

International Price Dispersion in State-Dependent Pricing Models

Technical Appendix

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The typical approach used in solving state-dependent pricing or inventory models is the simulation technique proposed by Krusell and Smith (1997) and used by Willis(2003) and Khan and Thomas(2003) in models with non-convexities. The method involves replacing the distribution of the firms' last period's prices with a vector of its moments, postulating a linear relationship between aggregate prices and quantities and the state of the world, and solving for the unknown coefficients of these approximants by minimizing their in-sample forecast errors.

We depart slightly from the standard method, and use a solution technique free of simulations, one that draws heavily on collocation, a residual-based functional approximation method discussed at length in Miranda and Fackler (2002). A simulation-free solution technique used to solve models with heterogeneous agents was originally suggested by den Haan (1997) in the context of an uninsurable idiosyncratic risks model. The advantage of this solution method is its explicit reliance on numerical theory, as the nodes at which equilibrium conditions are solved, and the basis functions used in approximation, are chosen to ensure optimality of the approximants.

Before discussing how we solve for the aggregate prices and quantities, we turn to the solution of the firm's problem.

We solve the system of two functional equations that characterize the firm's problem in (8) using collocation. More specifically, we approximate each of the two value functions using a linear combination of N Chebyshev polynomials. To solve for the $2N$ unknown coefficients, we require that the Bellman equations hold at $2N$ nodes in the state space. This condition yields $2N$ equations we use to solve for the unknown coefficients. In addition, one needs to solve the firm's maximization problem in (8) and evaluate the expectations on the RHS of the Bellman equation by discretizing the distribution of shocks and integrating using Gaussian quadrature. We evaluate the accuracy of our solution method by calculating the difference in the two sides of the Bellman equations for points other than the collocation nodes. These residuals are small (less than 5×10^{-4} in absolute

value) and are equioscillatory, a property typical of Chebyshev approximations.

We next turn to the solution of the equilibrium conditions. We have found that the most efficient approximation to the distribution of last period's prices (conditional on trade costs) is the mean of the deviations of the firms' last period's prices from their steady-state optimum, $\hat{\mu}_t = \frac{\tilde{p}_{t-1}(z)}{\tilde{p}_{ss}(z)}$ as there is little variation in this moment across firms of different types and introducing a higher moment adds little precision to our approximation to the aggregate functions. The following five functional equations (in five unknown functions: $C(s), \tilde{P}(s), \tilde{W}(s), \tilde{e}(s), \Gamma(s)$ where $s = (g, \hat{\mu})$ is the state of the world, and $\Gamma(s)$ the law of motion of μ , are sufficient to characterize the equilibrium of this economy:

$$\begin{aligned}\frac{\tilde{W}(s)}{\tilde{P}(s)} &= \psi C(s)^\kappa \\ \tilde{P}(s)C(s) &= 1 \\ \frac{C(s)^{-k}}{\tilde{P}(s)} &= \frac{\tilde{\lambda}}{\tilde{e}(s)} \\ \tilde{P}(s) &= \left(\int_0^1 \left[\tilde{p}(z; s)^{1-\gamma} + (\tau(z)\tilde{e}(s)p^*(z))^{1-\gamma} \right]^{\frac{1}{1-\gamma}} dz \right)^{\frac{1}{1-\theta}} \\ \Gamma(s) &= \int_0^1 \frac{\tilde{p}_{-1}(z; s)}{\tilde{p}(z)} dz,\end{aligned}$$

where $\tilde{p}(z; s)$ are the prices that solve the firms' problems, and $\tilde{p}(z)$ the firm's steady-state optimal price.

We solve for the aggregate functions by replacing them with a combination of Chebyshev polynomials. Given an initial guess for the coefficients on these polynomials, we solve the firm's problems and recompute a new set of aggregate quantities and prices at each state of the world used to discretize the state-space. We approximate the unknown functions using a relatively small number of basis functions (typically $K = 64; 8$ for each dimension in the aggregate state-space) but solving the model at a larger number of nodes (typically $M = 144$) and retrieving the unknown coefficients by minimizing the sum of squared residuals. For example, letting \tilde{P} be a $M \times 1$ vector of home prices that satisfy the equilibrium conditions at each node, Φ be a $M \times K$ matrix of K Chebyshev polynomials evaluated at M nodes, we find the K unknown coefficients c by solving

$$\min_c \left(\tilde{P} - \Phi c \right)' \left(\tilde{P} - \Phi c \right)$$

This set of coefficients for all aggregate variables is used to re-solve the firm's problem, obtain a set of new aggregate variables at each node and calculate a new c . The convergence criterion is the norm of the difference between the last two sets of c and we typically stop when $norm < 10^{-5}$. Once the algorithm converges, our approximants produce accurate out-of sample forecasts, and explain 95% of the variation of aggregate variables in simulations of the model.

References

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