1 Monopolistic Competition and Menu Costs

We consider an extension to the economy studied in Hopenhayn (1992) to allow for imperfect competition in the goods market. We assume a role for money and study the effect of frictions that prevent adjustment of prices to changes in monetary policy. We consider an economy with menu costs of changing prices. As in the models we have studied earlier, firms are heterogeneous, here because of differences in the timing of their price changes. We show how the response of aggregate quantities to changes in monetary policy critically depend on the distribution of prices at the firm level. The references here are Blanchard and Kiyotaki (1987), Caplin and Spulber (1987) and Caplin and Leahy (1991).

1.1 Environment

Consumers

The representative consumer has preferences over consumption, c_t, and work, n_t. Let us assume as earlier linear disutility from work. Consumption here is a CES bundle of goods purchased from the different firms in this economy. Unlike in Hopenhayn's model, we no longer assume that goods produced by different firms are perfect substitutes. Rather, letting $c_t\left(i\right)$ denote consumption of good produced by firm i and normalizing the mass of firms to 1, we have

$$C_t = \left(\int_0^1 c_t\left(i\right)^{\frac{\theta-1}{\theta}}\, di\right)^{\frac{\theta}{\theta-1}}$$

The consumer maximizes

$$\max_{c_t(i),n_t,\mathbf{B}'} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - n_t\right]$$

subject to

$$C_t = \left(\int_0^1 c_t\left(i\right)^{\frac{\theta-1}{\theta}}\, di\right)^{\frac{\theta}{\theta-1}}$$

$$\int_0^1 p_t\left(i\right) c_t\left(i\right) di + \mathbf{q}\cdot\mathbf{B}' \leq W_t n_t + \Pi_t + B_t$$

where $p_t\left(i\right)$ is the price of good sold by firm i, W_t is the wage, and the consumer's other sources of income are the bonds from the previous period, and Π_t the profits it receives from its ownership of firms. We assume complete markets here, $\mathbf{q} \cdot \mathbf{B}'$ is the expenditure on state contingent bonds that pay 1 unit if a particular state is realized next period, and $\mathbf{q}$ is a vector of prices for these securities

We introduce demand for money in an ad-hoc fashion, by simply assuming that that demand for money is interest inelastic and equal to

$$M_t = \int_0^1 p_t(i)\, c_t(i)\, di$$

Think of "cash-in-advance" type constraint where goods must be paid for with cash, and the consumer goes to the central bank and demands M_t units and then repays (at no interest) at the end of the period out of labor and dividend income. We assume We assume the central bank controls the money supply. In particular, assume

$$\log M_t - \log M_{t-1} = g_t$$

and $g_t \sim iidN(g, \sigma^2)$. This is the only source of uncertainty in this economy.

Characterize consumer decision rules

It is helpful to split the consumer's problem into two stages: a) decide how to allocate her income across the different goods and b) the dynamic part of deciding how much money/bonds to carry into the next period and how much to consume. You can verify if you would like that the direct approach yields identical results.

a) static problem.

Suppose that in the second stage the.consumer decides she wants to allocate E_t of its income for consumption. The problem is: how does she allocate E_t among the different goods? Clearly, she does this so as to maximize c_t subject to the constraint that she spends less than E_t.

$$\max \left(\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} + \lambda \left(E_t - \int p_t(i)\, c_t(i)\, di \right)$$

The FOC w.r.t. $c_t(i)$ is

$$c_t^{\frac{1}{\theta}} c_t(i)^{\frac{-1}{\theta}} = \lambda P_t(i)$$

A similar Foc must hold for any other good j. Dividing the two we have

$$\frac{c_t(i)^{\frac{-1}{\theta}}}{c_t(j)^{\frac{-1}{\theta}}} = \frac{P_t(i)}{P_t(j)}$$

or

$$c_t(j) = \left(\frac{P_t(j)}{P_t(i)}\right)^{-\theta} c_t(i)$$

Let us now use the definition of c_t to express $c_t(i)$ as a function of c_t rather than $c_t(j)$:

$$\begin{aligned} c_t(j)^{\frac{\theta-1}{\theta}} &= \left(\frac{P_t(j)}{P_t(i)}\right)^{1-\theta} c_t(i)^{\frac{\theta-1}{\theta}} \text{ (raise both sides to } \frac{\theta-1}{\theta}\text{)} \qquad (1) \\ \int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj &= P_t(i)^{\theta-1} c_t(i)^{\frac{\theta-1}{\theta}} \int_0^1 (P_t(j))^{1-\theta} dj \text{ (integrate over } dj\text{)} \\ \left(\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} &= c_t = P_t(i)^{\theta} c_t(i) \left(\int_0^1 (P_t(j))^{1-\theta} dj\right)^{\frac{\theta}{\theta-1}} \text{ (raise to } \frac{\theta-1}{\theta}\text{)} \end{aligned}$$

The final step is to define an aggregate price index. Let

$$P_t \equiv \int P_t(i) \frac{c_t(i)}{c_t} di$$

be an average of prices in this economy (weighted by relative consumption). From the previous expression we have

$$P_t(i)c_t(i) = \frac{c_t}{\left(\int_0^1 (P_t(j))^{1-\theta} dj\right)^{\frac{\theta}{\theta-1}}} P_t(i)^{1-\theta}$$

so

$$\int P_t(i)c_t(i)di = P_t c_t = \frac{c_t}{\left(\int_0^1 (P_t(j))^{1-\theta} dj\right)^{\frac{\theta}{\theta-1}}} \int P_t(i)^{1-\theta} di$$

which implies

$$P_t \equiv \left(\int P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$$

Plugging this into the last expression in (1) we have

$$c_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} c_t = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} \frac{E_t}{P_t}$$

b) armed with the allocations in a) we can now rewrite the problem in a more familiar form, using the fact that $\int P_t(i)c_t(i)di = P_t c_t$:

$$\max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - n_t\right]$$

subject to:

$$P_t c_t \leq W_t n_t + \Pi_t$$

The foc says:

$$\frac{W_t}{P_t} = c_t$$

Firms

The technology for producing output is $y_t(i) = n_t(i)$. [CRS does not generate unbounded profits now. Why?]

Consider now the firm's problem in the absence of any frictions on price adjustment. We have the firm choosing the price to charge (alternatively how much to sell) to maximize static profits:

$$\max P_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} c_t - W_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta} c_t$$

where we have made use of the demand function derived above as well as the assumption of linear technology. Clearly, the Focs is

$$(1-\theta) P_t(i)^{-\theta} + \theta W_t P_t(i)^{-\theta} = 0 \tag{2}$$

$$P_t(i) = \frac{\theta}{\theta-1} W_t$$

The firm charges a markup $\frac{\theta}{\theta-1}$ over its marginal cost, with the markup decreasing (1 in the limit) as θ increases and thus goods become more substitutable. Clearly this allocation is inefficient as the consumer will pay more for the good than the marginal cost to the society of producing it.

Equilibrium

I leave the definition of an equilibrium for you as an exercise. Notice that in equilibrium $W_t = P_t c_t = M_t$. So wages are proportional to the money stock in this economy.

Quadratic adjustment costs

Consider next what happens if we assume the firm has to pay a quadratic adjustment cost, $\gamma\left(P_t(i) - P_{t-1}(i)\right)^2$ for changing its price. The problem it solves is

$$\max_{P_t(i)} E_t \sum_t Q_t P_t^{\theta} c_t \left[P_t(i)^{1-\theta} - W_t P_t^{-\theta} - \gamma\left(P_t(i) - P_{t-1}(i)\right)^2\right]$$

where $Q_t = \beta^t \frac{u'(c_t)/P_t}{u'(c_0)/P_0}$ is the relative price of an Arrow-Debreu security that pays 1 unit of currency at t (in terms of date 0 currency). Let $q_t = Q_t P_t^{\theta} c_t$. and the foc is

$$q_t \left[(1-\theta) P_t(i)^{-\theta} + \theta W_t P_t(i)^{-\theta-1} - \gamma \left(P_t(i) - P_{t-1}(i) \right) \right] + q_{t+1} \gamma \left(P_{t+1}(i) - P_t(i) \right) = 0$$

This simplifies to

$$\begin{aligned} P_t(i) - P_{t-1}(i) &= E_t \frac{q_{t+1}}{q_t} \left[P_{t+1}(i) - P_t(i) \right] + \frac{\phi_t(i)}{\gamma} \left[P_t(i) - \frac{\theta}{\theta-1} W_t \right] \\ \phi_t(i) &= (1-\theta) P_t(i)^{-\theta-1} \end{aligned}$$

This is a second-order difference equation that says that the change in firm $i's$ price at t is a linear combination of its optimal rate of inflation next period and the deviation of its price from the flexible-optimum. This is the so-called Partial Adjustment Equation: the firm adjusts partially its price to equalize its price to the the flexible-optimum.

Aggregation is trivial in this economy because all firms are identical: $P_t = \left(\int P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} = P_t(i)$. So we can write

$$\pi_t = \beta E_t \frac{u'(c_{t+1}) P_{t+1}^{\theta} c_t}{u'(c_t) P_t^{\theta} c_t} \pi_{t+1} + \frac{(1-\theta) P_t^{-\theta-1}}{\gamma} \left[P_t - \frac{\theta}{\theta-1} W_t \right]$$

which is a second-order difference equation in P_t. Recall $W_t = M_t$ and $c_t = \frac{M_t}{P_t}$ so we can write

$$P_t^{\theta} \pi_t = \beta E_t (1+\pi_{t+1})^{\theta} \pi_{t+1} + \frac{(1-\theta)}{\gamma} \left[1 - \frac{\theta}{\theta-1} \frac{M_t}{P_t} \right]$$

In this economy unexpected shocks to M_t lead to increases in c_t because P_t does not immediately adjust.

Fixed adjustment costs

Assume next that changing the price from one period to another entails payment of a fixed cost κ, expressed in terms of units of labor.

To write the firm's problem recursively, first notice that a firm's current-period profits (valued at the Arrow-Debreu prices and normalizing $U'(c_0)/P_0$ to 1.

$$\Pi = \frac{U'(c)}{P} \left[p \left(\frac{p}{P} \right)^{-\theta} - W \left(\frac{p}{P} \right)^{-\theta} - \kappa \mathbf{1}_{\{p \neq p_{-1}\}} W \right]$$

where lower case denotes an individual firm's price and upper case denotes the aggregate price level and the last term is the menu cost. Let's impose the equilibrium condition $Pc = M = W$ to simplify things:

$$\Pi = \frac{p}{M}\left(\frac{p}{P}\right)^{-\theta} - \frac{W}{M}\left(\frac{p}{P}\right)^{-\theta} - \kappa\frac{W}{M}\mathbf{1}_{\{p\neq p_{-1}\}}$$

It is helpful to write the problem by expressing all nominal variables relative to the money supply in order to keep the problem bounded. So let a hat denote a normalized variable:

$$\Pi = \hat{p}\left(\frac{\hat{p}}{\hat{P}}\right)^{-\theta} - \left(\frac{\hat{p}}{\hat{P}}\right)^{-\theta} - \kappa\ \mathbf{1}_{\{\hat{p}\neq \hat{p}_{-1}\}}$$

As in previous economies we have studied, the distribution over firms of $\hat{p}_{-1}$ is part of the definition of a recursive equilibrium. Each firm's price depends on $\hat{p}_{-1}$: if that price is close to the firm's current optimum, $\frac{\theta}{\theta-1}\hat{W} = \frac{\theta}{\theta-1}$, there is no need to pay the menu cost and adjust, and $\hat{p} = \hat{p}_{-1}$; otherwise the firm will reset its price. Let μ denote this distribution. We can write the firm's problem recursively as:

$$V\left(\hat{p}_{-1};\mu\right) = \max_{p}\hat{p}\left(\frac{\hat{p}}{\hat{P}}\right)^{-\theta} - \left(\frac{\hat{p}}{\hat{P}}\right)^{-\theta} - \kappa\ \mathbf{1}_{\{\hat{p}\neq \hat{p}_{-1}\}} + \beta EV\left(\frac{\hat{p}}{\exp\left(g\right)};\mu'\right)$$

Notice that the current price $\hat{p}$ becomes $\frac{\hat{p}}{\exp(g)}$ next period in this normalized space: an increase in M will erode the firm's price relative to the wage and render it suboptimal.

In general, this problem is difficult to characterize, unless you work in continuous time (see the recent book of Nancy Stokey "The Economics of Inaction: Stochastic Control Models and Fixed Costs", Princeton University Press, 2009). But the solution is typically characterized by a triplet (s, x, S) such that if $\hat{p}_{-1} \in [s, S]$, it leaves its price unchanged, and if $\hat{p}_{-1} \notin [s, S]$, it adjusts it to $\hat{p} = x$. The return point, x, is, in general, not equal to $\frac{\theta}{\theta-1}$ (unless the profit function is quadratic and the mean growth rate of money equal to 0). If mean g (growth rate of money) is high the firm will choose x above $\frac{\theta}{\theta-1}$ and let inflation gradually erode it.

We next consider two aggregation theorems and show the dynamic response of this economy to a money change crucially depend on the distribution of prices across firms.

Caplin-Spulber

Proposition: Suppose that for some S

1) the initial distribution of (log) relative prices $\hat{p}_i(0)$ is uniform on $[-S, S]$

2) the triplet that characterizes firm's optimal policy is $(-S, S, S)$

3) ε_t is bounded below by g (so that the money growth is non-negative)

Then $M/P = c$ is constant over time (money is neutral).

Proof: For any change ΔM, the fraction of adjusting firms is $\frac{\Delta M}{2S}$ and each adjusts its price by $2S$. Therefore the aggregate price level changes by $\Delta P = \frac{\Delta M}{2S} \times 2S = \Delta M$, thus proportionally to the money stock.

Caplin-Leahy

Suppose. Suppose for some S,

1) the initial distribution of (log) relative prices $\hat{p}_i(0)$ is uniform on $\left[-\frac{S}{2}, \frac{S}{2}\right]$

2) the triplet that characterizes firm's optimal policy is $(-S, 0, S)$

3) $g = 0$ (no trend money growth)

Then an increase in ΔM either results in no change in the price level $\Delta P = 0$ and $\Delta c = \Delta M$ or $\Delta P = \Delta M$, depending on the distribution of $\hat{p}_{it}(0)$ in the period of the shock.