

Reductive Theories of Modality*

THEODORE SIDER

M. J. Loux and D. W. Zimmerman, eds., *The Oxford Handbook of Metaphysics* (OUP, 2003): 180-208

I Modality

Logic begins but does not end with the study of truth and falsity. Within truth there are the *modes* of truth, ways of being true: necessary truth and contingent truth. When a proposition is true, we may ask whether it *could* have been false. If so, then it is contingently true. If not, then it is necessarily true; it *must* be true; it could not have been false. Falsity has modes as well: a false proposition that could not have been true is *impossible* or necessarily false; one that could have been true is merely contingently false. The proposition that some humans are over seven feet tall is contingently true; the proposition that all humans over seven feet tall are over six feet tall is necessarily true; the proposition that some humans are over seven feet tall and under six feet tall is impossible, and the proposition that some humans are over nine feet tall is contingently false.

Of these four modes of truth, let us focus on necessity, plus a fifth: possibility. A proposition is possible if it is or could have been true; hence propositions that are either necessarily true, contingently true, or contingently false are possible.

Notions that are similar to the modes of truth in being concerned with what might have been are called modal. Dispositions are modal notions, for example the disposition of fragility. Relatedly, there are counterfactual conditionals, for example “if this glass were dropped, it would break.” And the notion of supervenience is modal.¹ But let us focus here on necessity and possibility.

Modal words are notoriously ambiguous (or at least context-sensitive²). I may reply to an invitation to give a talk in England by saying “I can’t come; I have to give a talk in California the day before”. This use of “can’t” is perfectly appropriate. But it would be equally appropriate for me to say that I *could* cancel my talk in California (although that would be rude) and give the talk in England instead. What I cannot do is give both talks. But wait: it also seems appropriate to say, in another context, that given contemporary transportation,

*Thanks to Phillip Bricker, John Hawthorne, Michael Loux, Peter Momtchiloff, Daniel Nolan, Brian Weatherson and Dean Zimmerman for helpful comments.

¹ See Kim (1993, part I), and Lewis (1986a, 14-17).

² See Kratzer (1977).

one can give a talk in California one day and England the next. It may be very exhausting, but one *can* do it. What one cannot do is give a talk in California and then give a talk in England the next *hour*. But in yet another context one could say the following: “Given the limits on travel faster than the speed of light, one cannot give a talk on Earth, and then another on Alpha Centauri an hour later. But one *could* give a talk in California and then an hour later give a talk in England.” Finally, even this performance seems appropriate: “The laws of nature could have been different. Supra-luminal travel might have permitted by the laws of nature. One could (if the laws had indeed allowed supra-luminal travel) have given a talk on Earth, and then another an hour later on Alpha Centauri, 4.12×10^{13} km away. What is impossible is giving talks on Earth and Alpha Centauri at the very same time.”

There are, therefore, different “strengths” of necessity and possibility, which can be signified by modal words (like ‘can’) in different contexts. Philosophers have tended to concentrate on a very broad sort, so-called “metaphysical” possibility and necessity. According to many, it is metaphysically possible that the laws of nature be different, that the past be different from what it actually was, and so on.³ All of the scenarios in the last paragraph—giving a talk in England, giving a talk in California one day and England the next, giving a talk in California at one moment and a talk in England an hour later, giving a talk on Earth one moment and on Alpha Centauri an hour later—are metaphysically possible. What is *not* metaphysically possible? Almost everyone agrees that contradictions are metaphysically impossible—it is metaphysically impossible to both give a talk in California and also not to give a talk in California. And everyone who accepts the legitimacy of the notion of analyticity—of truth that is in some sense guaranteed by meaning—agrees that the negations of analytic sentences like ‘all bachelors are unmarried’ are impossible. But it is usually thought that there exist further impossibilities. Examples might include the existence of a round square, someone’s being taller than himself, someone’s being in two places at once, George W. Bush’s being a donkey, there existing no numbers, and there existing some water that is not made up of H₂O. Exactly what is metaphysically impossible beyond logical and analytic contradictions is unclear; this unclarity is what makes the analysis of metaphysical possibility and necessity so difficult. But it is metaphysical possibility and necessity that most concerns philosophers, and so from now on it is on the metaphysical sense of

³ Although see Kneale (1949) and Shoemaker (1998) on the necessity of the laws of nature, and Prior (1955) on the necessity of the past.

the modal notions that I will focus.

It is common to distinguish between *de re* and *de dicto* modality. The contrast may be brought out with this example:

(*De dicto*) Necessarily, the number of the planets is odd
 $\Box[(\text{the } x: Nx) Ox]$

(*De re*) The number of the planets is such that it is necessarily odd
 $(\text{The } x: Nx) \Box[Ox]$

The *de dicto* sentence is false. It claims that it is necessary that the number of the planets is odd, whereas there clearly might have been 6 or 8 planets. The *de re* sentence, however, is presumably true. It claims of the number that actually numbers the planets—namely, 9—that it is necessarily odd. Assuming with orthodoxy that mathematical facts are necessary, this is true: the number 9 itself is necessarily odd. The *de dicto* sentence claims that a certain *descriptive* claim is necessary: it is necessary that the number picked out by the description ‘the number of the planets’, whatever that might turn out to be, is odd. In each possible world, whatever number is the number of the planets in that world must be odd. In contrast, the *de re* sentence uses the description ‘the number of the planets’ to single out a certain individual, the number 9, but then goes on to make a modal claim about that number itself; the description used to single out 9 plays no role in evaluating the modal claim about 9. In each possible world, 9 itself must be odd, never mind whether 9 is the number of the planets in that world.

There is a grammatical contrast between the *de re* and the *de dicto* sentences that is made clearer by the symbolically regimented versions of those sentences. In the *de re* sentence there is a variable in the scope of the modal operator \Box (symbolizing ‘it is necessary that’) that is bound to a quantifier outside the scope of the \Box whereas in the *de dicto* sentence no quantification into the scope of modal operators occurs. A further example: the false sentence ‘Possibly, some bachelor is unmarried’, or ‘ $\Diamond \exists x(Bx \& \sim Mx)$ ’ is *de dicto*, whereas the true sentence ‘Some bachelor is possibly unmarried’, or ‘ $\exists x(Bx \& \Diamond \sim Mx)$ ’ is *de re*, since the variable x occurs inside the scope of the \Diamond but is bound to the quantifier $\exists x$ which occurs outside the scope of the \Diamond . This grammatical or syntactic way of drawing the *de re/de dicto* distinction is common, and can be extended to natural language given the existence of natural language analogs of modal operators and variable binding. However, for present purposes it will

be useful to (somewhat stipulatively) draw the distinction slightly differently. Specifically, in addition to sentences with quantification into modal contexts, let us count as *de re* modal sentences in which “directly referential terms” occur within the scope of modal operators. Directly referential terms are terms whose propositional contributions are simply their referents, for example proper names and indexicals. The reason for counting these sentences as *de re* is that they attribute modal properties to objects *simpliciter*, rather than under descriptions.⁴

Modality is important to philosophy for many reasons. A first reason derives from philosophy’s traditional association with logic. Advances in modal logic in the middle of the 20th century provided a reason to be interested in the modalities. Moreover, propositions that are logically true seem necessarily true. Another source of modality’s importance is that necessary truth, according to one tradition, demarcates philosophical from empirical inquiry. Science identifies contingent aspects of the world, whereas philosophical inquiry reveals the essential nature of its objects; philosophical propositions are therefore necessarily true when true at all.

But the most important source of importance derives from modality’s connections with epistemology and philosophy of language. These connections are at the core of analytic philosophy. The propositions identified by traditional epistemology as those that can be known *a priori*, independent of sensory experience, seem necessary. These are generally agreed to include the propositions of logic as well as analytic truths. Whether there are other *a priori* propositions was one of the great questions of 17th and 18th century epistemology, and the debate continues to this day. But it was generally agreed until recently that all *a priori* propositions are necessarily true. Indeed, before the publication of Saul Kripke’s *Naming and Necessity*, it was not uncommon to identify *a prioricity* with necessity.

Given the compelling examples of necessary *a posteriori* propositions given by Kripke, and by Hilary Putnam (1975a), as well as Kripke’s examples of contingent *a priori* propositions, this identification is no longer made. And given W. V. O. Quine’s (1951) critique of analyticity, some have doubted the

⁴ *De re* modal claims are often explained etymologically as those that attribute necessity to an object, a *res*, e.g., the number 9, rather than to a proposition, a *dictum*, e.g., the proposition that the number of the planets is odd. But this way of drawing the distinction is misleading. In a perfectly good sense of “object,” propositions are objects. Moreover, modal sentences containing directly referential terms inside the scopes of modal operators would attribute modal properties to (singular) propositions, but would nevertheless be *de re* on my usage.

connection between analyticity and necessity, others the sense of the notion of necessity itself. But despite this, many of the important traditional connections remain. It is still relatively common to claim that *some* necessary propositions are *a priori*; thus, the nature of necessity is relevant to epistemology, for what is necessary truth, that it can be ascertained without sensory input? And despite Quine, there remains an overwhelming temptation to think that the notion of linguistic convention has *some* legitimate application, and *some* connection with the traditional notion of necessity.

2 Reduction

Our topic is the reduction of modality, whether modal notions can be reductively defined. Certainly some modal notions can be defined in terms of other modal notions. Take as undefined any of the five modalities of truth discussed above, and the rest may be defined in terms of it. For example, symbolizing 'It is necessary that ϕ ' as ' $\Box\phi$ ', one can define the other modalities thus:

it is possible that ϕ :	$\sim\Box\sim\phi$
it is contingently true that ϕ :	$\phi \ \& \ \sim\Box\phi$
it is contingently false that ϕ :	$\sim\phi \ \& \ \sim\Box\sim\phi$
it is impossible that ϕ :	$\Box\sim\phi$
it is contingent whether ϕ :	$\sim\Box\phi \ \& \ \sim\Box\sim\phi$

But these definitions are not reductive, since necessity is utilized without being defined. The more interesting question is whether a *reductive* definition of modality is possible, a definition of the modal in terms of the non-modal.

Why seek such a thing? One traditional motivation lies in modality's connection to epistemology. Many modal claims are known *a priori*, and it is a puzzle how this is possible, how we manage to know modal claims without the benefit of sensory experience. The epistemology of the modal can be secured if modal notions are defined in terms of notions whose epistemology is secure.

There are also reasons from metaphysics to seek reduction. Reductionism is required by any ontology that claims to give a comprehensive account of reality in terms of primitive entities and notions that do not include modal notions. And some metaphysical programs without quite so high ambitions require reductionism, for example, the extensionalism of Quine and Davidson.⁵

⁵ See, for example, Quine (1951); Davidson (1967). Of course, many extensionalists prefer eliminativism to reductionism.

It is easy to get into a frame of mind according to which modal notions should not be taken as “rock bottom”, ontologically speaking. The frame of mind is not unlike Hume’s when he confronted causation. One can see the prior event, and also the later one, but where is the causation? Likewise: I can see that this colored thing is extended, and indeed that all colored things I have examined are extended, but where is the necessity, that colored things *must* be extended? Part of the puzzlement here is of course epistemic, and epistemic reasons for reductionism have already been mentioned. But there is a particularly metaphysical puzzlement here as well. In metaphysics one seeks an account of the world in intelligible terms, and there is something elusive about modal notions. Whether something *is* a certain way seems unproblematic, but that things might be otherwise, or must be as they are, seems to call out for explanation.

Accepting necessity or possibility as a primitive feature of reality would be like accepting tensed facts as primitive, or accepting dispositions as primitive, or accepting counterfactuals as primitive. While some are willing to make these posits, others seek to reduce “hypothetical” notions to “categorical” notions— notions which are in a sense “self-contained” and do not “point beyond themselves” as the hypothetical notions do.⁶

Parsimony is a final metaphysical reason to seek reduction. The metaphysician prefers desert landscapes when she can get them; when it is possible to reduce, we should. Of course the reduction might fail; parsimony gives us reason to search but does not guarantee success. *Primitivism*, the view that modality is unanalyzable, is an important and legitimate alternative to reductionism, and is favored by many because of the difficulty of finding an adequate reduction. But reduction is the topic of the present essay.

Primitivism and reductionism are not exhaustive alternatives: one might prefer *eliminativism* to each.⁷ The eliminativist denies that there is any such thing as modality. I will not take on eliminativism here except to say that it is a position of last resort, given the embedding of modality in ordinary and philosophical talk and practice. At any rate, the availability of the eliminativist position is no reason to bypass our inquiry into reductionism, for if a tenable reduction does exist then there is no reason to be an eliminativist.

A reduction of modality is an analysis of modal propositions in certain other

⁶ See my Sider (2001b, chapter 2, section 3).

⁷ Perhaps we can classify Simon Blackburn (1987), Hartry Field (1989, 38-45), and Quine (1951, 1953c) as eliminativists about metaphysical necessity and possibility.

terms. By analysis I mean identity: if the proposition that **it is necessary that everything colored is extended** is analyzed as the proposition that p , then the proposition that **it is necessary that everything colored is extended** *just is* the proposition that p . A proposed reduction must at least be extensionally adequate in that the modal propositions and the reducing propositions must have the same truth values. In addition, to be non-circular, or genuinely reductive, the terms in which the reducing propositions are expressed must be “non-modal”. But there is a bit of awkwardness here: in a sense the reducing terms must indeed be modal if the reduction is successful, since if the reduction is successful then the reducing propositions *are* modal propositions, given that analysis is identity. The awkwardness should be resolved as follows. Any reductionist program takes certain notions as being “acceptable”. What acceptability amounts to depends on what is driving the reduction—it may be epistemic acceptability, or categoricity, or extensionality, or something else. “Non-modal”, then, means “acceptable”—a reduction is non-circular or genuinely reductive if the notions it employs are acceptable according to its standards, whatever those may be.

There are several strategies for reduction. In section 3 I discuss reductions based on possible worlds, both abstract (sections 3.1-3.4) and concrete (sections 3.5-3.10); in section 4 I discuss conventionalism.⁸

3 Possible worlds

The analysis foremost in the contemporary consciousness is the possible-worlds analysis, which reduces possibility and necessity via the Leibnizian biconditionals:⁹

A proposition is necessary iff it is true in all possible worlds

A proposition is possible iff it is true in some possible world

A possible world is a complete possible history of the entire universe. One possible world is actual—this is the totality of what actually occurs. The other

⁸ This survey of reductive strategies is incomplete in at least two ways: it concentrates exclusively on recent theories, and it ignores the quasi-epistemic theory of modality currently being developed by David Chalmers.

⁹ These are characterizations of metaphysical possibility and necessity. Narrower sorts of possibility and necessity may be characterized by restricting the quantifier over possible worlds in various ways. For example, a proposition is nomically possible iff it is true in some possible world that obeys the laws of the actual world.

worlds are merely possible—they are non-actual ways things might have been.

3.1 Reductionism about modality and the ontology of possible worlds¹⁰

Provided one believes in possible worlds at all, the *truth* of the Leibnizian biconditionals is hard to question. But if talk of truth in possible worlds must itself be defined in terms of possibility and necessity then these biconditionals will not constitute reductive analyses. One might still employ possible worlds in a reduction of some of the other modal notions, thus providing a partial reduction of some modal notions to possibility and necessity. One could give definitions of counterfactuals, statements about dispositions, claims of supervenience, and so on, in terms of possible worlds, even if talk of possible worlds must ultimately be understood in terms of possibility and necessity. But if the possible worlds reduction of modality is to be complete, the notion of truth in possible worlds must be non-modal. And whether this is so depends on the ontological status of possible worlds.

It is important to separate the general contemporary interest in possible worlds from their use in reducing modality. Possible worlds are ubiquitous in metaphysics, and are frequently utilized in semantics,¹¹ ethics,¹² probability theory,¹³ philosophy of mind,¹⁴ and many other contexts.¹⁵ The suitability of possible worlds for these other purposes is largely independent of their ontological status. Not so for their use in reducing modality.

According to David Lewis, the most prominent defender of the possible worlds analysis of modality, possible worlds are “concrete” entities of a kind with the world we live in.¹⁶ The totality of cows that are spatiotemporally related to us does not exhaust the totality of cows in existence. There are in addition all the cows that exist in other possible worlds. Thus, for Lewis, reality

¹⁰ For more on the ontology of worlds see Kit Fine’s contribution to this volume.

¹¹ See Carnap (1947), the exposition of Montague grammar in Dowty et al. (1981), Lewis (1970), and Cresswell (1973).

¹² See Feldman (1986).

¹³ See Lewis (1980).

¹⁴ See Chalmers (1996).

¹⁵ The purely formal “possible worlds” in the model theory for modal logic (see Cresswell and Hughes (1996); Chihara (1998)) must be distinguished from the genuine possible worlds useful in metaphysics, semantics, philosophy of mind, and so on. See Lewis (1986a, 17–20), Sider (2001c), and the beginning of Sider (2002).

¹⁶ The main defense of this theory is in Lewis (1986a); see also Lewis (1968, 1971, 1973). Lewis has serious reservations about the term ‘concrete’; see his Lewis (1986a, section 1.7).

includes purple cows, talking cows, flying cows, and so on: purple, talking and flying cows are possible, and so by the Leibnizian biconditionals exist in other possible worlds, and so according to Lewis's metaphysics of possible worlds exist simpliciter. In calling these cows "cows" Lewis means to be speaking completely literally; they are no less real or concrete or cow-ey than actual cows. The actual world is just one world among many, and has no privileged ontological status. Its actuality consists merely in being *our* world. Inhabitants of other worlds call their worlds actual and they speak truly as well: 'actual,' according to Lewis, is an indexical word that refers to the possible world of the speaker.

For most of us this is too much to take. Those who wish nevertheless to speak of possible worlds must therefore provide some kind of reduction of possible worlds talk that does not require ontological commitment to the Lewisian multiverse. But as we will see, this is difficult to achieve without presupposing modality. Many accept this limitation, and consequently do not attempt to reduce the modal to the non-modal.

3.2 Linguistic ersatzism

Many of the leading reductions of possible worlds talk identify possible worlds with certain abstract entities. The abstract possible worlds of these so-called "abstractionist" views are allegedly much easier to believe in than Lewis's concrete worlds. One proposal, which Lewis labels "linguistic ersatzism," identifies worlds with sets of sentences.¹⁷ A possible world in which donkeys talk and fish walk would be identified with a set of sentences that includes the sentences 'donkeys talk' and 'fish walk,' in addition to other sentences describing the rest of what occurs in this possible world. Possible individuals inhabiting these possible worlds may be constructed as well—as sets of formulas containing free variables. For example, a talking donkey might be constructed as a set containing, among others, the formulas 'x is a donkey' and 'x talks'.

The problem then becomes how to distinguish sets of sentences that de-

¹⁷ See for instance Jeffrey (1965, 196-197). Probably the clearest statement is in Lewis (1986a, chapter 3), which criticizes linguistic ersatzism as well as other reductions of possible worlds. Subsequent literature on this topic has been largely a reaction to Lewis's critique. See Bricker (1987) for a further critique; see Heller (1998); Roy (1995); Sider (2002) for responses. Terminological note: what I call linguistic ersatzism is what Lewis calls linguistic ersatzism with a *rich* world-making language; what I call "combinatorial" conceptions of possible worlds are what Lewis calls versions of linguistic ersatzism with a poor world-making language.

scribe possible worlds from sets of sentences that describe states of affairs that could not possibly occur. Call a set, *S*, of sentences an *ersatz world* iff *S* is *maximal* in that for each sentence, either it or its negation is a member of *S*. We cannot say that a proposition is possible iff it expressed by some sentence in some ersatz world, for this would count *every* (expressible) proposition as possible—every sentence is a member of the set of *all* sentences, which is obviously maximal.¹⁸ The proposition that some fish are not fish would turn out possible. We must, it seems, restrict our attention to the *ersatz possible worlds*, where an ersatz world is *possible* iff it would be possible for all the members of the set to be true together. But this characterization of an ersatz possible world makes use of the notion of possibility. Therefore, a possible-worlds analysis of modality that makes use of ersatz possible worlds in this way is circular.¹⁹

Many other abstractionist theories have the same limitation. I will illustrate the point by discussing the reduction of worlds to states of affairs proposed by Alvin Plantinga (1974, 1976), but parallel points can be made about similar theories defended by Robert M. Adams (1974) and Robert Stalnaker (1976).²⁰ Plantinga's states of affairs are abstract entities much like propositions (indeed, it is hard to see how they differ from propositions). States of affairs are to be distinguished from sentences since distinct sentences in different languages may all assert that the very same state of affairs obtains. States of affairs are abstract, necessarily existing entities. A given state of affairs may "obtain," or it may fail to obtain: the state of affairs of *Plantinga's being tall* obtains since Plantinga is indeed tall; the state of affairs *Plantinga's being an atheist* does not obtain since Plantinga is not, in fact, an atheist. (Thus, Plantinga's conception of states of affairs is unlike another conception according to which there are no false or non-obtaining states of affairs or facts.²¹) Some definitions: say that a state of affairs, *S*, is *possible* iff it is (metaphysically) possible for *S* to obtain; say that *S includes T* when it is impossible for *S* to obtain without *T* obtaining; say that *S precludes T* when it is impossible for *S* and *T* to both obtain; finally, say that *S is maximal* iff for any state of affairs, *T*, either *S includes T* or *S*

¹⁸ Let us here ignore Cantorian problems with the existence of a set of all sentences.

¹⁹ Even setting aside the reduction of modality, linguistic ersatzism (like most other reductive theories of worlds) has trouble accounting for possibilities involving non-actual individuals. See Bricker (1987, 349-353), Fine's contribution to this volume, Lewis (1986a, 157-165), McMichael (1983), and Sider (2002).

²⁰ See Lewis (1986a, section 3.4) for criticisms of these views; see van Inwagen (1986) for a response.

²¹ See for example Wittgenstein (1961); Armstrong (1997).

precludes *T*. Given these definitions, Plantinga identifies the possible worlds with the maximal possible states of affairs. Various objections can be raised against this account, for example to the assumptions that states of affairs exist necessarily and that maximal states of affairs exist.²² But the main point to notice here is that Plantinga's possible worlds cannot be utilized in a completely reductive account of modality since Plantinga uses the notion of possibility in defining his possible worlds. Plantinga does not pretend otherwise; he accepts that possibility and necessity must remain unanalyzed.

3.3 Combinatorialism

Some abstractionists avoid circularity by giving a combinatorial definition of a possible world. Consider, for example, the identification of possible worlds with sets of spacetime points, each set representing the possibility that all and only the points in the set are occupied by matter.²³ I call this a "combinatorial" conception of possible worlds because the multiplicity of worlds results from the combinatorial nature of set theory: for any combination of spacetime points there exists a set containing all and only those points. This definition of 'possible world' is clearly non-modal. Modality is not needed to rule out impossible representations of worlds because it is intuitively plausible that any pattern of occupation of space-time points is possible.

Recall the Leibnizian biconditionals used to define possibility and necessity: "a proposition is necessary iff it is true in all possible worlds" and "a proposition is possible iff it is true in some possible world." In addition to the notion of a possible world, these biconditionals make use of the notion of propositions being *true in* possible worlds. Identifying worlds with sets of space-time points may eliminate modality from the definition of 'possible world,' but as Lewis has argued, modality reappears in the definition of 'true in'.²⁴ What would it mean to say that it is true in a certain set, *S*, of space-time points that there exists a talking donkey? The set does not literally contain a talking donkey; rather, it in some sense "represents" the existence of a talking donkey. But 'represents' is as in need of explanation as is 'true in'.

If we could analyze 'talking donkey' in terms of occupied points of space-time then we could determine precisely which patterns of occupation would suffice for the existence of a talking donkey, and then we could say that it is true

²² See Fine (1985); Chihara (1998, 120-141).

²³ Compare Cresswell (1972); Heller (1998); Quine (1968); Lewis (1986a, 146-148).

²⁴ See Lewis (1986a, 150-157), and also Lewis (1992).

in S that there is a talking donkey iff S contains one of these patterns. But no one knows how to provide this sort of analysis of ‘talking donkey’. Moreover, a *general* analysis of modality requires a general definition of ‘proposition *p* is true in set S’ for arbitrary propositions *p*; a series of one-off definitions for a few chosen propositions is no progress toward a general analysis.

We might define ‘*p* is true in possible world *w*’ as meaning ‘necessarily: if all and only the points in *w* are occupied by matter then *p* is true.’ But this definition uses necessity. No other definition seems available; ‘true in’, therefore, renders the account of modality circular.

Similar remarks apply to other combinatorial accounts of possible worlds. On one proposal, possible worlds are combinations (sets) of fundamental states of affairs (compare Armstrong (1989, 1997); Skyrms (1981); Wittgenstein (1961)). On another, worlds are combinations of primitive atomic sentences, where a primitive atomic sentence is an atomic sentence involving only primitive vocabulary (compare Carnap (1947)). Each of these theories has the following form: a possible world is a combination of “elements”. The hope is that “elements” may be construed so that *all* combinations of elements are possible, thus eliminating the need for a modal definition of a *possible* combination of elements. But the problem is then how to say when a *non-element* is true-in a possible world. Suppose that a donkey’s talking is not itself a fundamental state of affairs and that ‘donkey’ and ‘talk’ are not primitive predicates; in virtue of what then does a donkey talk *in* a set of elements? Only the circular modal definition suggests itself: a donkey talks relative to a set of elements iff, necessarily: if all and only the elements of that set are true, then a donkey talks.

The combinatorialist might enrich the conception of elements, and allow an element corresponding to ‘a donkey talks.’ But then it will no longer be true that all combinations of elements are possible. The presence of some elements that concern occurrences at the micro-level will be incompatible with the presence of certain other elements that concern talking and donkeys, and modality will again be needed to define ‘possible world’.²⁵

I have argued that abstractionists about possible worlds must appeal to modal notions in order to define ‘possible world’ and ‘true in.’ One might instead appeal to logical consequence and analyticity in these definitions. A linguistic ersatzer might, for example, define a possible world as a maximal set of sentences that does not logically imply any sentence that is analytically false. But there are commonly thought to exist impossibilities beyond those that are

²⁵ See Lewis (1986a, 150-157).

analytically false (see section 4.2 below). The definition thus counts certain sets of sentences as being ersatz possible worlds despite the impossibility of those sentences being true together. Perhaps one could appeal to other notions in addition to analyticity. But anyone able to do this is well along the way to a reduction of possibility and necessity that doesn't require possible worlds at all. (Of course one might use worlds to analyze other modal notions.) The theory we are considering is thus morphing into a theory of the sort to be discussed in section 4.2.

3.4 Fictionalism

A somewhat different reductive theory of worlds talk is Gideon Rosen's (1990) modal fictionalism. Unlike the theorists discussed so far, Rosen does not identify possible worlds with abstract entities. He rather regards talk of possible worlds as being like talk of characters in works of fiction. We speak of Sherlock Holmes despite his nonexistence: we say things like "Sherlock Holmes is a detective." This is legitimate because in uttering such a sentence one does not *really* intend discuss a real person; one really means something like this: "According to the Conan Doyle stories, Sherlock Holmes is a detective." Rosen views talk of possible worlds like talk of fictional characters: a sentence, P, about possible worlds should be regarded as meaning that "according to the fiction of possible worlds, P." Rosen goes on to suggest an analysis of modality. For any modal sentence, P, let P* be the possible-worlds analysis of P. For example, if P is 'possibly, there are talking donkeys,' then P* would be 'in some possible world there are talking donkeys.' Rosen's analysis of the modal sentence, P, is then this: according to the fiction of possible worlds, P*.

The status of this account as a *reduction* of modality is doubtful, since it is plausible that the locution 'according to' occurring in Rosen's analysis expresses a modal notion.²⁶ Rosen's analysis mentions a "fiction of possible worlds." This fiction is set out in Rosen's paper, and is in effect a short summary of David Lewis's theory of possible worlds given in *On the Plurality of Worlds*. But this summary, and indeed the entirety of *On the Plurality of Worlds*, falls far short of an exhaustive description of a plurality of worlds. This is entirely unsurprising (and not just because of limitations of finitude). If it were possible to explicitly lay out a description, D, of the possible worlds, then the theorists discussed above who identified worlds with abstract entities would not have needed modal notions

²⁶ See Rosen (1990, section 8).

to define their possible worlds. The linguistic ersatz, for example, could simply have defined the possible worlds as all and only the sets of sentences describing worlds in *D*. Explicitly constructing *D* is difficult precisely because of the difficulty of reductively analyzing modality. Metaphysical possibility is narrower than logical and analytical possibility, but exactly how much narrower is unclear. This is the fundamental obstacle to the reduction of modality, and it remains an obstacle whether one is telling a fiction about possible worlds or trying to construct worlds from abstract entities.

Rosen's fiction, therefore, falls far short of a complete description of the totality of possible worlds. Why, then, does Rosen suppose his reduction of worlds-talk adequate? According to Rosen, given the everyday notion of truth according to a fiction, much more is true in a fiction than what the fiction explicitly states. For example, it is presumably true in the Sherlock Holmes stories that Holmes had a liver, even though this is never explicitly mentioned. Likewise, Rosen claims, much more is true in his fiction of worlds beyond what he explicitly builds into it. But now, let *P* be any true modal sentence such that *P** (*P*'s worlds-analysis) is not explicitly entailed by Rosen's fiction. If Rosen's account is to be adequate then despite his fiction not explicitly entailing *P**, it must nevertheless be true according to that fiction that *P**. Thus, Rosen's 'according to' is doing much of the work of analyzing modality; accepting 'according to' without definition seems tantamount to accepting unreduced modality.²⁷

Another important reduction of worlds talk is defended by Kit Fine (1977); see also his (1981; 1980; 1982) and his contribution to this volume). Like Rosen, Fine does not identify worlds with abstract entities; but Fine makes no use of fictions. Rather, he translates sentences about possible worlds directly into a modal language. The rough idea is to interpret 'there is a possible individual such that ϕ ' as meaning 'possibly, there exists an individual such that ϕ '. Clearly, this account of talk about possible worlds cannot be used in a reductive theory of modality since it overtly employs modal notions.

3.5 Lewisian, "concrete," possible worlds

Hard as they are to accept, only Lewisian possible worlds allow a non-circular analysis of possibility and necessity; that is their great advantage. So for the remainder of this section I consider Lewisian possible worlds.

²⁷ For more details see Sider (2002, section V).

The possible worlds analysis of modality via the Leibnizian biconditionals is reductive only if the terms ‘possible world’ and ‘true in’ are non-modal. As we have seen, a Lewisian possible world is a concrete entity of the same kind as our own world. Such a possible world is simply the mereological sum or fusion of all the entities it contains. So our own world is the mereological sum of our surroundings and us. A possible world in which donkeys fly and pigs talk would be an object containing flying donkeys and talking pigs. But which fusions of possible objects count as possible worlds? How are Lewisian possible worlds individuated? If no answer could be given, the Lewisian analysis would not be reductive after all, for the predicate ‘possible world’ as applied to pieces of Lewis’s multiverse would remain a modal primitive.

Lewis’s answer is to define possible worlds spatiotemporally: x is a possible world iff (roughly) x is a *maximal spatiotemporally interrelated whole*, in that i) any two parts of x are spatiotemporally related to each other, and ii) anything that is spatiotemporally related to any part of x is itself part of x (1986a, section 1.6). This is a non-modal definition of a possible world.

We are now in a position to give the Lewisian reduction of *de dicto* necessity. Consider any sentence, ϕ , with no free variables or directly referential terms. Say that ϕ is *true in* a possible world, w , iff ϕ is true when all its quantifiers are restricted to parts of w . Then, we can say that ‘Necessarily, ϕ ’ is true iff ϕ is true in every possible world w .²⁸

According to Lewis, possible worlds do not overlap—no possible individual is (wholly) present in more than one possible world.²⁹ Therefore the account just given of *de dicto* necessity cannot be straightforwardly carried over to *de re* necessity. For to decide whether, e.g., George W. Bush is necessarily human we would have to ask whether the sentence ‘Bush is human’ is true in other possible worlds. But Bush exists only in the actual world and not in any other possible world. Instead, Lewis defends a *counterpart theory* of *de re* modal claims. To say that Bush is necessarily human is not to say that Bush himself exists in other possible worlds and is human in those worlds; rather, it is to say that all of Bush’s *counterparts* are human. And to say that Bush is possibly human is to say that some of Bush’s counterparts are human. A counterpart of Bush in a world, w , is an object in w that is similar enough to Bush, and which is at least as similar to Bush as are other objects in w . Exactly what sort of similarity

²⁸ Sorts of necessity narrower than metaphysical necessity may be analyzed by restricting the quantifier over worlds in various ways.

²⁹ See Lewis (1986a, chapter 4), for an extensive defense of this view.

is relevant can vary depending on the interests of the person uttering the *de re* modal sentence and her conversational audience. Thus, Lewis achieves a reduction of *de re* modality to the non-modal notion of similarity plus Lewisian possible worlds.³⁰

3.6 The incredulous stare³¹

Lewis's analysis of modality is compelling and comprehensive. Nevertheless, almost no one other than Lewis himself accepts it in its entirety. The chief reason is that most philosophers regard the existence of Lewisian possible worlds as being extremely implausible. According to Lewis, there exist golden mountains, unicorns, talking donkeys, and fire-breathing dragons. You've got to be kidding me! Lewis calls this reaction the incredulous stare, and takes it seriously, but argues that the intrinsic implausibility of his possible worlds are outweighed by the theoretical benefits of positing them. Here Lewis follows Quine's (1948) conception of ontological commitment: it is reasonable to postulate the entities over which one's best overall theory quantifies. According to Lewis, his theory of possible worlds provides the best systematic account of modal and other phenomena; its ontology is therefore reasonable (1986a, chapter 1). Lewis's argument is deliberately parallel to Quine's famous argument for the existence of sets: our best overall empirical theory, mathematical physics, quantifies over real numbers; therefore we have reason to posit real numbers, or the sets to which they may be reduced.³²

It is an interesting question why most philosophers so vehemently reject Lewisian worlds, especially since many accept the Quinean conception of ontological commitment and take the Quinean argument for sets seriously. Some may regard Quinean indispensability arguments as only being successful when applied to scientific theories; Lewis's argument concerns a theory whose alleged benefits are largely philosophical rather than empirical. This probably was Quine's reaction. But many philosophers have a more sympathetic attitude towards philosophy than did Quine, and yet regard Lewis's argument as a non-starter. Perhaps they reject Lewis's claim that his modal realism provides the most powerful systematic account of modal and other phenomena. Perhaps I speak for the majority when I say that I do not really know why I find the

³⁰ See Lewis (1968, 1971, 1986a, section 1.2 and chapter 4) on counterpart theory. Lewis gives possible-worlds analyses of other modal notions in his Lewis (1973, 1986a, chapter 1).

³¹ See also Bigelow and Pargetter (1987).

³² See Putnam (1971), Quine (1951, section 6), and Quine (1960, chapter 7).

incredulous stare compelling; I only know that I do.

3.7 The objection from actualism

Like the incredulous stare, the objection from actualism is directed against the existence of Lewisian possibilia. But whereas the objection behind the incredulous stare is that Lewisian worlds are *unlikely* (for reasons of parsimony), the objection from actualism is that Lewis's ontology is conceptually incoherent. It is not incoherent that there exist fire-breathing dragons and talking donkeys; and it is arguably not incoherent that there exist fire-breathing dragons and talking donkeys that are not spatiotemporally related to me. What is incoherent, according to the objection, is the existence of anything at all that is *non-actual*. The objector upholds *actualism*, the claim that it is a conceptual truth that everything is actual. According to the actualist, Lewis's *possibilism*, his acceptance of the existence of non-actual things, is conceptually incoherent since 'actual' is simply a blanket term for absolutely everything.³³ If Lewisian worlds—concrete entities spatiotemporally unrelated to us—really did exist, they would simply be part of actuality (since the actual includes absolutely everything), and would have nothing to do with possibility and necessity.

Lewis's own response to this objection is powerful. Lewis considers three claims:

- (1) Everything is actual
- (2) Actuality consists of everything that is spatiotemporally related to us, and nothing more.
- (3) Possibilities are not parts of actuality, they are alternatives to it.

According to Lewis, the actualist claims that (1) is analytically true, and that (2) is synthetic (i.e., non-analytic) and probably false, whereas Lewis's view commits him to (2) and the denial of (1). Lewis replies as follows:

...I think [all three theses] are on an equal footing. Together they fix the meaning of 'actual,' but they go far beyond just fixing meanings. I don't see any evidence that the analyticity is concentrated more in some of them than in others. (1986a, 99-100)

³³ See Lycan (1979, section VI), and Lewis (1986a, chapter 2.1).

I would expand on this reply by appeal to the following familiar picture of content determination. Linguistic convention supplies *defeasible* constraints on meaning, constraints that may be defeated by what candidate meanings exist. Example: convention supplies the defeasible constraint that things that are in *contact* have absolutely no space between them; in fact there is no relation of this sort between material bodies (at least, none that fits with other things we say about contact); as a consequence, the meaning of the term ‘contact’ is a relation that does not satisfy this conventional constraint. So in a sense, the sentence ‘things that are in contact have absolutely no space between them’ is analytic but false! It is something like analytic since it is a conventional constraint on the meaning of ‘contact,’ but it is false since this constraint is outweighed by other considerations; consequently ‘contact’ means something that does not obey the constraint. Arguably, the claim that a person has free will only if her actions are causally undetermined is another such claim. Given this picture of meaning, Lewis’s reply could be that (1), (2) and (3) are all partial conventional constraints on the meaning of ‘actual.’ In fact, the best candidate meaning for ‘actual’ (and other modal terms) is given by Lewis’s theory of modality, which vindicates (2) and (3) but not (1). So Lewis can grant the objector that (1) is in a sense analytic; it is nevertheless false.

3.8 Island universes

Other objections to Lewis’s modal realism may be raised, objections that focus on Lewis’s definition of a possible world as a spatiotemporally interrelated whole. This definition rules out the possibility of an absolutely empty possible world (though, as Lewis notes, it allows the possibility of a world containing nothing but empty spacetime (1986a, 73)). More worrisome is that the definition rules out the existence of a single possible world containing two disconnected spacetimes, for Lewis’s definition counts two disconnected spacetimes as two worlds rather than one. Intuitively, one would have thought it possible that there exist disconnected spacetimes, that there exist pairs of things that are in no way spatiotemporally related to each other.³⁴

One could modify Lewis’s definition of a possible world to avoid this difficulty. Suppose Lewis counted *every* possible individual as a possible world. Possible worlds, on this definition, would overlap extensively. I myself would count as a possible world, as would the aggregate of me plus the Eiffel Tower, as

³⁴ For arguments that this is possible, see Bigelow and Pargetter (1987, section 3), and Bricker (2001, 33-39). Bricker also discusses empty worlds.

would the aggregate of me, the Eiffel Tower and the Empire State Building, and so on. Now consider two maximal spatiotemporally interrelated sums—two Lewis-worlds. On the new definition the sum of these two possible individuals itself counts as a possible world. Hence there are possible worlds with disconnected spacetimes, and we have avoided the undesirable consequence that disconnected spacetimes are impossible.

On this view, I inhabit a possible world that contains a talking donkey—take any fusion of me, any possible talking donkey, and zero or more other things. And yet we certainly do not want it to turn out that an everyday utterance by me of the sentence ‘there exists a talking donkey’ would be true. Thus, we should add to the view the claim that in non-modal sentences, quantifiers are typically restricted in some way. (Most of us believe this anyway.) For Lewis, the quantifiers in non-modal sentences are restricted (at least) to objects in the actual world—i.e., the world of the speaker—but on this new view there is no such thing as *the* world of the speaker. Instead the view ought to be that the quantifiers in a non-modal sentence should be restricted (at least) to things spatiotemporally related to the speaker. Everyday utterances of ‘there exists a talking donkey’ no longer turn out true. But in its place there is a different odd consequence. Although the sentence ‘it is possible that there exist disconnected spacetimes’ turns out true, in no possible world could one truly utter ‘there (actually) exist disconnected spacetimes.’ I leave it to the reader to judge whether this is worse than the implausible consequence facing Lewis’s original theory that we were trying to avoid.

3.9 Shalkowski’s objection

An interesting objection of a rather different sort has been raised by Scott Shalkowski (1994).³⁵ Lewis’s analysis is that a (*de dicto*) proposition is possible iff it is true in all Lewis-worlds. But suppose that Lewis-worlds containing nine-foot-tall humans, or purple cows, are simply absent from Lewis’s multiverse. Then certain propositions will incorrectly turn out impossible. To rule out these “gaps in logical space,” Shalkowski argues, Lewis must require that there exists a Lewis-world for every possible way things could have been. But then his account would become circular, for he would need modal notions to characterize his multiverse. Relatedly, suppose a round square, or a box both empty and full, were present in Lewis’s multiverse. Then certain impossible propositions

³⁵ See also McGinn (2000, chapter 4).

would be judged possible by Lewis's analysis. To rule out impossibilia in his multiverse, so the objection goes, Lewis must again make circular use of modal notions—he must claim that his multiverse contains only *possible* objects.

To evaluate this objection we must get clear about the nature of circularity and the aims of analysis. Suppose reality is just the way Lewis thinks it is. That is, suppose that Lewis's multiverse exists, and that there is a possible world for all and only the possible ways things might have been. Never mind what reason there would be for thinking reality to be this way; just suppose for the moment that it does. There is then the question of whether there is room in this reality for modality. Within this multiverse, is there a candidate property we can identify with the property of being a possible proposition? The answer seems to be *yes*—it is the property of being a proposition that is true at some Lewis-world. As shown above, this property can be defined in entirely non-modal terms (in terms of spatiotemporal notions and the restriction of quantifiers). Thus, an adequate non-modal definition of 'possible' can be given, *if* Lewis's ontology is indeed correct. There is then the question of whether it is reasonable to believe that Lewis's ontology is correct. But here Lewis has his Quinean answer—we ought to believe in his ontology because of its theoretical utility. So: if reality is as Lewis says it is then a reductive analysis of modality is possible; moreover, Lewis has an argument that reality is indeed this way.

A reductive analysis of modality must be i) genuinely reductive, and ii) materially adequate. An analysis is genuinely reductive if the terms in the analysans are non-modal; it is materially adequate if the truth values it assigns to modal sentences are the correct ones. Since all that is required for an analysis to be genuinely reductive is that its analysans not contain modal notions, and since Lewis's analysans involves only spatiotemporal notions and quantifier restriction, Lewis's analysis is genuinely reductive, contrary to Shalkowski's claim that it is circular. It is certainly true that there are modal conditions Lewis's multiverse must obey if his analysis is to be materially adequate—as Shalkowski says, the multiverse must contain a maximal spatiotemporally-interrelated whole for each possibility, and it must contain no impossible objects. But the existence of this modal condition of material adequacy does not compromise the genuinely reductive character of the analysis. If the existence of an F-condition of adequacy on an analysis of F-ness would render that analysis circular, then no analysis of *anything* would be non-circular. It is an adequacy condition on the analysis of F-ness as G-ness that all and only *F*s are *G*s. I think, therefore, that Shalkowski's objection should be rejected.

3.10 The Humphrey objection to counterpart theory

Any argument against Lewisian possible worlds is also an argument against the reductive theory of *de dicto* modality that presupposes them. But rejecting Lewisian possible worlds does not require rejecting the counterpart-theoretic analysis of *de re* modality. One could reject Lewis's theory of possible worlds and individuals, supply some other reduction of *de dicto* modality or take it as primitive, and then utilize *de dicto* modality in a construction of ersatz possible worlds and individuals as discussed above. A counterpart relation could then be introduced over these ersatz possible individuals, and something very much like Lewis's analysis of *de re* modality could then be given.³⁶

But there are arguments against counterpart theory that do not turn on the Lewisian conception of the nature of possible individuals, the most famous of which is Saul Kripke's "Humphrey objection." After losing the 1972 presidential election to Richard Nixon, imagine Hubert Humphrey saying to himself "I might have won the election if only I had done such and such." According to counterpart theory, the analysis of Humphrey's claim "I might have won" is that a counterpart of Humphrey's in another possible world wins the election. But, Kripke argues, while Humphrey cares very much that he might have won, surely Humphrey "could not care less whether someone *else*, no matter how much resembling him, would have been victorious in another possible world" (Kripke, 1972, p. 45, n. 13). On one interpretation, the objection is that the counterpart-theoretic analysis of *de re* modal statements is inconsistent with the significance we invest in modal judgments. Another way of taking the objection is simply that counterpart theory gives an untuitively implausible analysis of everyday modal judgments.

I find these objections unconvincing. It must be granted that the average non-philosopher might find the counterpart-theoretic analysis unfamiliar, and perhaps surprising, but if this were an obstacle then few philosophical analyses of any sort would be possible. We must not demand of a correct analysis that it be immediately recognized as such by any competent speaker—we learned this from the paradox of analysis. Our demands must be more modest: the analysis must fit most of our usage of the term being analyzed, it must not be too *ad hoc*, it must presuppose no objectionable ontology or primitive notions, and so on. Whether counterpart theory best fits these desiderata is something that must be settled on the basis of a philosophical investigation into its merits and the merits of competing theories; counterpart theory should not be dismissed out

³⁶ See Stalnaker (1986); Sider (2002).

of hand simply because of the intuitions behind the Humphrey objection.³⁷

4 Conventionalism

4.1 Truth by convention

The old “linguistic” or “conventionalist” theory of necessity has few contemporary adherents, for the most part with good reason. In *Language, Truth and Logic*, A. J. Ayer gives a bald statement of conventionalism. Note that necessity and *a prioricity* are apparently equated, as was not atypical at the time (Ayer’s index entry for “Necessary propositions” reads “See *A priori* propositions”):

The views which are put forward in this treatise derive from the doctrines of Bertrand Russell and Wittgenstein, which are themselves the logical outcome of the empiricism of Berkeley and David Hume. Like Hume, I divide all genuine propositions into two classes: those which, in his terminology, concern “relations of ideas,” and those which concern “matters of fact.” The former class comprises the *a priori* propositions of logic and pure mathematics, and these I allow to be necessary and certain only because they are analytic. That is, I maintain that the reason why these propositions cannot be confuted in experience is that they do not make any assertion about the empirical world, but simply record our determination to use symbols in a certain fashion. (Ayer, 1936, p. 31)

A proposition is analytic, Ayer goes on to say, “when its validity depends solely on the definitions of the symbols it contains...” (p. 78). Analytic propositions can be known *a priori* because they are “devoid of factual content” (p. 78), because they merely “record our determination to use words in a certain fashion.”

Though Ayer is mostly concerned with epistemology, with claiming that logic and mathematics are *a priori* because analytic, he also says that a truth is necessary iff it is analytic in this sense. Analytic truths, for Ayer, “say nothing about the world”; this theory of necessity might, therefore, be thought congenial to metaphysical as well as epistemological reductionists.

Something like this view of necessity was once widely held, both by logical positivists and by ordinary language philosophers.³⁸ A language comes

³⁷ See Hazen (1979, 320-324), Forbes (1985, 143), and Sider (2001b, 194-196).

³⁸ Among conventionalists see Carnap (1937, §69), Carnap (1950), Malcolm (1940). For thorough (critical) discussion and references to conventionalists and their critics see Pap (1958,

equipped with certain rules, which language users conventionally adopt. Certain sentences, analytic sentences, will be true purely in virtue of these rules. Language users *make these sentences true*, by adopting the relevant conventions. The sentence 'It is necessary that ϕ ' is true iff sentence ϕ is one of these analytic sentences.

Some criticisms are familiar. Ayer says that an analytic statement "records our determination to use symbols in a certain way"; if this means that analytic statements are actually *about* language use, then analytic statements would seem to be contingent, since it is contingent that we use language the way we do (Broad, 1936, 107). A related argument may be based on the work of Casimir Lewy (1976, 9). The conventionalist supposes that something like (1) gives the meaning of (2):

- (1) 'Vixen' means the same as 'female fox'
- (2) Necessarily, something is a vixen if and only if it is a female fox

However, it is arguably possible for (2) to be true while (1) is false; imagine that we used 'vixen' as we actually do but used 'female fox' to refer to a kind of ice cream.³⁹

But the most profound objections, I think, are those that challenge the very idea of something's being "true by convention." In what sense is the truth of the following due to convention?

- (B) All bachelors are men

(B) is true in part because of the meanings we assign to the terms 'bachelor' and 'men,' and indeed, to 'all' and 'are.' But all sentences, however contingent and empirical, share this debt to convention: 'the acceleration due to gravity is 9.8 m/s²' would not be true if we used '9' to mean 8 or 'gravity' to mean rocket propulsion. As Quine says in "Truth by Convention," "...definitions are available only for transforming truths, not for founding them" (1966, 81). When one defines 'bachelor' as meaning the same as 'unmarried man,' one provides a new linguistic vehicle for expressing the (logical) truth that all unmarried men are men, namely sentence (B). But this does not make (B) true, for this explanation of its truth depended on the "prior" truth that all unmarried

chapter 7); see also Lewy (1976), especially chapter 5, Boghossian (1997) 1997.

³⁹ For further discussion see Ibberson (1979).

men are men. One can call (B) “analytic,” if this means just that it may be transformed into a logical truth by substituting synonyms for synonyms, but it has not been shown to be conventionally true unless logical truths can be shown to be conventionally true. And as Quine famously goes on to argue, logical truths do not in any important sense owe their truth to conventions.

Quine’s argument that logical truths are not true by convention is ingenious and extremely influential. Nevertheless, I think there is a more compelling case to be made.⁴⁰ What could it mean to say that we make logical truths true by convention? Imagine an attempt to legislate truth: “Let every sentence of the form ‘If P then P’ be true.” What would this accomplish? The legislator could be resolving to use the word ‘true’ in a new way; he could be listing the sentences to which this new term ‘true’ applies. But this isn’t making logic true by convention; it is legislating a new sense of ‘true’. On the other hand, the legislator could be singling out a meaning for ‘if..then’: ‘if..then’ is to stand for a relation, R, between propositions, such that for any proposition, p , the proposition that $R(p, p)$ is true. But this does not amount to logical truth by convention either, for it appeals to an antecedent notion of propositional truth. The propositions $R(p, p)$ are assumed to “already” be true; they are merely used to pick out the desired relation R.

There are a number of ways I can cause the proposition that my computer monitor has been thrown out the window to be true. I could throw the monitor out myself, pay or incite someone else to do it, and so on. I cannot, however, cause this proposition to be true simply by pronouncing. I can pronounce until I am blue in the face, and the computer will remain on my desk; my pronouncements do not affect the truth values of statements about computer monitors. Statements about conventions are different. These we, or at least our linguistic community, *can* make true by pronouncement. A convention consists of the activities of language users; that is why we can so easily make it true that

⁴⁰ Arguments somewhat similar to mine are given by the critics of conventionalism mentioned in note 38. Quine’s arguments in “Truth by Convention” were two. First, he shows that the conventionalist about logic can be aped by an obviously wrong-headed conventionalist about empirical science. This shows that *something* is wrong about logical conventionalism, but doesn’t show exactly what that is. Second, Quine argues that since there are infinitely many logical truths, but we can only give conventions in a finite way, logic itself will be required to infer non-basic logical truths from our conventions. But i) Quine has no adequate response to the claim that our conventions are implicit in our societal linguistic behavior, and ii) Quine has no adequate response to a finitary conventionalist who tries to introduce conventional truth in a language whose set of well-formed formulas is finite, nor to a conventionalist with an infinitary mind who legislates all the truths of logic individually.

conventions exist. (It is hard to say how explicit, recognized, or unanimous the pronouncement must be; indeed, ‘pronouncement’ is stretched.) Only statements *about pronouncements*, for example statements about conventions, seem to be made true by our pronouncements. Statements about monitors, or bachelors, or rain, are about a part of the world we cannot affect simply by pronouncement. That it is either raining or not raining is about rain; I cannot affect the world in the matter of rain simply by pronouncement; therefore I cannot make it the case that either it will rain or it will not rain simply by pronouncement.

A related argument is this. I cannot make it the case that it rains simply by pronouncing, nor can I make it the case that it does not rain simply by pronouncing. But if I cannot make it the case that p simply by pronouncement, nor can I make it the case that q simply by pronouncement, then I cannot make it the case that p or q simply by pronouncement.⁴¹ Therefore, I cannot make it the case that either it rains or it doesn’t rain, simply by pronouncement. Similarly for other logical truths. If ϕ expresses a logical truth, then I cannot in general make it the case that ϕ simply by pronouncement.

Seen in this light, Ayer’s claims that logical truths “depend solely on meaning” and “say nothing about the world” look misleading at best. In what sense does the truth of ‘it is raining or it is not raining’ depend solely on the meanings of its terms? Certainly, its truth depends on the fact that ‘or’ means disjunction and ‘not’ means negation; but doesn’t it also depend on a fact about disjunction and negation, to the effect that any disjunction of a proposition with its negation is true?⁴² In what sense do logical truths “lack factual content? ‘It is raining or it is not raining’ concerns the world, specifically concerning the matter of rain. After all, ‘It is raining’ and ‘It is not raining’ each concerns the world on the subject of rain, and the disjunction says that one or the other will hold. And how could ‘All bachelors are male’ not say anything about the world? It contains a quantifier over bachelors, and says of them that they are male. So it says something about the properties of bachelors – as worldly entities as one could ask for.

⁴¹ It is not true of all sentential operators O that $\sim O(p \text{ or } q)$ follows from $\sim Op$ and $\sim Oq$; the argument merely assumes this to be the case for the operator ‘I can make it the case simply by pronouncement that’.

⁴² It might be said that the truth of ‘it is raining or it is not raining’ does not *modally* depend on anything other than the meaning of this sentence, since it is necessarily true that if the sentence has its actual meaning then it is true. But this modal conception of truth by convention could not play a part in reducing modality.

4.2 Beyond truth by convention

Truth by convention should be rejected, along with conventionalist theories of modality based on that notion. But reductions similar in spirit to conventionalism may yet survive. For example, the claim that necessity is analyticity is separable from the claim that analytic truths are true by convention. One could reject the latter while accepting the former, as did C. I. Lewis.⁴³ This identification of necessity with analyticity was once very popular (though many defenders also subscribed to truth by convention).⁴⁴ Carnap (1947) held a hybrid view that granted an important role to analyticity, but which also appealed to possible worlds construed as sets of atomic sentences. A related view is that necessity is to be identified with some sort of logical truth or provability, the only meaning for ‘necessary’ to which Quine (1953*c*) was friendly. (This was, of course, a consequence of his unfriendliness towards analyticity.) Note that it is probably inaccurate to describe these philosophers as identifying what I have been calling metaphysical necessity with analyticity. It would be better to say that these philosophers rejected, or did not possess, the contemporary concept of metaphysical necessity; their claim was that analytic necessity is the only sensible sort of necessity in the neighborhood.⁴⁵

Few nowadays would identify necessity with analyticity, given the now well-known synthetic sentences that express necessary truths; it is from these cases that the contemporary notion of “metaphysical necessity” springs.⁴⁶

What are these synthetic necessary truths? Many would cite mathematical examples. On some conceptions of mathematical truth, the sentences of mathematics are not analytic, but are nevertheless necessarily true.

I would cite also the laws of mereology, whatever those are. There are some conditions, *C*, such that it is necessarily true that whenever objects satisfy conditions *C*, there exists an object that is composed of those objects, a “fusion” of those objects. I myself would claim that the conditions *C* are vacuous, that objects *always* have a fusion; but even defenders of restrictive mereology will want to claim that there are *some* conditions that necessarily suffice for fusion. However, it is very difficult to see how the sentence “if some objects are in

⁴³ Lewis (1946, Book 1). Lewis does not distinguish *a priori* from necessary truth. See also Lewis and Langford (1932, chapter VI), in which ‘ \diamond ’ is interpreted as “self-consistency,” meaning analytic consistency (see Lewis (1918, chapter V) for an earlier presentation).

⁴⁴ See Pap (1958, part two) for an excellent discussion.

⁴⁵ See Neale (2000) on the history of modal logic and the concept of necessity.

⁴⁶ See Plantinga (1974, p. 2).

conditions C then there exists something that is composed of those objects” could be analytic, for it asserts the *existence* of a thing, and how could a statement of existence be analytic?⁴⁷

Claims about the essences of particular things seem not to be analytic, but are necessary. That George W. Bush is human seems synthetic since names like ‘George W. Bush’ seem to lack definitions, but many would claim it is necessarily true. Likewise, Kripke and Putnam’s cases of the necessary *a posteriori* seem synthetic. Putnam (1975*a*), for example, argues that any liquid not made of H₂O would not be water, and concludes that it is necessary that all water is made of H₂O. But it is not part of the meaning of ‘water’ that water is made of H₂O, since ‘water’ was used long before anyone knew of the chemical makeup of water, and surely ‘water’ has not since changed its meaning.

A final obstacle to the identification of necessity with analyticity is *de re* modality. If ‘ $\Box\phi$ ’ means that the sentence ϕ is an analytic truth, then it is hard to make sense of quantification into modal contexts. Given the usual Tarskian treatment of quantification, ‘ $\exists x\Box Fx$ ’ is true iff there is some object, o , such that ‘ $\Box Fx$ ’ is true relative to an assignment of o to the variable ‘ x ’; ‘ $\forall x\Box Fx$ ’ is true iff for every o , ‘ $\Box Fx$ ’ is true relative to an assignment of o to the variable ‘ x .’ In each case we must make sense of the application of ‘ \Box ’ to ‘ Fx ’—a sentence with a free variable. Given necessity as analyticity, ‘ $\Box Fx$ ’ is true relative to an assignment of o to the variable ‘ x ’ iff ‘ Fx ’ is analytic, relative to an assignment of o to the variable ‘ x .’ But how could ‘ Fx ’ be analytic, relative to an assignment of o to ‘ x ’? Analyticity is a function of meaning, not merely reference, whereas with open sentences relative to assignments we have only reference. This (together with skepticism about a more metaphysical conception of modality, under which quantification into modal contexts *would* make sense) was the heart of Quine’s attack on *de re* modality.⁴⁸

For these reasons many doubt the identification of necessity with analyticity; and as mentioned above, many reject the conventionalist theory of necessity as well. Nevertheless, analyticity and convention may yet play an important role in the reduction of modality. I take it that logical, analytic and mathematical truths do not owe their truth to convention, except in the uninteresting sense in which every true sentence partly owes its truth to the conventions that give that sentence its meaning. It might still be a convention to call logical, analytic

⁴⁷ On mereology see Lewis (1986*a*, 212-213); Sider (2001*b*, chapter 4, section 9.1); and van Inwagen (1990). For an argument that statements of existence are not analytic see Sider (2001*a*) and the introduction to Sider (2001*b*). The chief opposing view is that of Carnap (1950).

⁴⁸ See Quine (1953*b*); Burgess (1997); Neale (2000).

and mathematical truths necessary. It would be analytic to ‘necessary’ that logical, analytic and mathematical truths are necessary. ‘Necessary’ would be a word used for truths of certain kinds.

Pretend for the moment that only logical, analytic and mathematical truths are necessary. One could then hold that ‘necessary’ just means ‘is either a logical, analytic or mathematical truth.’ This theory is reductive, and similar in spirit to conventionalism, but it makes no objectionable assumptions about “truth by convention.” On this theory, there is a convention to call logical, analytic and mathematical truths necessary. So, provided ‘ $2+2=4$ ’ is a mathematical truth, the following sentence will be true:

Necessarily, $2+2=4$

Convention can do this much. It need not play any role in making it true that $2+2=4$, or in making this be a mathematical truth. ‘ $2+2=4$ ’ is made true by whatever makes mathematical truths true generally (perhaps facts about numbers in Platonic heaven); its status as a *mathematical* truth is made true by whatever generally makes mathematical truths mathematical (perhaps the fact that its subject matter is mathematics).

The contingency objection was that conventionalism turns statements about logic and mathematics into statements *about* conventions, which then inherit the contingency of conventions. The present theory has no such consequence, nor does it imply that statements that logical, analytic or mathematical truths are necessary are about conventions, nor does it imply that such statements are mind-dependent. It is a convention to call logical, analytic and mathematical truths necessary, but the *content* of a statement of, say, mathematics is just mathematical, and the content of a statement of necessity is just that a certain sentence or proposition is a logical, analytic or mathematical truth, which has nothing to do with convention.

Conventionalists granted a special status to convention as a source of truth. This was an essential part of the epistemology of many conventionalists. Ayer, for instance, used it to explain our knowledge of logical and mathematical truths. But if we are not trying to fit modality into an overly demanding epistemology, we do not need convention to play this role.

Conventionalists also seemed to regard truth by convention as an essential ingredient to a reductive theory of necessity. As Paul Boghossian (1997, 336) puts it:

Guided by the fear that objective, language-independent, necessary connections would be metaphysically odd, [the positivists] attempted to show

that all necessities could be understood to consist in linguistic necessities...Linguistic meaning, by itself, was supposed to generate necessary truth; *a fortiori*, linguistic meaning, by itself, was supposed to generate truth.

But the theory just sketched shows we do not need truth by convention to account for modality without appealing to anything “metaphysically odd.” Moreover, truth by convention would not have demystified modality on its own anyway. Like many writers, Boghossian seems to presuppose that if linguistic meaning generates truth, then it automatically generates *necessary* truth. This does not follow without further assumptions about necessity. Some account of necessity is still required to bridge the gap between ‘true by convention’ and ‘necessary.’ If necessity were truth in all Lewisian worlds then the gap would be bridged, for then conventional truth would suffice for necessary truth (provided that the same conventions were adopted for talk about worlds other than our own). But no positivist had *this* in mind. The thought must rather have been to bridge the gap by a convention involving ‘necessary’: ‘necessary’ just means ‘made true by convention.’ But now it is clear that truth by convention was never needed, for ‘necessary’ could just as well be regarded as obeying a different convention, one that does not require truth by convention, for example the convention that ‘necessary’ applies to the truths of mathematics and logic, and to analytic truths.

The theory just sketched is far too simple to be plausible. Some necessities are neither logical nor mathematical nor analytic; moreover, the theory as stated has nothing to say about *de re* modality. Its existence nevertheless makes a point: theories that claim that necessary truths are truths *of a certain kind* enjoy some of the virtues of traditional conventionalism while avoiding many of the standard objections. Whether a workable theory of this sort can be developed thus becomes an interesting and important question. There are obstacles, but I believe that this approach should be taken seriously by reductionist-minded philosophers of modality.⁴⁹

Rutgers University
sider@philosophy.rutgers.edu

⁴⁹ I hope to pursue this strategy in a future paper. A somewhat related reductive account is defended by Christopher Peacocke (1997, 1999).

References

- Adams, Robert Merrihew (1974). "Theories of Actuality." *Noûs* 8: 211–231.
Reprinted in Loux 1979: 190–209.
- Armstrong, David M. (1989). *A Combinatorial Theory of Possibility*. New York: Cambridge University Press.
- (1997). *A World of States of Affairs*. Cambridge: Cambridge University Press.
- Ayer, Alfred Jules (1936). *Language, Truth and Logic*. London: Victor Gollancz Ltd., 1946. Second edition.
- Bigelow, John and Robert Pargetter (1987). "Beyond the Blank Stare." *Theoria* 53: 97–114.
- Blackburn, Simon (1987). "Morals and Modals." In *Fact, Science and Value, Essays in Honour of A.J. Ayer's Language, Truth and Logic*. Oxford: Blackwell.
Reprinted in Blackburn 1993.
- (1993). *Essays in Quasi-Realism*. Oxford: Oxford University Press.
- Boghossian, Paul (1997). "Analyticity." In Bob Hale and Crispin Wright (eds.), *A Companion to the Philosophy of Language*, 331–368. Oxford: Blackwell.
- Bricker, Phillip (1987). "Reducing Possible Worlds to Language." *Philosophical Studies* 52: 331–355.
- (2001). "Island Universes and the Analysis of Modality." In Gerhard Preyer and Frank Siebelt (eds.), *Reality and Humean Supervenience: Essays on the Philosophy of David Lewis*, 27–55. Lanham, MD: Rowman & Littlefield.
- Broad, C. D. (1936). "Are there Synthetic *A Priori* Truths?" *Aristotelian Society Supplementary Volume* 15: 102–117.
- Burgess, John P. (1997). "Quinus ab Omni Naevo Vindicatus." *Canadian Journal of Philosophy* Supplementary volume 23: 25–65.
- Carnap, Rudolf (1937). *The Logical Syntax of Language*. London: Routledge & Kegan Paul.

- (1947). *Meaning and Necessity: A Study in Semantics and Modal Logic*. Chicago: University of Chicago Press.
- (1950). “Empiricism, Semantics and Ontology.” *Revue Internationale de Philosophie* 4: 20–40. Reprinted in *Meaning and Necessity: A Study in Semantics and Modal Logic*. 2nd edn. Chicago: University of Chicago Press, 1956.
- Chalmers, David (1996). *The Conscious Mind*. Oxford: Oxford University Press.
- Chihara, Charles S. (1998). *The Worlds of Possibility*. Oxford: Clarendon.
- Cresswell, M. J. (1973). *Logics and Languages*. London: Methuen.
- Cresswell, M.J. (1972). “The World Is Everything That Is The Case.” *Australasian Journal of Philosophy* 50: 1–13. Reprinted in Loux 1979: 129–145.
- Cresswell, M.J. and G.E. Hughes (1996). *A New Introduction to Modal Logic*. London: Routledge.
- Davidson, Donald (1967). “Truth and Meaning.” *Synthese* 17: 304–323.
- Dowty, David R., Robert E. Wall and Stanley Peters (1981). *Introduction to Montague Semantics*. Dordrecht: Kluwer.
- Feldman, Fred (1986). *Doing the Best We Can*. Dordrecht: D. Reidel.
- Field, Hartry (1989). *Realism, Mathematics and Modality*. Oxford: Blackwell.
- Fine, Kit (1977). “Postscript.” In *Worlds, Times, and Selves*. London: Duckworth.
- (1980). “First-Order Modal Theories, Part II—Propositions.” *Studia Logica* 39: 159–202.
- (1981). “First-Order Modal Theories, Part I—Sets.” *Noûs* 15: 117–206.
- (1982). “First-Order Modal Theories, Part III—Facts.” *Synthese* 53: 43–122.
- (1985). “Plantinga on the Reduction of Possibilist Discourse.” In J. Tomberlin and Peter van Inwagen (eds.), *Alvin Plantinga*, 145–186. Dordrecht: D. Reidel.
- Forbes, Graeme (1985). *The Metaphysics of Modality*. Oxford: Oxford University Press.

- French, Peter, Theodore E. Uehling, Jr. and Howard K. Wettstein (eds.) (1986). *Midwest Studies in Philosophy XI: Studies in Essentialism*. Minneapolis: University of Minnesota Press.
- Hazen, Allen (1979). "Counterpart-Theoretic Semantics for Modal Logic." *Journal of Philosophy* 76: 319–38.
- Heller, Mark (1998). "Property Counterparts in Ersatz Worlds." *Journal of Philosophy* 95: 293–316.
- Ibberson, John (1979). "Necessity by Convention." *Mind* 88: 554–571.
- Jeffrey, Richard (1965). *The Logic of Decision*. New York: McGraw-Hill.
- Kim, Jaegwon (1993). *Supervenience and Mind*. Cambridge: Cambridge University Press.
- Kneale, W. C. (1949). *Probability and Induction*. Oxford: Clarendon.
- Kratzer, Angelika (1977). "What 'Must' and 'Can' Must and Can Mean." *Linguistics and Philosophy* 1: 337–355.
- Kripke, Saul (1972). "Naming and Necessity." In Donald Davidson and Gilbert Harman (eds.), *Semantics of Natural Language*, 253–355, 763–769. Dordrecht: Reidel. Revised edition published in 1980 as *Naming and Necessity* (Cambridge, MA: Harvard University Press).
- Lewis, C. I. (1918). *A Survey of Symbolic Logic*. Berkeley: University of California Press.
- (1946). *An Analysis of Knowledge and Valuation*. LaSalle, IL: Open Court.
- Lewis, C. I. and C. H. Langford (1932). *Symbolic Logic*. New York: Century Company.
- Lewis, David (1968). "Counterpart Theory and Quantified Modal Logic." *Journal of Philosophy* 65: 113–126. Reprinted in Lewis 1983: 26–46.
- (1970). "General Semantics." *Synthese* 22: 18–67. Reprinted in Lewis 1983: 189–232.
- (1971). "Counterparts of Persons and their Bodies." *Journal of Philosophy* 68: 203–211. Reprinted in Lewis 1983: 26–46.

- (1973). *Counterfactuals*. Oxford: Blackwell.
- (1980). “A Subjectivist’s Guide to Objective Chance.” In R. C. Jeffrey (ed.), *Studies in Inductive Logic and Probability*, volume 2, 263–93. Berkeley: University of California Press. Reprinted in Lewis 1986b.
- (1983). *Philosophical Papers, Volume 1*. Oxford: Oxford University Press.
- (1986a). *On the Plurality of Worlds*. Oxford: Basil Blackwell.
- (1986b). *Philosophical Papers, Volume 2*. Oxford: Oxford University Press.
- (1992). “Critical Notice of Armstrong’s *A Combinatorial Theory of Possibility*.” *Australasian Journal of Philosophy* 70: 211–224. Reprinted in Lewis 1999: 196–214.
- (1999). *Papers in Metaphysics and Epistemology*. Cambridge: Cambridge University Press.
- Lewy, Casimir (1976). *Meaning and Modality*. Cambridge: Cambridge University Press.
- Loux, Michael J. (ed.) (1979). *The Possible and the Actual*. Ithaca: Cornell University Press.
- Lycan, William (1979). “The Trouble with Possible Worlds.” In Loux (1979), 274–316.
- Malcolm, Norman (1940). “Are Necessary Propositions Really Verbal?” *Mind* 49: 189–203.
- McGinn, Colin (2000). *Logical Properties*. Oxford: Oxford University Press.
- McMichael, Alan (1983). “A Problem For Actualism About Possible Worlds.” *Philosophical Review* 92: 49–66.
- Neale, Stephen (2000). “On a Milestone of Empiricism.” In Alex Orenstein and Petr Kotatko (eds.), *Knowledge, Language and Logic*, 237–346. Dordrecht: Kluwer.
- Pap, Arthur (1958). *Semantics and Necessary Truth*. New Haven: Yale University Press.

- Peacocke, Christopher (1997). "Metaphysical Necessity: Understanding, Truth and Epistemology." *Mind* 521–574.
- (1999). *Being Known*. Oxford: Clarendon.
- Plantinga, Alvin (1974). *The Nature of Necessity*. Oxford: Oxford University Press.
- (1976). "Actualism and Possible Worlds." *Theoria* 42: 139–160. Reprinted in Loux 1979: 253–273.
- Prior, A. N. (1955). "Diodoran Modalities." *Philosophical Quarterly* 5: 205–213.
- Putnam, Hilary (1971). *Philosophy of Logic*. New York: Harper & Row.
- (1975a). "The Meaning of Meaning." In Keith Gunderson (ed.), *Language, Mind and Knowledge*, volume 7 of *Minnesota Studies in the Philosophy of Science*, 131–193. Minneapolis: University of Minnesota Press. Reprinted in Putnam 1975b: 215–271.
- (1975b). *Mind, Language and Reality: Philosophical Papers, volume 2*. Cambridge: Cambridge University Press.
- Quine, W. V. O. (1936). "Truth by Convention." In O. H. Lee (ed.), *Philosophical Essays for A. N. Whitehead*, 90–124. New York: Longmans. Reprinted in Quine 1966: 70–99.
- (1948). "On What There Is." *Review of Metaphysics* 2: 21–38. Reprinted in Quine 1953a: 1–19.
- (1951). "Two Dogmas of Empiricism." *Philosophical Review* 60: 20–43. Reprinted in Quine 1953a: 20–46.
- (1953a). *From a Logical Point of View*. Cambridge, Mass.: Harvard University Press.
- (1953b). "Reference and Modality." In Quine (1953a), 139–159.
- (1953c). "Three Grades of Modal Involvement." In *Proceedings of the 11th International Congress of Philosophy*, volume 14, 65–81. Amsterdam: North-Holland. Reprinted in Quine 1966: 156–174.

- (1960). *Word and Object*. Cambridge, Mass.: MIT Press.
- (1966). *The Ways of Paradox*. New York: Random House.
- (1968). “Propositional Objects.” *Critica* 2: 3–22. Reprinted in Quine 1969: 139–160.
- (1969). *Ontological Relativity and other Essays*. New York: Columbia University Press.
- Rosen, Gideon (1990). “Modal Fictionalism.” *Mind* 99: 327–354.
- Roy, Tony (1995). “In Defense of Linguistic Ersatzism.” *Philosophical Studies* 80: 217–242.
- Shalkowski, Scott (1994). “The Ontological Ground of the Alethic Modality.” *Philosophical Review* 103: 669–688.
- Shoemaker, Sydney (1998). “Causal and Metaphysical Necessity.” *Pacific Philosophical Quarterly* 79: 59–77.
- Sider, Theodore (2001a). “Criteria of Personal Identity and the Limits of Conceptual Analysis.” *Philosophical Perspectives* 15: 189–209.
- (2001b). *Four-Dimensionalism*. Oxford: Clarendon.
- (2001c). “Review of Charles Chihara, *The Worlds of Possibility*.” *Philosophical Review* 110: 88–91.
- (2002). “The Ersatz Pluriverse.” *Journal of Philosophy* 99: 279–315.
- Skyrms, Brian (1981). “Tractarian Nominalism.” *Philosophical Studies* 40: 199–206. Reprinted in Armstrong 1989: 145–152.
- Stalnaker, Robert (1976). “Possible Worlds.” *Noûs* 10: 65–75. Reprinted in Loux 1979: 225–234.
- (1986). “Counterparts and Identity.” In French et al. (1986), 121–140.
- van Inwagen, Peter (1986). “Two Concepts of Possible Worlds.” In French et al. (1986), 185–213. Reprinted in van Inwagen 2001: 206–242.
- (1990). *Material Beings*. Ithaca, NY: Cornell University Press.

— (2001). *Ontology, Identity and Modality*. Cambridge: Cambridge University Press.

Wittgenstein, Ludwig (1961). *Tractatus Logico-Philosophicus*. London: Routledge & Kegan Paul. Translation by D. F. Pears and B. F. McGuinness; first published in 1921.