag{2}$$

The worker faces the life-time budget constraint $\int_0^1 c_t dt \leq \int_0^1 w_t n_t dt$. An optimal consumption, working schedule prescribes a constant consumption path and a fraction $T^\star$ of a lifetime devoted to work.

The worker is indifferent about the timing of her labor supply. Therefore, we are free to assume that the worker frontloads her work at the beginning of life, so that the present value of labor income for someone who works a fraction T of her lifetime is

$$\int_0^T \Phi t^{\phi}\, d\,t = \Phi \frac{T^{\phi+1}}{\phi+1} \equiv c(T;\Phi,\phi). \tag{3}$$

Since the worker consumes all income over her lifetime, this will equal the present value of consumption, which over the unit interval of life also equals the constant instantaneous consumption level, denoted $c(T;\Phi,\phi)$. Thus, the optimal lifetime labor supply that solves

$$\max_{T\in[0,1]} \Big\{\log[c(T;\Phi,\phi)] - BT\Big\}, \tag{4}$$

[5] We retain the assumption of Ljungqvist and Sargent (2006) that the worker's subjective discount rate and the market interest rate are equal and, for expositional simplicity, set both to zero.

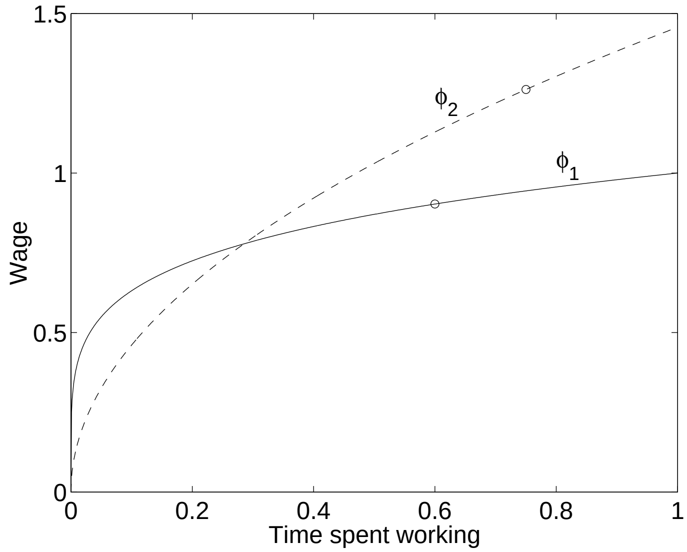

Figure 1: Two wage profiles with $\phi_1 = 0.2$ and $\phi_2 = 0.5$, respectively. For a disutility of work $B = 2$, the circle on each profile denotes the optimal fraction of lifetime spent working. As normalizations, we have picked $\Phi_1 = 1$ and Φ_2 so that $c(T_1^\star; \Phi_2, \phi_2) = c(T_1^\star; \Phi_1, \phi_1)$.

is

$$T^\star(\phi) = \min\left\{\frac{\phi+1}{B}\,,\ 1\right\}. \tag{5}$$

Notice that $T^*(\phi)$ is independent of the level parameter Φ, but depends on the curvature parameter ϕ.

3 Profile elasticity and career length

An elasticity parameter $\phi = 0$ means constant earnings, $w_t = \Phi$, while $\phi > 0$ indicates an earnings profile that increases in cumulated time worked, but at a decreasing rate (except for the linear specification, $\phi = 1$). A higher value of ϕ implies a slower relative decay in the slope of the earnings profile over the time worked. Evidently, a worker confronting a higher ϕ responds by working longer.

As an illustration, for a disutility of work $B = 2$, figure 1 depicts two earnings profiles, $\phi_1 = 0.2$ and $\phi_2 = 0.5$, with the optimal fraction of lifetime spent working, $T^\star(\phi_i) \equiv T_i^\star$, marked by a circle on each profile. As a normalization, we set $\Phi_1 = 1$ and then set Φ_2 to assure that $c(T_1^\star; \Phi_2, \phi_2) = c(T_1^\star; \Phi_1, \phi_1)$, i.e., so that both earnings profiles yield the same present value of labor income when the same fraction $T_1^\star$ is devoted to work. But while that choice is optimal for a worker with profile ϕ_1, the agent with the higher ϕ_2 will choose to work a greater fraction $T_2^\star$ of her lifetime.

4 Concluding remarks

Ljungqvist and Sargent (2006, section 6.5) used their basic time-averaging model to start thinking about patterns in their figure 3.6, reproduced from Rogerson (2006, figure 37): labor force participation rates in Europe now roughly equal those in the U.S. for workers of ages 30-50, but are less for workers younger than 30 and older than 50.[6] Why do some workers in Europe choose shorter careers than others? What accounts for lower participation of young workers in Europe? Who *pays* for older workers to withdraw from work? Are early retirements self-financed, or does the government chip in? And what difference does the financing make in terms of the choices of career lengths? These are exciting and important questions when it comes to understanding cross-country and cross-time aggregate labor market outcomes. Viewed as a model of self-financed retirement, the streamlined model with the interior solutions presented here asserts that the workers who retire later will be those with earnings profiles that are more elastic to accumulated working time. How this outcome is to be modified when other features affect the worker's budget set, such as retirement programs and generous government financed unemployment and disability benefits that depend on past earnings, is the subject of our ongoing research.[7]

[6] Prescott et al. (2009, figure 3) also present the same figure.

[7] See Kitao et al. (2008), and Ljungqvist and Sargent (2008), a companion paper to Ljungqvist and Sargent (2006).

A Equivalence between employment lotteries and time averaging?

Ljungqvist and Sargent (2006) found that in models with indivisible labor, a high disutility of labor is the source of a high aggregate labor supply elasticity, not the Rogerson aggregation theory based on employment lotteries and complete markets. The time-averaging model with indivisible labor and a high disutility of labor yields a high aggregate labor supply elasticity for a variety of specifications, including ones in which experience affects earnings.

But an *exact* equivalence of aggregate outcomes under individual time-averaging, on the one hand, and employment lotteries with complete markets, on the other hand, hinges on work experience not affecting earnings. Ljungqvist and Sargent (2006, sections 3.5, 3.6) analyze an increasing earnings-experience profile that is a step function with two flat spots and show that the equivalence between the lotteries and time-averaging models breaks down. It also break down for the specification that we have adopted in this paper. An increasing earnings-experience profile creates a nonconvexity over careers and allows a representative family to achieve aggregate allocations with employment lotteries that individuals cannot attain by time averaging.

Thus, consider a representative family consisting of a continuum $j \in [0,1]$ of ex ante identical workers like those in section 2. The family chooses a consumption and employment allocation $c_t^j \geq 0$, $n_t^j \in \{0,1\}$ to maximize

$$\int_0^1 \int_0^1 \Big[\log(c_t^j) - B n_t^j\Big] dt\, dj \tag{6}$$

subject to

$$\int_0^1 \int_0^1 \Big[w_t^j n_t^j - c_t^j\Big] dt\, dj \geq 0\,, \tag{7}$$

where w_t^j is the potential earnings of worker j at time t which depends on her past work experience, as described in (2).

As in Ljungqvist and Sargent (2006, section 3.6), the family solves this problem by setting $c_t^j = \bar{c}$ for all j, $t \in [0,1]$ and by administering a lifetime employment lottery once and for all before time 0 that assigns a fraction $N \in [0,1]$ of people to work always ($n_t^j = 1$ for all $t \in [0,1]$ for these unlucky people) and a fraction $1-N$ always to enjoy leisure ($n_t^j = 0$ for all $t \in [0,1]$ for these lucky ones). An individual who works throughout her lifetime generates present-value labor income equal to $c(1;\Phi,\phi)$, as defined in (3). Thus, the family's optimal labor supply that solves

$$\max_{N\in[0,1]} \Big\{\log[N\, c(1;\Phi,\phi)] - BN\Big\}, \tag{8}$$

is $N^\star = \min\{B^{-1}, 1\}$. Hence, members of the representative family on average work less than individuals who are left to 'time average', as characterized by (5).[8] The latter individuals

[8]Ljungqvist and Sargent (2006, sections 3.5, 3.6) obtain a similar outcome with their two-flat-spot

confront a difficult choice between enjoying leisure and earning additional labor income at the peak of their lifetime earnings potential. This tension is not experienced by the individuals who follow the instructions of the family planner who uses lotteries to convexify the indivisibility brought by careers. Of course, in the special ($\phi = 0$) case when work experience does not affect earnings, the aggregate labor supplies are exactly the same across a Rogerson (1988) employment-lottery model and a Ljungqvist and Sargent (2006) time-averaging model, and people enjoy the same expected lifetime utilities.

We can concisely summarize the message of this appendix by comparing the responses of aggregate time spent employed to labor tax rate τ for the time-averaging model,[9]

$$T^\star(\tau;\phi) = \min\left\{\frac{(1-\tau)(\phi+1)}{B}\,,\ 1\right\}, \tag{9}$$

and for the employment-lottery model,

$$N^\star(\tau) = \min\left\{\frac{1-\tau}{B}\,,\ 1\right\}. \tag{10}$$

As noted above, individuals in the time-averaging model choose a longer career length than the average lifetime labor supply in the employment lottery model, at an interior solution. Therefore, if the equilibria without taxation are characterized by a corner solution (with individuals working until some mandatory retirement age), successive increases in taxation will first reduce employment in the economy with employment lotteries while the labor supply in the economy with time averaging is more robust. However, when taxes are so high that the response of the individuals in the time-averaging model is to shorten their careers, the elasticity of aggregate labor supply with respect to the net-of-tax rate, $[(1-\tau)/L]\,\partial L/\partial(1-\tau)$, is equal to one in both the time-averaging ($L = T$) and employment-lottery ($L = N$) models. So, yes, the exact equivalence between the models breaks down, but nevertheless with a high disutility of labor like those calibrated in the real business cycle literature, a high labor supply elasticity can still come through in both frameworks.

experience-earnings profile, regarding workers in the time-averaging model who choose to acquire the higher level of experience (while the career length of any workers who do not aspire for the higher experience level in the time-averaging model is the same as the average lifetime labor supply in the employment-lottery model).

[9] See Ljungqvist and Sargent (2006, sections 4.2, 4.3) for the same exercise applied to a different earnings function.

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