# Macroeconomic Theory II, Spring 2009: Homework 2

**The assignment is due next Wednesday, April 8**

**Problem 1**

Exercise 16.2 in L-S, page 561.

**Problem 2**

A consumer chooses her consumption stream by solving

$$
\begin{aligned}
& \max_{\{c_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\
& s.t. \\
c_t + a_{t+1} &= (1+r)\, a_t + y_t, \\
a_{t+1} &\geq -\phi
\end{aligned}
$$

The income process $y_t \in Y = \{y_1, y_2, ..., y_N\}$ follows a Markov chain $\pi(y', y) = \Pr(y_{t+1} = y' | y_t = y)$. The consumer also faces a "natural borrowing constraint", i.e. $\phi = y_1/r$, where $r$ is the constant real interest rate.

a) Can you think of a joint condition on the period-utility $u$ and on the transition matrix $\pi$ such that the natural borrowing constraint is never binding for the agent (i.e. the solution of the problem above is always interior for all $t$)?

b) Suppose that the agent does *not* face any borrowing constraint. What joint condition on $u$ and on the income process (i.e., on $Y$ and on $\pi$) do we need to impose to insure that $a_t > 0$ at every $t$ ?

**Problem 3**

Assume that the consumer has CRRA period utility over consumption given by

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

with $\gamma > 0$, discounts the future at rate $\beta \in (0,1)$ and faces a constant interest rate $R$. Assume that consumption is conditionally log-normal with mean $E_t \ln(c_{t+1}) = \mu_t$ and variance $v_t$.

a) Show that the optimal consumption path follows

$$E_t(\Delta \ln c_{t+1}) = \frac{1}{\gamma} \ln(\beta R) + \frac{1}{2}\gamma v_t. \tag{1}$$

b) Based on the equation above, does the agent display precautionary saving behavior?

c) Suppose we tested the Permanent Income Hypothesis (PIH) by running the regression

$$\Delta \ln c_{t+1} = \alpha_0 + \alpha_1 y_t + \varepsilon_{t+1},$$

where $y_t$ is past income and we found that $\alpha_1$ is significantly different from zero. Based on your analysis above, could you say that this result means necessarily a rejection of the PIH?

**Problem 4: Consumption with CARA utility**

Consider the "income fluctuation problem" of an agent with CARA utility. Precisely, assume that the agent is infinitely lived, discounts the future at the factor $\beta$, faces i.i.d. income shocks $y_t$, can save/borrow through a risk-free asset with constant gross interest rate $R$ (ignore borrowing limits), and has period utility

$$u(c_t) = -\frac{1}{\sigma} e^{-\sigma c_t}.$$

Guess that the optimal consumption allocation takes the following form

$$c_t = B(Ra_t + y_t) + D$$

where $B$ and $D$ are constants and have to be determined.

a) Solve for the consumption allocation in closed form (i.e., determine the two constants) and argue that the "precautionary saving motive" is constant across all agents, i.e., it is independent of the individual pair of state variables $(a, y)$. Explain your answer.