Macroeconomic Theory II, Spring 2009: Homework 3

Problem 1

Consider the consumption-saving problem discussed in class,

$$\begin{aligned} V(a,y) &= \max_{\{c,a'\}} \left\{ u(c) + \beta \sum_{y' \in Y} \pi(y',y) V(a',y') \right\} \\ & \quad s.t. \\ c + a' &\leq (1+r)a + y \\ a' &\geq -\phi \end{aligned}$$

Assume that:

- The income process is iid and y is $U[0,1]$
- The borrowing limit is $\phi = 0$, i.e. the agent is not allowed to accumulate any debt.
- Period utility is given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 2$, and the discount factor is $\beta = 0.95$.
- The interest rate is $r = 0.02$.

a) Rewrite the Bellman equation in one single state variable, $w = (1+r)a + y$

b) With the help of Matlab, compute the decision rule $a'(w)$, $c(w)$

c) Simulate agent decision rules. Say 10,000 agents with initial conditions of your choice and simulate their paths using decision rules above by using a random number generator (rand) . Simulate this economy for $T = 100$ periods and check if the distribution of w has converged. This is the invariant distribution.

d) What is the Gini coefficient in this economy? Share of wealth accounted for by top 5, 1%? Aggregate supply of assets?

e) Redo the exercise for $r = .0, r = .01, r = .02, r = .03$ and plot the resulting aggregate supply of assets curve.

Problem 2

Recall that in Problem Set 2 you have solved the "income fluctuation problem" of an agent with CARA utility. You have assumed that the agent is infinitely lived, discounts

the future at the factor β, faces i.i.d. income shocks y_t, can save/borrow through a risk-free asset with constant gross interest rate $(1+r)$ (ignore borrowing limits), and had period utility

$$u(c_t) = -\frac{1}{\sigma}e^{-\sigma c_t}.$$

You have shown that the optimal consumption allocation takes the following form

$$c_t = Bw_t + D$$

where B and D are constants you have determined and $w_t = (1+r)\,a_t + y_t$

a) Solve for the consumption allocation numerically, by choosing whatever parameter values you like, and verify the solution against the one you have obtained in closed form. Work with one state variable only.

For both problems, you should submit a print-out of your code as well as a short note discussing the solution. Feel free, as always, to work in groups.