

# Macroeconomic Theory II, Spring 2009: Homework 4

## Problem 1

Time is discrete and indexed by  $t \in \{0, 1, \dots, \infty\}$ . The economy is populated by a continuum of measure one of infinitely-lived households indexed by  $i$  on  $[0, 1]$ . Preferences for individual  $i$  are given by

$$U(c_0^i, c_1^i, \dots; h_0^i, h_1^i, \dots) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, 1 - h_t^i),$$

where  $\beta \in (0, 1)$  is the time discount factor,  $c \geq 0$  denotes consumption and  $h \geq 0$  hours worked. Household  $i$  at time  $t$  supplies the optimal amount of hours worked  $h_t^i$  to a competitive labor market, taking the hourly wage  $\omega_t$  as given.

Each household owns a single private firm, i.e. firm  $i$  is identified with household  $i$ . Each firm produces the final good  $y$  (which can be used for consumption or investment) through

$$y_t^i = F(z_t^i, k_t^i, n_t^i),$$

where  $F$  is a production technology displaying constant-returns in capital and labor. The variable  $z_t^i$  represents a firm-specific productivity shock which follows the Markov process  $\pi(z', z) = \Pr\{z_{t+1}^i = z' | z_t^i = z\}$  with  $z_{\min} = 0$ .

Households can freely hire labor on the market for their firm, but there is no market to trade claims on physical capital, thus households can only invest the capital they own in their firm. Moreover,  $k_t^i \geq 0$ . Capital depreciates at rate  $\delta$ . Let  $\pi_t^i$  denote the profits accruing to individual  $i$  from operating her firm at time  $t$ .

Households can trade a risk-free bond  $b$  in zero-net supply with return  $R_t$  and they can accumulate debt up to the “natural borrowing constraint”.

Assume that the economy is in a stationary equilibrium.

a) Do firms operate all with the same capital-labor ratio? Can you aggregate individual firms into a unique aggregate production technology  $F(Z, K, N)$ ? Explain your answer.

b) What is the natural borrowing constraint faced by each household?

c) List explicitly the individual state variables and write down the problem of the household in recursive formulation.

d) Define a Recursive Competitive Equilibrium (RCE) for this economy.

e) Discuss the existence and uniqueness of the RCE for this economy.

Imagine now that a new financial market opens allowing agents to invest their own  $k$  in other households' firms.

f) What is a simple way to represent the new economy with this additional market? In particular, list what the individual state variables become in this case.

## Problem 2

Consider an economy where households have period utility  $u(c, 1 - h)$  where  $c$  is consumption and  $h$  is hours worked. Households can be in either one of two states  $s \in \{u, e\}$  denoting employment ( $e$ ) and unemployment ( $u$ ). Employed workers supply hours  $h$  to the market and receive wages  $w$  per hour. Unemployment means being out of work for the period, with no earnings but unemployment insurance (UI)  $b$  provided by a left-wing government. This employment-status shock follows a Markov chain  $\Gamma(s', s)$  and is uninsurable directly on the market.

There is no other individual productivity shock and no aggregate uncertainty. Households can save through a risk-free bond, and borrow up to the natural limit (which is what?). The aggregate production technology is a standard CRS production function. Labor and capital markets are competitive. The government finances UI expenditures by taxing employed workers proportionately to their labor earnings at rate  $\tau$  and balances its budget every period.

1) State the household problem recursively. Define a recursive stationary equilibrium for this economy.

2) Suppose that the right-wing party is thinking about running for elections on a platform based on “Less Taxes for Everyone” and asks you (the economist) to compute the welfare change of completely eliminating UI benefits (and associated taxes). How would you make this computation? What would you expect to happen to equilibrium prices and allocations during the transition?

3) Suppose now that the left-wing party, worried about being taken out of office, is thinking of lowering taxes by establishing a time-limit to UI benefits (i.e. after  $n$  periods of unemployment, UI benefits drop to  $\underline{b} < b$ ). How would you change the answer to point 1) in case this is the way UI gets implemented?

### Problem 3

Households are infinitely lived, have period utility  $u(c_{it}, 1 - h_{it})$  and discount factor  $\beta$ . A household can be either a worker or a retiree. Workers' efficiency units of labor are given by

$$\begin{aligned}\varepsilon_{it} &= \pi_{it} \exp(y_{it}), \text{ with} \\ y_{it} &= \rho y_{i-1,t-1} + v_{it},\end{aligned}$$

with  $\pi_{it} = 1$ , while a retiree has  $\pi_{it} = 0$ . Every period, with probability  $\delta$ , households switch their type between worker and retiree and viceversa. When they switch type, households retain their wealth and their history of labor productivity shocks.

Households save through a risk-free asset with rate of return  $r_t$ . Working households are free to borrow up to a limit  $b$ , while retirees cannot borrow. Wages per efficiency unit are  $w_t$  and are determined in the competitive labor market. Firms operate a standard CRS production function.

The government implements a pay-as-you-go social security system that pays to each retiree, every period, benefits  $d_{it}$  equal to a fraction  $\rho$  of her average earnings computed over her last lifetime spell as a worker. The system is financed through a labor income tax  $\tau_t$  paid by current workers. The government budget is balanced every period.

1) State the household problem in dynamic programming formulation. Define a recursive competitive stationary equilibrium for this economy.

2) As the economy is peacefully resting in its steady-state, the government announces that will privatize social-security (i.e., set  $\tau = 0$  and  $\rho = 0$ ) starting from the next day. Explain how you would compute the equilibrium of this economy during the transition induced by this announcement.

3) What would you expect to happen to equilibrium prices and allocations during the transition?